This first set (set 0) is designed to acquaint you with using WeBWorK. Your score on this set will not be counted toward your final grade.

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

1. (1 pt) set0/prob1.pg

This problem demonstrates how you enter numerical answers into WeBWorK.

Evaluate the expression \(3(-10)(10 - 6 - 2(9))\):

In the case above you need to enter a number, since we’re testing whether you can multiply out these numbers. (You can use a calculator if you want.)

For most problems, you will be able to get WeBWorK to do some of the work for you. For example:

Calculate \((-10) \times (10)\):

The asterisk is what most computers use to denote multiplication and you can use this with WeBWorK. But WeBWorK will also allow you to use a space to denote multiplication. You can either \(-10 \times 10\) or \(-100\) or even \(-10 \times 10\). All will work. Try them.

Now try calculating the sine of 45 degrees (that’s sine of \(\pi/4\) in radians and numerically \(\sin(\pi/4)\) equals \(0.707106781885747\) or, more precisely, \(1/\sqrt{2}\)).

We said you should enter \(\sin(x)\) even though WeBWorK will also accept \(\sin x\) or even \(\sin x\) because you are less likely to make a mistake. Try entering \(\sin(2x)\) without the parentheses and you may be surprised at what you get. Use the Preview button to see what you get. WeBWorK will evaluate functions (such as \(\sin\)) before doing anything else, so \(\sin 2x\) means first apply \(\sin\) which gives \(\sin(2)\) and then multiply by \(x\). Try it.

2. (1 pt) set0/prob1a.pg

This problem demonstrates how you enter function answers into WeBWorK.

First enter the function \(\sin x\). When entering the function, you should enter \(\sin(x)\), but WeBWorK will also accept \(\sin x\) or even \(\sin x\). If you remember your trig identities, \(\sin(x) = -\cos(x + \pi/2)\) and WeBWorK will accept this or any other function equal to \(\sin(x)\), e.g. \(\sin(x) + \sin(x)*2 + \cos(x)*2\).

We said you should enter \(\sin(x)\) even though WeBWorK will also accept \(\sin x\) or even \(\sin x\) because you are less likely to make a mistake. Try entering \(\sin(2x)\) without the parentheses and you may be surprised at what you get. Use the Preview button to see what you get. WeBWorK will evaluate functions (such as \(\sin\)) before doing anything else, so \(\sin 2x\) means first apply \(\sin\) which gives \(\sin(2)\) and then multiply by \(x\). Try it.

Now enter the function \(2 \cos t\). Note this is a function of \(t\) and not \(x\). Try entering \(2 \cos x\) and see what happens.

3. (1 pt) set0/prob1b.pg

This problem will help you learn the rules of precedence, i.e. the order in which mathematical operations are performed. You can use parentheses (and also square brackets \([\ ]\) and/or curly braces \(\{\}\)) if you want to change the normal way operations work.

So first let us review the normal way operations are performed.

The rules are simple. Exponentiation is always done before multiplication and division and multiplication and division are always done before addition and subtraction. (Mathematically we say exponentiation takes precedence over multiplication and division, etc.). For example what is \(1+2*3\)?

and what is \(2 \cdot 3^2\)?

Now sometimes you want to force things to be done in a different way. This is what parentheses are used for. The rule is: whatever is enclosed in parentheses is done before anything else (and things in the inner most parentheses are done first).

For example how do you enter

\[
\frac{1 + \sin(3)}{2 + \tan(4)}
\]

Hint: this is a good place to use \(\{\}\)’s and also to use the “Preview” button.

Here are some more examples:

\[
(1+3)^9 = 36, (2*3)^2 = 6^2 = 36, 3^{2*2} = 3^4 = 81, (2+3)^{2*2} = 5^2 = 25, 3^{2*2} = 3^4 = 81
\]
Think of / as the horizontal line in a fraction. Ask yourself what 1/2/2 should mean. WeBWorK and most other computers read things from left to right, i.e. 2/3*4 means (2/3)*4 or 8/3, it also written as a line with two dots, but unfortunately, this "line with two dots" symbol is not on computer keyboards. Don't think of / as the horizontal line in a fraction. Ask yourself what 1/2/2 should mean.) WeBWorK and most other computers read things from left to right, i.e. 2/3*4 means (2/3)*4 or 8/3, IT DOES NOT MEAN 2/12. Some computers may do operations from right to left. If you want 2/(3*4) = 2/12, you have to use parentheses. The same thing happens with addition and subtraction. 1-3+2 = 0 but 1-(3+2) = -4. This is one case where using parentheses even if they are not needed might be a good idea, e.g. write (2/3)*4 even though you could write 2/3*4. This is also a case where previewing your answer can save you a lot of grief since you will be able to see what you entered. Enter 2/3*4 and use the Preview button to see what you get.

4. (1 pt) set0/prob2.pg
This problem demonstrates a WeBWorK True/False question.

Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false. You must get all of the answers correct to receive credit.

_1. _-1 < -6
_2. _\pi \geq 3.1416
_3. _10 -1 \leq 10
_4. _-3 < -3

Notice that if one of your answers is wrong then, in this problem, WeBWorK will tell you which parts are wrong and which parts are right. This is the behavior for most problems, but for true/false or multiple choice questions WeBWorK will usually only tell you whether or not all the answers are correct. It won’t tell you which ones are wrong. The idea is to encourage you to think rather than to just try guessing.

In every case all of the answers must be correct before you get credit for the problem.

5. (1 pt) set0/prob3.pg
This problem demonstrates a WeBWorK Matching question.

Match the statements defined below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

_1. _x \text{ is greater than or equal to 8}
_2. _x \text{ is greater than 8}
_3. _x \text{ is less than or equal to 8}
_4. _The distance from x to 8 is more than 1

6. (1 pt) set0/prob4/prob4.pg
This problem demonstrates a WeBWorK problem involving graphics.

The simplest functions are the linear (or affine) functions — the functions whose graphs are a straight line. They are important because many functions (the so-called differentiable functions) “locally” look like straight lines. (“locally” means that if we zoom in and look at the function at very powerful magnification it will look like a straight line.)

Enter the letter of the graph of the function which corresponds to each statement.

_1. _The graph of the line is increasing
_2. _The graph of the line is decreasing
_3. _The graph of the line is constant
_4. _The graph of the line is not the graph of a function

This is another problem where you aren’t told if some of your answers are right. (With matching questions and true false questions, this is the standard behavior – otherwise it is too easy to guess your way to the answer without learning anything.)

If you are having a hard time seeing the picture clearly, click on the picture. It will expand to a larger picture on its own page so that you can inspect it more closely.

Some problems display a link to a web page where you can get additional information or a hint:

7. (1 pt) set0/prob5.pg
This problem demonstrates a WeBWorK question that requires you to enter a number or a fraction.
Evaluate the expression \( \frac{\frac{148}{265}}{12} \). Give your answer in decimal notation correct to three decimal places or give your answer as a fraction.

Now that you have finished you can use the "Prob. List" button at the top of the page to return to the problem list page. You’ll see that the problems you have done have been labeled as correct or incorrect, so you can go back and do problems you skipped or couldn’t get right the first time. Once you have done a problem correctly it is ALWAYS listed as correct even if you go back and do it incorrectly later. This means you can use WeBWorK to review course material without any danger of changing your grade.
Complete the sentence:
________ world!

You can view the source for this problem.

2. (1 pt) setMAAtutorial/standardexample.pg

**Standard Example**

Complete the sentence:

________ world;
Enter the sum of these two numbers:
3 + 5 =
Enter the derivative of 

\[ f(x) = x^5 \]

\[ f'(x) = \]

You can view the source for this problem.

3. (1 pt) setMAAtutorial/simplemultiplechoiceexample.pg

**Multiple choice example**

To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 543

What is the derivative of \( \tan(x) \)?

A. \(-\cot(x)\)
B. \(\sec^2(x)\)
C. \(\tan(x)\)
D. \(\cosh(x)\)
E. \(\sin(x)\)

Enter the letter corresponding to the correct answer:

You can view the source for this problem.

4. (1 pt) setMAAtutorial/multiplechoiceexample.pg

**Multiple choice example**

To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 2646

What is the derivative of \( \tan(x) \)?

- A. \(\cos^3(x)\)
- B. \(\tan(x)\)
- C. \(\sech(x)\)
- D. \(-\cot(x)\)
- E. \(\sec^2(x)\)
- F. \(\cosh(x)\)
- G. \(\sin(x)\)

You can view the source for this problem.

5. (1 pt) setMAAtutorial/matchinglistexample.pg

**Matching list example**

To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 189

Place the letter of the derivative next to each function listed below:

1. \(\sin(x)\)
2. \(x^{20}\)
3. \(\cos(x)\)
4. \(\tan(x)\)

A. \(\cos(x)\)
B. \(-\sin(x)\)
C. \(20x^{19}\)
D. \(\sec^2(x)\)

Let's print the questions again, but insist that the first two questions (about sin and cos) always be included. Here is a second way to format this question, using tables:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\sin(x))</td>
<td>A. (-\sin(x))</td>
</tr>
<tr>
<td>2. (x^{20})</td>
<td>B. (\cos(x))</td>
</tr>
<tr>
<td>3. (\cos(x))</td>
<td>C. (20x^{19})</td>
</tr>
</tbody>
</table>

And below is yet another way to enter a table of questions and answers:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\sin(x))</td>
<td>A. (-\sin(x))</td>
</tr>
<tr>
<td>2. (x^{20})</td>
<td>B. (\cos(x))</td>
</tr>
<tr>
<td>3. (\cos(x))</td>
<td>C. (20x^{19})</td>
</tr>
</tbody>
</table>

D. The derivative is not provided

You can view the source for this problem.

6. (1 pt) setMAAtutorial/truefalseexample.pg

**True False Example**

To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 2509

Enter T or F depending on whether the statement is true or false. (You must enter T or F – True and False will not work.)

1. All differentiable strictly increasing functions have non-negative derivatives at every point
2. All differentiable functions are continuous.
3. All closed sets are compact
4. All polynomials are differentiable.

You can view the source for this problem.

7. (1 pt) setMAAtutorial/popuplistexample.pg

True False Pop-up Example
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 4961

Indicate whether each statement is true or false.
1. All functions with positive derivatives are increasing.
2. All increasing functions have positive derivatives
3. All differentiable strictly increasing functions have non-negative derivatives at every point
4. All polynomials are differentiable.

You can view the source for this problem.

8. (1 pt) setMAAtutorial/ontheflygraphicsexample1.pg

On-the-fly Graphics Example 1
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 4087

WARNING: Use SHIFT reload when refreshing to make sure that the image is refreshed!!!!!!

Identify the graphs A (blue), B (red) and C (green) as the graphs of a function and its derivatives (click on the graph to see an enlarged image):

is the graph of the function
is the graph of the function’s first derivative
is the graph of the function’s second derivative

You can view the source for this problem.

9. (1 pt) setMAAtutorial/ontheflygraphicsexample2.pg

On-the-fly Graphics Example 2
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 11

WARNING: Use SHIFT reload when refreshing to make sure that the image is refreshed!!!!!!

Identify the graphs A (blue), B (red) and C (green) as the graphs of a function and its derivatives (click on the graph to see an enlarged image):

is the graph of the function
is the graph of the function’s first derivative
is the graph of the function’s second derivative

You can view the source for this problem.

10. (1 pt) setMAAtutorial/staticgraphicsexample/staticgraphicsexample.pg

Static graphics Example
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.
This is a graph of the function $F(x)$: (Click on image for a larger view)

Enter the letter of the graph below which corresponds to the transformation of the function.

1. $SF(x)$
2. $-F(-x)$
3. $F(x^2)$
4. $F(-x)$

You can view the source for this problem.

11. (1 pt) setMAAtutorial/hermitegraphexample.pg

Hermite polynomial graph example
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 1308

We are developing other ways to specify graphs which are to be created 'on the fly'. All of these new methods consist of adding macro packages to WeBWorK. Since they do not require the core of WeBWorK to be changed, these enhancements can be added by anyone using WeBWorK.

These two piecewise linear graphs were created by specifying the points at the nodes.
Click on the graph to view a larger image.

If the black function is written as $f(x)$, then the orange function would be written as $f(\_\_\_\_\_)$.
List the internal local minimum points in increasing order:

You can view the source for this problem.

12. (1 pt) setMAAtutorial/htmllinksexample/htmllinksexample.pg

**HTML links example**

This example shows how to link to resources outside the problem itself.

Linking to other web pages over the internet is easy. For example, you can get more information about the Buffon needle problem and how it is used by ants to find new nest sites by linking to Ivars Peterson’s column on the MAA site.

All of the files in the html directory of your WeBWorK course site can be read by anyone with a web browser and the URL (the address of the file). This is a good place to put files that are referenced by more than one problem in your WeBWorK course.

Here is the link to the to the calculator page stored in the top level of the html directory of the tutorialCourse.

Finally there are files, such as picture files, which are stored with the problem itself in the same directory.

And the table below has three more graphs which are stored in the directory containing the current problem.

You can view the source for this problem.

13. (1 pt) setMAAtutorial/javascriptexample1.pg

**JavaScript Example 1**

To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 1809
13. (1 pt) setMAAtutorial/javascriptexample1.pg
Find the derivative of the function f(x). The windows below will tell you the value of f for any input x. (I call this an “oracle function”, since if you ask, it will tell.)
\[ f'(2) = \] 
You may want to use a calculator to find the result. You can also enter numerical expressions and have WeBWorK do the calculations for you.
The java Script calculator was displayed here
You can view the source for this problem.

14. (1 pt) setMAAtutorial/javascriptexample2.pg
JavaScript Example 2
To see a different version of the problem change the problem seed and press the ’Submit Answer’ button below.
Problem Seed: Change the problem seed to change the problem:3659

15. (1 pt) setMAAtutorial/vectoreldexample.pg
To see a different version of the problem change the problem seed and press the ’Submit Answer’ button below.
Problem Seed: Change the problem seed to change the problem:1661

Match the following equations with their direction field. Clicking on each picture will give you an enlarged view. While you can probably solve this problem by guessing, it is useful to try to predict characteristics of the direction field and then match them to the picture.

Here are some handy characteristics to start with – you will develop more as you practice.
A. Set y equal to zero and look at how the derivative behaves along the x axis.
B. Do the same for the y axis by setting x equal to 0
C. Consider the curve in the plane defined by setting y’=0 – this should correspond to the points in the picture where the slope is zero.
D. Setting y’ equal to a constant other than zero gives the curve of points where the slope is that constant. These are called isoclines, and can be used to construct the direction field picture by hand.

1. \( y' = e^{-x} + 2y \)
2. \( y' = -2 + x - y \)
3. \( y' = y + 2 \)
4. \( y' = 2 \sin(x) + 1 + y \)
You can view the source for this problem.

16. (1 pt) setMAAtutorial/conditionalquestionexample.pg

Conditional questions example
If \( f(x) = 6x + 25 \), find \( f'(-5) \).

17. (1 pt) setMAAtutorial/javaappletexample.pg

Java applet example
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.
Problem Seed: Change the problem seed to change the problem: 3083
WARNING: Use SHIFT reload when refreshing to make sure that the image is refreshed!!!!!

This problem illustrates how you can embed Java applet code in a WeBWorK example to create an interactive homework problem that could never be provided by a text book.

WeBWorK can use existing JavaScript and Java code to augment its capabilities.

The java applet was displayed here
The graph above represents the function
\[ f(x) = x^2 + ax + b \]
where \( a \) and \( b \) are parameters.
For each value of \( a \) find the value of \( b \) which makes the graph just touch the x-axis.
if \( a = 1.5 \) then __________
if \( a = 1 \) then __________
if \( a = 0.5 \) then __________

Does this relationship between \( a \) and \( b \) specify \( b \) as a function of \( a \)? ___ (Yes or No)
Does this relationship between \( a \) and \( b \) specify \( a \) as a function of \( b \)? ___ (Yes or No)
Write a formula for calculating this value of \( b \) from \( a \).
\[ b = \]

You can view the source for this problem.
1. (1 pt) setSampleAnswers/sample_num_ans.pg
This problem demonstrates various WeBWorK procedures for dealing with numerical answers. In particular, by entering syntactically incorrect answers, you can see the error messages generated by WeBWorK. Note that exponentiation can be denoted by ** or ^ . See Answer Evaluators for documentation on these procedures.

Enter the number 52.1. You are only allowed to enter a number (e.g. 52.1, 521E1, 521E-1, etc.)
This uses strict_num_cmp

Enter the number 52.1 again. This time you can enter a number or fraction (e.g. 52.1, 521/10, 5.21/1 etc.)
This uses frac_num_cmp

Enter the number 52.1 a third time. This time you can enter any arithmetic expression equaling 52.1 (e.g. 52.1, 100/2+3-.9, (5*10**2+21)/10 etc.)
This uses arith_num_cmp

Finally enter the number 52.1 a fourth time. Now you can enter any expression involving elementary functions which equals 52.1 (e.g. 52.1, 50.1+ln(e**2), tan(pi/4) + ln(exp(2)) + cosh(0) - 1.9 + arcsin(0) + 5*sqrt(10**2), etc.). See Available Functions for details on entering expressions involving elementary functions.
This uses std_num_cmp

You can view the source for this problem.

2. (1 pt) setSampleAnswers/sample_str_ans.pg
This problem demonstrates various WeBWorK procedures for dealing with string answers. See Answer Evaluators for documentation on these procedures.

Enter the string “Hi there.” without the quotes but don’t forget the period. Spaces before the “H” and after the “.” are ignored, but otherwise you must enter the string exactly as given with 2 spaces between the words. Actually, viewing this in html, you will not see the two spaces, but they are really there.
This uses strict_str_cmp

Now enter the string “Hi there.” again. This time all multiple spaces are treated as a single space. E.g. you can enter as many spaces as you want between the “Hi” and the “there.” and your answer will still be accepted as correct.
This uses std_str_cmp

Finally enter the string “Hi there.” a third time. This time case is ignored and all multiple spaces are treated as a single space. E.g. “hi there.” and “hi ThErE.” are valid answers.
This uses std_str_cmp

3. (1 pt) setSampleAnswers/sample_fun_ans.pg
This problem demonstrates various WeBWorK procedures for dealing with answers involving functions. In particular, by entering syntactically incorrect answers, you can see the error messages generated by WeBWorK. Note that exponentiation can be denoted by ** or ^ . See Answer Evaluators for documentation on these procedures. See Available Functions for details on entering expressions involving elementary functions.

Enter the derivative of sin(x) (e.g. cos(x), sin(x+pi/2), cos(x)**2 + cos(x) + sin(x)**2 -1, etc.)
This uses function_cmp

Next enter the derivative of sin(t) on the interval (-pi/2, pi/2). Note that now the variable is t and that sqrt(cos(t)**2) is a valid answer.
This uses function_cmp with parameters specifying the variable and interval

Finally enter the antiderivative of x + sin(x) (e.g. .5*(x**(2)) - cos(x), (1/2)*(x**(2)) - cos(x) + 3, sin(x)**2 + cos(x)**2 + sec(pi/4) - cos(x) + (x/sqrt(2))**2 +ln(e**x) -x, etc.)
This uses function_cmp with parameters specifying the variable and interval

You can view the source for this problem.

4. (1 pt) setSampleAnswers/sample_units_ans.pg
This problem demonstrates how WeBWorK handles numerical answers involving units. WeBWorK can handle all units that are used in elementary physics courses. See answers with units for more details.
Two perpendicular sides of a triangle are 5.05 m and 7.33 m long respectively. What is the length of the third side of the triangle?
You can answer this in terms of m’s, cm’s, km’s, in’s, ft, etc. but you must enter the units.
Check "Show Hint" and then "Submit Answer" if you don’t remember the Pythagorean theorem.

You can view the source for this problem.

5.(1 pt) setSampleAnswers/sample_myown_ans.pg
This problem demonstrates how you can write your own procedure to check answers. The procedure is embedded right in the problem. If you wanted to use it for several problems, you could put it in a file similar to "PGanswermacros.pl" and load it into the problem. See Answer Evaluators for documentation on how to write these procedures.

This problem asks you to enter a palindrome, a word, number, or phrase that is the same when read backwards or forward. For example, madam or Hannah. For us a standard palindrome will ignore spaces and case, but a strict palindrome will not. So e.g. Hannah is a standard but not a strict palindrome. We will write a test for both types. You can see the code for the test by viewing the source for this problem.

Enter a standard palindrome such as ”Hannah”, ”1234321”, or ”Mom”.
This uses std_palindrome_test

Now enter a strict palindrome such as ”1234321”, or ”mom”.
This uses std_palindrome_test
1. (1 pt) setSampleGraders/sample_default_grader.pg
This problem demonstrates WeBWorK procedures for grading multipart problems. This problem will be graded by the default grading method. The default grading method is set in the webworkCourse.ph file (or if not set there, in the Global.pm file). Usually the default method is the average grading method which grants partial credit giving all parts of a multipart problem equal weight.
You can see how this is done by viewing the source for this problem.
   - Enter the number 1.
   - Enter the number 2.
   - Enter the number 3.

2. (1 pt) setSampleGraders/sample_std_grader.pg
This problem demonstrates WeBWorK procedures for grading multipart problems. This problem will be graded by the standard grading method which does not grant partial credit.
You can see how this is done by viewing the source for this problem.
   - Enter the number 1.
   - Enter the number 2.
   - Enter the number 3.

3. (1 pt) setSampleGraders/sample_avg_grader.pg
This problem demonstrates WeBWorK procedures for grading multipart problems. This problem will be graded by the average grading method which grants partial credit giving all parts of a multipart problem equal weight.
You can see how this is done by viewing the source for this problem.
   - Enter the number 1.
   - Enter the number 2.
   - Enter the number 3.

4. (1 pt) setSampleGraders/sample_myown_grader.pg
This problem demonstrates WeBWorK procedures for grading multipart problems. This problem will be graded by an ad hoc grading procedure written in the code of the problem. In this three part question, the procedure gives 50 per cent of the weight to part 1 and 25 per cent to parts 2 and 3. It also outputs a message to the student telling them how the problem will be graded.
You can see how this is done by viewing the source for this problem.
   - Enter the number 1.
   - Enter the number 2.
   - Enter the number 3.
1. (1 pt) setGenericQuestions/CheckboxMult.pg

Check ALL correct answers.

Question

- A. WrongAnswer2
- B. Answer1
- C. Answer2
- D. WrongAnswer1
- E. Answer3

2. (1 pt) setGenericQuestions/Multiple.pg

Check ALL correct answers.

Question

- A. WrongAnswer1
- B. WrongAnswer2
- C. Answer
- D. All of the above
- E. None of the above

3. (1 pt) setGenericQuestions/TF.pg

Enter "Answer" or "Answerprime".

1. Question3
2. Question1

4. (1 pt) setGenericQuestions/Table.pg

Complete the following truth table by filling in the blanks with T or F as appropriate.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>Expr1</th>
<th>Expr2</th>
<th>Expr3</th>
<th>Expr4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. (1 pt) setGenericQuestions/standard.pg

Complete the sentence:

_________ world;
Enter the sum of these two numbers:

3 + 6 = _______

Enter the derivative of

\( f(x) = x^6 \)

\( f'(x) = \) _______

6. (1 pt) setGenericQuestions/simpleinput.pg

For each of the following angles, find the degree measure of the angle with the given radian measure:

\[ \frac{3\pi}{4} \]
\[ \frac{2\pi}{3} \]
\[ \frac{\pi}{4} \]
\[ \frac{3\pi}{2} \]
\[ 4\pi \]
1. (1 pt) setSampleGraphs/c0s2p2/c0s2p2.pg
This is a graph of the function $F(x)$: (Click on image for a larger view)

Enter the letter of the graph below which corresponds to the transformation of the function.

- 1. $5F(x)$
- 2. $F(x + 3)$
- 3. $-F(-x)$
- 4. $F(3x)$

You can view the source for this problem or consult the documentation for more details on the PG language.

2. (1 pt) setSampleGraphs/c0s1p8/c0s1p8.pg
Many of the problems we use in our calculus classes include 'static' graphs. One or more graphs are prepared ahead of time using a graphing program such as Mathematica or Matlab or some other graphing calculator.

The graph below was created using the program Xfunctions developed by David Ecks.

To see a different version of the problem change the problem seed and press the 'Submit Answer' button.

Problem Seed: Change the problem seed to change the problem: 725

Click on the image to obtain a larger version of the graph.

The function to the left represents the velocity of a race car as it travels a linear track. Negative velocities mean the car is backing up. (The letters in each answer must be in alphabetical order with no spaces between the letters.)

- 1. The interval from a to b
- 2. The interval from b to c
- 3. The interval from c to d
- 4. The interval from d to e
- 5. The interval from e to f

A. The car is moving forward on this interval
B. The car is backing up on this interval.
C. The forward velocity of the car is increasing on this interval.
D. The forward velocity of the car is decreasing on this interval.
E. The distance from the starting point is increasing on this interval.
F. The distance from the starting point is decreasing on this interval.

You can view the source for this problem or consult the documentation for more details on the PG language.

3. (1 pt) setSampleGraphs/prob3.pg
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.

Problem Seed: Change the problem seed to change the problem: 2734

This graph was created 'on the fly'. This function was created from the expression

$$\cos(4x) + .1*\exp(x)$$

for $x$ in [-1,4) using color:red and weight:3

Click on the graph to see an enlarged image.
4. (1 pt) setSampleGraphs/prob4.pg  
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.  
Problem Seed: Change the problem seed to change the problem: 3950  
We are developing other ways to specify graphs which are to be created 'on the fly'. All of these new methods consist of adding macro packages to WeBWorK. Since they do not require the core of WeBWorK to be changed, these enhancements can be added by anyone using WeBWorK.  
These two piecewise linear graphs were created by specifying the points at the nodes.  
Click on the graph to view a larger image.  

5. (1 pt) setSampleGraphs/prob5.pg  
To see a different version of the problem change the problem seed and press the 'Submit Answer' button below.  
Problem Seed: Change the problem seed to change the problem: 1773  
This problem illustrates how you can embed JavaScript code in a WeBWorK example to create an interactive homework problem that could never be provided by a text book.  
WeBWorK can use existing JavaScript and Java code to augment its capabilities.  

By typing any value x into the left hand window and pressing the −f−> button you can determine the value of f(x).  
Using this 'oracle' function, calculate the derivative of f at x = 2.  

You can use a calculator  

The JavaScript calculator was displayed here  
You can view the source for this problem. or consult the documentation for more details on the PG language.
6. (1 pt) setSampleGraphs/prob6.pg
This problem requires a browser capable of running Java.
To see a different version of the problem change the problem
seed and press the ’Submit Answer’ button below.
Problem Seed: Change the problem seed to change the prob-
lem: 662
This problem illustrates how you can Java applets in a WeB-
WorK example. This Java applet and data were created using
Geometer’s Sketchpad. WeBWorK can use existing JavaScript
and Java code to augment its capabilities.

First view the figure in the java applet. (This may take a
minute to load.)
The java applet figure illustrates a triangle and its median. You
can drag the corners of the triangle to change its shape.

Calculate the ratio of the areas of the two colored triangles.
The ratio of the areas = ___
Will this ratio be different for different triangles?

We are working on allowing java applets to be included di-
rectly in WeBWorK problems without using an external link.
This merely requires that we be able to specify the directory in
which the java applet resides; for security reasons, it can’t reside
in the same directory as the problem.

You can view the source for this problem. or consult the
documentation for more details on the PG language.

7. (1 pt) setSampleGraphs/prob8.pg
Identify the graphs A (blue), B (red) and C (green) as the
graphs of a function and its derivatives (click on the graph to
see an enlarged image):
___ is the graph of the function
___ is the graph of the function’s first derivative
___ is the graph of the function’s second derivative
You can view the source for this problem. or consult the
documentation for more details on the PG language.
1. (0 pts) setSampleQuestionnaires/WeBWorK,evaluation/WeBWorK,questionnaire_fall02.pg
Please answer the questionnaire on-line. Thank you.

2. (0 pts) setSampleQuestionnaires/TA,evaluation/TA,evaluation_fall02.pg
Please answer the questionnaire on-line. Thank you.
1. Consider the two points (3, -1) and (10, 6). The distance between them is:________
   The x co-ordinate of the midpoint of the line segment that joins them is:________
   The y co-ordinate of the midpoint of the line segment that joins them is:________

2. Consider the two points (1, -4) and (9, 11). The distance between them is:________
   The x co-ordinate of the midpoint of the line segment that joins them is:________
   The y co-ordinate of the midpoint of the line segment that joins them is:________

3. Consider the two points (4, -1) and (-3, -9). The distance between them is:________
   The x co-ordinate of the midpoint of the line segment that joins them is:________
   The y co-ordinate of the midpoint of the line segment that joins them is:________

4. Consider the two points (5, -4) and (-6, -8). The distance between them is:________
   The x co-ordinate of the midpoint of the line segment that joins them is:________
   The y co-ordinate of the midpoint of the line segment that joins them is:________

5. Find the distance between (9, 7) and (-1, -3).

6. Find the perimeter of the triangle with the vertices at (3, 1), (-3, 6), and (-5, -3).

7. Find the perimeter of the triangle with the vertices at (2, -1), (-4, 6), and (-6, -6).

8. Find the point (0, b) on the y-axis that is equidistant from the points (4, 4) and (6, -5).
   b =________

9. Find the distance between the two points, (7, 6) and (6, -6).
   d =________

10. Find the distance between the two points, (-5, -7) and (5, 1).
    d =________

11. Find the distance between the two points, (-4, 8) and (4, 1).
    d =________

12. Find the distance between the two points, (1, -6) and (-8, -3).
    d =________

13. Plot the points A = (0, 1), B = (3, 5), and C = (-8, 7). Notice that these points are vertices of a right triangle (the angle A is 90 degrees).
    Find the distance between A and B:________
    Find the distance between A and C:________
    Find the area of the triangle ABC:________
The equation of the line with slope $-7$ that goes through the point $(8, -1)$ has a $y$-intercept at ___.

An equation of a line through $(-5, 6)$ which is parallel to the line $y = 4x + 1$ has slope: ___ and $y$-intercept at: ___.

The equation of the line with slope 2 that goes through the point $(10, 7)$ can be written in the form $y = mx + b$ where $m$ is: ___ and where $b$ is: ______

The equation of the line with slope 5 that goes through the point $(3, 5)$ and is parallel to the line 5$x + 4y = 3$ can be written in the form $y = mx + b$ where $m$ is: ___ and where $b$ is: ______

The line whose equation is $4x - 7y = -10$ goes through the point $(-8, t)$ for $t = ___$

The line through $(8, 9)$ and $(9, -9)$ also goes through the point $(t, -3)$ for $t = ___$
1. (1 pt) setGeometry3Conics/ur_geo_3_1.pg

Match each graph to its equation.
(For all graphs on this page, if you are having a hard time seeing the picture clearly, click on it. It will expand to a larger picture on its own page so that you can inspect it more closely.)
A. \( x^2 = -2y \)
B. \( y^2 = 2x \)
C. \( x^2 = 2y \)
D. \((x - 1)^2 = -2(y - 1)\)
E. \((x - 1)^2 = 2(y + 1)\)
F. \( y^2 = -2x \)

Find an equation of the parabola that has a focus at \((4, 10)\) and a vertex at \((4, 4)\):

\[ y = \ldots \]

Find an equation of its directrix:

\[ y = \ldots \]

Find the vertex, focus, and directrix for the following functions.

(a) \((y - 6)^2 = 12(x - 2)\)
   vertex: \((\ldots, \ldots)\)
   focus: \((\ldots, \ldots)\)
   directrix \(x = \ldots\)

(b) \(y^2 - 4y = 20x - 2^2\)
   vertex: \((\ldots, \ldots)\)
   focus: \((\ldots, \ldots)\)
   directrix \(x = \ldots\)

(c) \((x - 6)^2 = 20(y - 3)\)
   vertex: \((\ldots, \ldots)\)
   focus: \((\ldots, \ldots)\)
   directrix \(y = \ldots\)

(d) \(x^2 + 24x = 4y - 24\)
   vertex: \((\ldots, \ldots)\)
   focus: \((\ldots, \ldots)\)
   directrix \(y = \ldots\)

Write equations for each parabola (If you have a hard time seeing the picture clearly, click on the picture so that you can inspect it more closely.)

Match each graph to its equation.
(For all graphs on this page, if you are having a hard time seeing the picture clearly, click on it. It will expand to a larger picture on its own page so that you can inspect it more closely.)
A. $\frac{x^2}{16} + \frac{y^2}{4} = 1$
B. $\frac{x^2}{4} + \frac{y^2}{16} = 1$
C. $\frac{(x+1)^2}{4} + (y-1)^2 = 1$
D. $\frac{x^2}{4} + y^2 = 1$
E. $x^2 + \frac{y^2}{4} = 1$
F. $x^2 + \frac{(y-1)^2}{4} = 1$
Find the center, vertices, and foci of each ellipse.

(a) \( \frac{x^2}{36} + \frac{y^2}{4} = 1 \)

Center: (____, ____)
Right vertex: (____, ____)
Left vertex: (____, ____)
Top vertex: (____, ____)
Bottom vertex: (____, ____)
Right focus: (____, ____)
Left focus: (____, ____)

(b) \( \frac{(x + 7)^2}{4} + \frac{(y - 6)^2}{49} = 1 \)

Center: (____, ____)
Right vertex: (____, ____)
Left vertex: (____, ____)
Top vertex: (____, ____)
Bottom vertex: (____, ____)
Top focus: (____, ____)
Bottom focus: (____, ____)

(c) \( 9x^2 + 16y^2 - 36x - 224y + 676 = 0 \)

Center: (____, ____)
Right vertex: (____, ____)
Left vertex: (____, ____)
Top vertex: (____, ____)
Bottom vertex: (____, ____)
Right focus: (____, ____)
Left focus: (____, ____)

The equation of the ellipse that has a center at (4, 9), a focus at (7, 9), and a vertex at (9, 9), is

\[ \frac{(x - C)^2}{A^2} + \frac{(y - D)^2}{B^2} = 1 \]

where
\( A = \) ____
\( B = \) ____
\( C = \) ____
\( D = \) ____

Match each graph to its equation.

(a)

\[ \frac{(y - A)^2}{B^2} + \frac{(x - C)^2}{D^2} = 1 \]

where
\( A = \) ____
\( B = \) ____
\( C = \) ____
\( D = \) ____

(b)

\[ \frac{(y - A)^2}{B^2} + \frac{(x - C)^2}{D^2} = 1 \]

where
\( A = \) ____
\( B = \) ____
\( C = \) ____
\( D = \) ____

Match each graph to its equation.

(a)
The equation of the hyperbola that has a center at (4, 10), a focus
at (9, 10), and a vertex at (7, 10), is

$$\frac{(x - C)^2}{A^2} - \frac{(y - D)^2}{B^2} = 1$$

where

\[
A = \\
B = \\
C = \\
D = 
\]

11. (1 pt) setGeometry3Conics/ur_geo_3_11.png
Write equations for each hyperbola (If you have a hard time seeing the picture clearly, click on the picture so that you can inspect it more closely.)

(a) \[
\frac{(y - A)^2}{B^2} - \frac{(x - C)^2}{D^2} = 1
\]

where \[A = \] \\
where \[B = \] \\
where \[C = \] \\
where \[D = \]

(b) \[
\frac{(x - A)^2}{B^2} - \frac{(y - C)^2}{D^2} = 1
\]

where \[A = \] \\
where \[B = \] \\
where \[C = \] \\
where \[D = \]

12. (1 pt) setGeometry3Conics/ur_geo_3_12.png
Solve the system by graphing each equation and finding the point of intersection.

\[
\begin{align*}
y &= \frac{12}{x+4} + 3 \\
y - 14 &= (x + 6)^2
\end{align*}
\]

\[x = \] \\
\[y = \]

13. (1 pt) setGeometry3Conics/ur_geo_3_13.png
The parabola given by the equation \(x = y^2 + 8y + 15\) has its vertex at \((h, k)\) for:
\[h = \] \\
and \\
\[k = \]

14. (1 pt) setGeometry3Conics/ur_geo_3_14.png
The parabola given by the equation \(y = -x^2 + 10x - 3\) has its vertex at \((h, k)\) for:
\[h = \] \\
and \\
\[k = \]

15. (1 pt) setGeometry3Conics/ur_geo_3_15.png
The parabola given by the equation \(x = -2y^2 + 28y - 110\) has its vertex at \((h, k)\) for:
\[h = \] \\
and \\
\[k = \]

16. (1 pt) setGeometry3Conics/ur_geo_3_16.png
The parabola given by the equation \(y = 3x^2 - 24x + 16\) has its vertex at \((h, k)\) for:
Match each equation for a parabola with the direction that the parabola opens.

IMPORTANT!! You only have 4 attempts to get this problem right!

1. \( y = \frac{1}{3}(x - 9)^2 + 5 \)
2. \( x = -\frac{1}{5}(y - 9)^2 + 5 \)
3. \( x = \frac{1}{7}(y - 9)^2 + 5 \)
4. \( y = -\frac{1}{5}(x - 9)^2 + 5 \)

A. right
B. left
C. up
D. down

Match each equation for a parabola with the direction that the parabola opens.

IMPORTANT!! You only have 4 attempts to get this problem right!

1. \( y = -\frac{1}{3}(x + 3)^2 + 2 \)
2. \( x = \frac{1}{7}(y + 3)^2 + 2 \)
3. \( x = -\frac{1}{9}(y + 3)^2 + 2 \)
4. \( y = \frac{1}{7}(x + 3)^2 + 2 \)
A. up
B. left
C. right
D. down

Match each equation for a parabola with the direction that the parabola opens.

IMPORTANT!! You only have 4 attempts to get this problem right!

1. \( y = 3(x - 4)^2 - 9 \)
2. \( x = -3(y - 4)^2 - 9 \)
3. \( y = -3(x - 4)^2 - 9 \)

A. up
B. left
C. right
D. down
1. (1 pt) setGeometry4SubsetsOfR2/ur_geo_4_1.pg
For each graph, determine its system of linear inequalities.

1.

• A. \[
\begin{align*}
  y & \leq -x - 2 \\
  y & \leq 2
\end{align*}
\]
• B. \[
\begin{align*}
  x & \leq y - 2 \\
  y & \geq 2
\end{align*}
\]
• C. \[
\begin{align*}
  y & \leq x + 2 \\
  y & \geq 2
\end{align*}
\]
• D. \[
\begin{align*}
  y & \leq x \\
  y & \leq 2
\end{align*}
\]
• E. \[
\begin{align*}
  y & \leq x + 2 \\
  x & \geq 2
\end{align*}
\]
• F. None of the above

2. (1 pt) setGeometry4SubsetsOfR2/ur_geo_4_2.pg
Graph the system:
\[
\begin{align*}
  x & \geq 0 \\
  y & \geq 0 \\
  y & \leq 5 \\
  5x + 3y & \leq 40
\end{align*}
\]
List the vertices of the region clockwise starting with (0,0):
(0,0); (___, ___);
(____, ___);
(____, ___);
(____, ___);
F. None of the above

3. (1 pt) setGeometry4SubsetsOfR2/ur_geo_4_3.pg
Graph the system:
\[
\begin{align*}
  x & \leq 5 \\
  y & \leq 1 \\
  x & \geq -7 \\
  4x - 6y & \leq 26 \\
  5x + 6y & \geq -35
\end{align*}
\]
List the vertices of the region clockwise starting with the vertex whose y-coordinate is the lowest:
(____, ___);
(____, ___);
(____, ___);
(____, ___);
F. None of the above
Choose 3 inequalities that form a system whose graph is the shaded region shown above.

- A. \( x \geq -1 \)
- B. \( y \geq -1 \)
- C. \( y \leq 1 \)
- D. \( 6x - 5y \geq -25 \)
- E. \( 6x + 5y \geq 25 \)
- F. \( 6x + 5y \leq 25 \)
- G. \( 6x - 5y \leq -25 \)
- H. \( y \leq -1 \)

Maximize \( z = 6x + y \) subject to \( x \geq -5, \ y \geq -4, \ 9x - 8y \leq 5, \ 2x + 10y \leq 60 \)

Answer: 

Minimize \( f(x,y) = 2y - 3x + 7 \) subject to \( x \geq -3, \ x \leq 6, \ 7x + 9y \leq 60, \ 3x + 9y \geq -36. \)

Answer: 

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
1. (1 pt) setDiscrete1Logic/ur_dis_1_1.pg

Enter "T" for each true proposition, "F" for each false proposition, and "N" for each statement which is not a proposition.

1. x+y=y+x for every pair of real numbers x and y.
2. x+1=5 if x=1.
3. 2+3=5.
4. 5+7=10.
5. Do not pass go.
6. All insects are ants.
7. This statement is false.
8. All ants are insects.

2. (1 pt) setDiscrete1Logic/ur_dis_1_2.pg

For each of the following sentences, determine whether an "inclusive or" or an "exclusive or" is usually what is meant by the sentence. Enter "I" for the inclusive case and "E" for the exclusive case.

1. Publish or perish.
2. To enter the country you need a passport or a voter registration card.
3. Lunch includes soup or salad.
4. Experience with C++ or Java is required.

3. (1 pt) setDiscrete1Logic/ur_dis_1_3.pg

Complete the following truth table by filling in the blanks with T or F as appropriate.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ⊕ p</th>
<th>p ⊕ ~q</th>
<th>(p ⊕ q) ∨ (p ⊕ ~q)</th>
<th>(p ⊕ q) ^ (p ⊕ ~q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The proposition in the final column is

A. a contingency
B. a tautology
C. a contradiction

4. (1 pt) setDiscrete1Logic/ur_dis_1_4.pg

What is the value of x after each of the following statements is encountered in a computer program, if x=4 before the statement is reached?

(a) if x < 3 then x := x + 1

(b) if (1 + 1 = 3) OR (3 > x) then x := x + 1

(c) if (2 + 3 = 5) AND (3 + 4 = 7) then x := x + 1

(d) if (1 + 1 = 2) XOR (x = 7) then x := x + 1

5. (1 pt) setDiscrete1Logic/ur_dis_1_5.pg

Evaluate each of the following expressions:

(a) 11000 ^ (01011 v 11011)

6. (1 pt) setDiscrete1Logic/ur_dis_1_6.pg

Fuzzy Logic is used in artificial intelligence. In fuzzy logic, a proposition has a truth value that is a number between 0 and 1 inclusive. A proposition with a truth value of 0 is false and one with truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.8 can be assigned to the statement "Fred is happy," since Fred is happy most of the time, and the truth value 0.35 can be assigned to the statement "John is happy," since John is happy slightly less than half the time.

The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. The truth value of a conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions.

What are the truth values of the statements:

(a) "Fred and John are happy."

(b) "Neither Fred nor John is happy."

7. (1 pt) setDiscrete1Logic/ur_dis_1_7.pg

Complete the following truth table by filling in the blanks with T or F as appropriate.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>¬p</th>
<th>p ∨ q</th>
<th>¬p ∧ (p ∨ q)</th>
<th>¬p ∧ (p ∨ q) → q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The propositions in the last two columns are

A. not logically comparable
B. not logically equivalent

8. (1 pt) setDiscrete1Logic/ur_dis_1_8.pg

Complete the following truth table by filling in the blanks with T or F as appropriate.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ∧ q</th>
<th>¬p ∧ ¬q</th>
<th>(p ∧ q) ∨ (¬p ∧ ¬q)</th>
<th>p ↔ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
9. (1 pt) setDiscrete1Logic/ur_dis_1_9.pg
Complete the following truth table by filling in the blanks with T or F as appropriate.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p → q</th>
<th>¬p</th>
<th>¬q</th>
<th>¬q → ¬p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“p → q” and “¬q → ¬p” are

- A. not logically comparable
- B. logically equivalent
- C. not logically equivalent

10. (1 pt) setDiscrete1Logic/ur_dis_1_10.pg
Complete the following truth table by filling in the blanks with T or F as appropriate.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ≡ q</th>
<th>¬(p ≡ q)</th>
<th>p ⇔ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
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</tbody>
</table>

The propositions in the last two columns are

- A. logically equivalent
- B. not logically equivalent
- C. not logically comparable
1. (1 pt) setDiscrete2Quantifiers/ur_dis_2_1.pg

Let $P(x)$ be the statement “the word $x$ contains the letter ‘a’”. What are the truth values of the following?

1. $P(true)$
2. $P(orange)$
3. $P(lemon)$
4. $P(false)$

2. (1 pt) setDiscrete2Quantifiers/ur_dis_2_2.pg

Determine the truth value of the following statements if the universe of discourse of each variable is the set of real numbers.

1. $\forall x \exists y(x + y = 1)$
2. $\exists x \forall y((x + 2y = 2) \land (2x + 4y = 5))$
3. $\exists x \forall y \neq 0(xy = 1)$
4. $\exists x \forall y(xy = 0)$
5. $\forall x \exists y(x^2 = y)$
6. $\exists x(x^2 = 2)$

3. (1 pt) setDiscrete2Quantifiers/ur_dis_2_3.pg

The notation $\exists! x P(x)$ denotes the proposition “There exists a unique $x$ such that $P(x)$ is true.”

If the universe of discourse is the set of integers, what are the truth values of the following?

1. $\exists! x(x + 3 = 2x)$
2. $\exists! x(x = x + 1)$
3. $\exists! x(x > 1)$
4. $\exists! x(x^2 = 1)$
1. (1 pt) setDiscrete3SetTheory/ur_st_1_5.pg

Suppose that
\[ A = \{2, 4, 6\}, B = \{2, 6\}, C = \{4, 6\} \text{ and } D = \{4, 6, 8\}. \]
Determine which of these sets are subsets of which other of these sets.
Check ALL correct answers below.

- A. \( A \subseteq C \)
- B. \( D \subseteq C \)
- C. \( D \subseteq B \)
- D. \( B \subseteq C \)
- E. \( A \subseteq D \)
- F. \( C \subseteq D \)
- G. \( C \subseteq A \)
- H. \( D \subseteq A \)
- I. \( B \subseteq D \)
- J. \( B \subseteq A \)
- K. \( A \subseteq B \)

2. (1 pt) setDiscrete3SetTheory/ur_st_1_6.pg

What is the cardinality of each of the following sets?
(a) \( \emptyset \)
(b) \( \{0\} \)
(c) \( \{0, \{0\}\} \)
(d) \( \{0, \{0\}, \{\{0\}\}\} \)

3. (1 pt) setDiscrete3SetTheory/ur_st_1_7.pg

\( A = \{1, 3, 5\}, B = \{2, 3\} \)
Check ALL of the following Cartesian products to which the following elements belong:
(a) \( \{1, 2\} \)
- A. \( B \times A \)
- B. \( B \times B \)
- C. \( A \times B \)
- D. \( A \times A \)
(b) \( \{3, 1\} \)
- A. \( A \times A \)
- B. \( A \times B \)
- C. \( B \times B \)
- D. \( B \times A \)
(c) \( \{1, 1\} \)
- A. \( A \times B \)
- B. \( B \times B \)
- C. \( A \times A \)
- D. \( B \times A \)
(d) \( \{3, 3\} \)

4. (1 pt) setDiscrete3SetTheory/ur_st_1_8.pg

\( A = \{1, 3, 5\}, B = \{2, 3\} \)
Check ALL elements of the following sets:
(a) \( A \cap B \)
- A. \( 1 \)
- B. \( 3 \)
- C. \( 4 \)
- D. \( 5 \)
- E. \( 2 \)
(b) \( A \cup B \)
- A. \( 4 \)
- B. \( 2 \)
- C. \( 5 \)
- D. \( 3 \)
- E. \( 1 \)
(c) \( A \setminus B \)
- A. \( 3 \)
- B. \( 5 \)
- C. \( 4 \)
- D. \( 2 \)
- E. \( 1 \)

5. (1 pt) setDiscrete3SetTheory/ur_st_1_9.pg

Complete the following membership table by filling in the blanks with 1 or 0 as appropriate.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( B - A )</th>
<th>( A \cup B )</th>
<th>( A \cap (B - A) )</th>
<th>( A \cup (B - A) )</th>
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</table>

Use the membership table above to answer the following questions.
For each part, check the answer that most completely describes the general situation.
(1) \( A - B \)
- A. \( A \)
- B. \( A \cap B \)
- C. \( C \subseteq A \)
- D. \( D \subseteq B \)
(2) \( A \cap (B - A) \)
- A. \( \emptyset \)
- B. \( A \)
- C. \( A \cap B \)
- D. \( B \)
(1) \( A \cup (B - A) \)
Complete the following membership table by filling in the blanks with 1 or 0 as appropriate.

<p>| | | | | |</p>
<table>
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</tbody>
</table>

Check the statement above that **MOST COMPLETELY** describes the relationship between the two sets:

- A. \( (A \cap B) \cup (A \cap B) \subseteq A \)
- B. \( A \subseteq (A \cap B) \cup (A \cap B) \)
- C. \( (A \cap B) \cup (A \cap B) \subseteq A \)
- D. \( (A \cap B) \cup (A \cap B) = A \)
- E. \( A \subseteq (A \cap B) \cup (A \cap B) \)

Suppose that the universal set is \( U = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \). Express each of the following subsets with bit strings (of length 10) where the ith bit (from left to right) is 1 if i is in the subset and zero otherwise.

- (a) 3, 4, 5
- (b) 1, 3, 6, 10
- (c) 2, 3, 4, 7, 8, 9

Check ALL elements of the following sets:

- (a) \( A \cap B \)
  - A. 4
  - B. 5
  - C. 2
  - D. 1
  - E. 3
- (b) \( A \cup B \)
  - A. 1
  - B. 2
  - C. 5

Fuzzy sets are used in artificial intelligence. Each element in the universal set \( U \) has a degree of membership, which is a real number between 0 and 1 (including 0 and 1 as possibilities), in a fuzzy set \( S \). The fuzzy set \( S \) is denoted by listing the elements with their degrees of membership (elements with 0 degree of membership are not listed).

For example, we write \( F = \{0.65 \text{Alice}, 0.9 \text{Brian}, 0.6 \text{Rita}, 0.1 \text{Oscar}, 0.4 \text{Fred} \} \) for the (fuzzy) set \( F \) of famous people to indicate that Alice has a 0.65 degree membership to \( F \), that Brian has a 0.9 membership to \( F \) and so on. (For example Brian is the most famous of these people while Oscar is the least famous.)

Also suppose that \( R \) is the (fuzzy) set of rich people given by \( R = \{0.2 \text{Alice}, 0.5 \text{Brian}, 0.2 \text{Rita}, 0.55 \text{Oscar}, 0.3 \text{Fred} \} \). The complement of a fuzzy set \( S \) is the fuzzy set \( \overline{S} \), where the degree of membership of an element in \( \overline{S} \) is 1 minus the degree of membership of that element in \( S \).

Thus for example we have:
\[
\overline{F} = \overline{0.65 \text{Alice}, 0.9 \text{Brian}, 0.6 \text{Rita}, 0.1 \text{Oscar}, 0.4 \text{Fred}}.
\]

The intersection of two fuzzy sets \( S \) and \( T \) is the fuzzy set \( S \cap T \), where the degree of membership of an element in \( S \cap T \) is the minimum of the degrees of membership of this element in \( S \) and in \( T \). Thus the fuzzy set \( F \cap R \) of the rich and famous people is:
\[
\overline{F \cap R} = \overline{0.65 \text{Alice}, 0.9 \text{Brian}, 0.6 \text{Rita}, 0.1 \text{Oscar}, 0.4 \text{Fred}}.
\]
Determine whether \( f \) is a function from \( \mathbb{Z} \) to \( \mathbb{R} \). Enter "Y" for yes and "N" for no.

1. \( f(n) = \sqrt{n^2 + 7} \)
2. \( f(n) = 1/(n^2 + 8) \)
3. \( f(n) = \pm n \)
4. \( f(n) = 1/n^2 - 25 \)

Find the following values.
(a) \([1.1]\]
(b) \([1.1]\]
(c) \([-0.1]\]
(d) \([-0.1]\]
(e) \([2.99]\]
(f) \([-2.99]\]
(g) \([\frac{1}{2} + \frac{1}{2}]\]
(h) \([\frac{1}{2} + \frac{1}{2} + \frac{1}{2}]\]

Determine if each of the following functions from \( \{a, b, c, d\} \) to itself is one-to-one and/or onto.
Check ALL correct answers.
(a) \( f(a) = d, f(b) = a, f(c) = c, f(d) = b \)
   - A. onto.
   - B. neither one-to-one nor onto.
   - C. one-to-one.
   \( f(a) = b, f(b) = a, f(c) = b, f(d) = c \)
   - A. neither one-to-one nor onto.
   - B. onto.
   - C. one-to-one.
   \( f(a) = c, f(b) = d, f(c) = a \)
   - A. onto.
   - B. neither one-to-one nor onto.
   - C. one-to-one.

Given that \( f(x) = 5x^2 + 9 \) and \( g(x) = 8x + 6 \) are functions from \( \mathbb{R} \) to \( \mathbb{R} \), find
(a) \( f \circ g \).

Find the number of bytes required to encode \( n \) bits of data where \( n \) equals:
(a) 4.
(b) 10.
(c) 500.
(d) 3000.

Determine if each of the following functions is \( O(x^2) \).
Answer Y for yes and N for no.

Find the least integer \( n \) such that \( f(x) \) is \( O(x^n) \) for each of the following functions:
(a) \( f(x) = 2x^2 + x^3\log(x) \)
(b) \( f(x) = 3x^2 + (\log x)^4 \)
(c) \( f(x) = \frac{2^x + x^2 + 1}{x^2 + 1} \)
(d) \( f(x) = \frac{x^2 + x + 1}{x + 1} \)

Check the answer that best describes the relationship between \( f(x) \) and \( x \).
(For example if \( f(x) \) is \( \Theta(x) \) check that as your answer and not \( O(x) \) or \( \Omega(x) \) even though these are true also in this case.)
(a) \( f(x) = 10 \) is
   - A. \( O(x) \)
   - B. \( \Omega(x) \)
   - C. \( \Theta(x) \)
(b) \( f(x) = 3x + 7 \) is
   - A. \( \Theta(x) \)
   - B. \( O(x) \)
   - C. \( \Omega(x) \)
(c) \( f(x) = x^2 + x + 1 \) is
   - A. \( \Theta(x) \)
   - B. \( O(x) \)
   - C. \( \Omega(x) \)
(d) \( f(x) = 5\log(x) \) is
   - A. \( \Theta(x) \)
In this problem it will be useful to recall the following properties of logarithms: \( \log(xy) = \log(x) + \log(y) \) and \( \log(x^a) = a \log(x) \).

Find the least integer \( k \) such that \( f(n) \) is \( O(n^k) \) for each of the following functions:

(a) \( f(n) = n \log(4^n) \)
(b) \( f(n) = 1^8 + 2^8 + \cdots + n^8 \)
(c) \( f(n) = \log(n!) \)
(d) \( f(n) = \frac{\log(n^n)}{n^2+1} \)
1. (1 pt) setDiscrete5Algorithms/ur_dis_5_1.pg

Check ALL correct statements about the following algorithms.

(a) \{ procedure \ double \ (n: \text{positive integer}) \}
while \ n \not\subseteq 0 \ begin \n := 2n \ end
output(n) \} 

- A. When \( n=5 \) is the input, the while loop is infinite.
- B. When \( n=5 \) is the input, \( n=10 \) is the final output of the algorithm.
- C. This algorithm lacks finiteness.

(b) \{ procedure \ divide \ (n: \text{positive integer}) \}
while \ n \geq 0 \ begin \m := 1/n \n := n - 1 \end
output(m) \} 

- A. This algorithm works and outputs \( 1/n \).
- B. This algorithm lacks definiteness since division by zero occurs.
- C. When \( n=1 \) is the input, on the second iteration of the while loop a division by zero occurs.
- D. When \( n=1 \) is the input, the algorithm exits the while loop after the first iteration and outputs \( m=1 \).
- E. When \( n=1 \) is the input, after the first iteration of the while loop we have \( m=1 \) and \( n=0 \).
- F. This algorithm works and always outputs 1.

(c) \{ procedure \ sum \ (n: \text{positive integer}) \}
\sum := 0 \ while \ i \not\subseteq 10 \ begin \sum := \sum + i \end
output(sum) \} 

- A. If \( i \) is initialized to the value 1 in the beginning of the algorithm, the algorithm works and outputs \( 1 + 2 + \cdots + 10 \).
- B. If \( i \) is initialized to the value 1 in the beginning of the algorithm, the algorithm works and outputs \( 1 + 2 + \cdots + 9 \).
- C. This algorithm does not seem to use the input \( n \).

2. (1 pt) setDiscrete5Algorithms/ur_dis_5_2.pg

This exercise refers to the binary search algorithm given below.

\{ procedure \ binary \ search \ (x : \text{integer}, a_1, a_2, \ldots, a_n: \text{increasing integers}) \}
i := 1 \ \{ i \ \text{is left endpoint of search interval} \}
j := n \ \{ j \ \text{is right endpoint of search interval} \}
while \ i < j \ begin \m := \lfloor (i + j)/2 \rfloor \if \ x > a_m \ \text{then} \ i := m + 1 \else \ j := m \ \text{end} \if \ x = a_i \ \text{then} \ location := i \else location := 0 \ \text{end} \{ \text{location is the subscript of term equal to } x, \text{ or } 0 \text{ if } x \text{ is not found} \}

Suppose our list of increasing integers is shown in the table below:

\begin{array}{cccccccc}
1 & 4 & 6 & 9 & 12 & 13 & 14 & 15 \\
\end{array}

Suppose we conduct the binary search algorithm on this list where we search for 14.

(a) In the language of the algorithm above, enter the correct values for the following variables for this particular search:
\( x = \ldots \ \text{n = \ldots a}_2 = \ldots a_4 = \ldots \)

(b) After the first iteration of the while loop, what are the values of the following variables?
\( i = \ldots \ \text{j = \ldots m = \ldots} \)

(c) Intuitively, after the first iteration of the while loop, we have cut down our search to a set \( S \) of roughly half of the original numbers on the list. Check the numbers that are in the set \( S \) after the first iteration of the while loop:

- A. 9
- B. 4
- C. 13
- D. 12
- E. 6
- F. 1
- G. 14
- H. 15
(d) After the second iteration of the while loop, some of the variables have altered values. Enter the values of the following variables after the second while loop iteration:

\[ i = \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ ..
1. Are the following integers primes? Enter "1" for a prime and "0" otherwise.
   1. 77
   2. 66
   3. 98
   4. 1
   5. 86
   6. 88

2. Find the prime factorization of the following numbers:
   (write \( p^d \) if a prime does not appear in the given number.)
   \[900 = 2^3 \cdot 3^2 \cdot 5^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 26 \]
   where
   \[a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \quad e = \_ \quad f = \_ \quad g = \_ \quad h = \_ \]
   \[9800 = 2^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 26 \]
   where
   \[a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \quad e = \_ \quad f = \_ \quad g = \_ \quad h = \_ \]
   \[2992 = 2^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 26 \]
   where
   \[a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \quad e = \_ \quad f = \_ \quad g = \_ \quad h = \_ \]
   \[5909761 = 2^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 26 \]
   where
   \[a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \quad e = \_ \quad f = \_ \quad g = \_ \quad h = \_ \]

3. The value of the Euler \( \phi \) function (\( \phi \) is the Greek letter phi) at the positive integer \( n \) is defined to be the number of positive integers less than or equal to \( n \) that are relatively prime to \( n \). For example for \( n = 14 \), we have \( \{1, 3, 5, 9, 11, 13\} \) are the positive integers less than or equal to 14 which are relatively prime to 14. Thus \( \phi(14) = 6 \). Find:
   \( \phi(3) = \_ \)
   \( \phi(9) = \_ \)
   \( \phi(6) = \_ \)
   \( \phi(12) = \_ \)

4. What are the greatest common divisors of the following pairs of integers?
   (a) \( 2^5 \cdot 3^2 \cdot 5^2 \) and \( 2^4 \cdot 3^3 \cdot 5^2 \)
   (b) \( 2^6 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \)
   (c) \( 2^5 \cdot 7 \) and \( 2^2 + 3^2 \)

5. Find the first few terms of the sequence of pseudorandom numbers generated using the linear congruential generator
   \[ x_{n+1} = (7x_n + 2) \mod 5 \]
   with seed \( x_0 = 5 \).
   \[ x_1 = \_ \quad x_2 = \_ \quad x_3 = \_ \quad x_4 = \_ \quad x_5 = \_ \quad x_6 = \_ \]
   Now find the first few terms of the sequence of pseudorandom numbers generated using the linear congruential generator
   \[ x_{n+1} = (4x_n + 5) \mod 9 \]
   with seed \( x_0 = 1 \).
   \[ x_1 = \_ \quad x_2 = \_ \quad x_3 = \_ \quad x_4 = \_ \quad x_5 = \_ \quad x_6 = \_ \]

6. Encrypt the message HALT by translating the letters into numbers
   (via \( A = 0, B = 1, C = 2, D = 3, E = 4, F = 5, G = 6, H = 7, I = 8 \),
   \( J = 9, K = 10, L = 11, M = 12, N = 13, O = 14, P = 15, Q = 16, R = 17, S = 18, T = 19, U = 20, V = 21, W = 22, X = 23, Y = 24, Z = 25 \))
   and then applying the encryption function given, and then translating the numbers back into letters.
   (a) \( f(p) = (p + 2) \mod 26 \)
   (b) \( f(p) = (p + 4) \mod 26 \)
   (c) \( f(p) = (4p + 7) \mod 26 \)

7. Decrypt the following messages encrypted using the Caesar cipher:
   \( f(p) = (p + 3) \mod 26 \)
   (a) FUDCB KDWV
   (b) EOXH MHDQV
   (c) PIKTCFGW

8. Books are identified by an International Standard Book Number (ISBN), a 10-digit code \( x_1x_2 \ldots x_{10} \), assigned by the publisher. These 10 digits consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing
company, and finally, a 1-digit check digit that is either a digit or the letter X (used to represent 10). This check digit is selected so that $\sum_{i=1}^{10} d_i x_i \equiv 0 \mod 11$ and is used to detect errors in individual digits and transposition of digits. 
(a) The ISBN for a book is 6 – 06 – 031205 – Q where Q is the check digit. What is Q?
(b) The ISBN of another book is 0 – 640 – 89M57 – 7. Find the digit M.

10. (1 pt) setDiscrete6Integers/ur_dis_f_10.pg
Convert the following integers from decimal notation to binary notation.
(Do not put extra zeros in front of your binary notation or it might confuse WebWorK. So write 101 versus 0101 etc.)
(a) 441
(b) 1279
(c) 196257

11. (1 pt) setDiscrete6Integers/ur_dis_f_11.pg
Convert the following integers from binary notation to decimal notation:
(a) 1010010001
(b) 11110011
(c) 1111001111

12. (1 pt) setDiscrete6Integers/ur_dis_f_12.pg
Convert each of the following integers from binary notation to octal and hexadecimal notation.
(a) 111100101
    octal: __________ hexadecimal: __________
(b) 10111011000
    octal: __________ hexadecimal: __________
(c) 1111001111
    octal: __________ hexadecimal: __________

13. (1 pt) setDiscrete6Integers/ur_dis_f_13.pg
One’s complement representations of integers are used to simplify computer arithmetic. To represent positive and negative integers with absolute value less than $2^{n-1}$, a total of n bits is used. The leftmost bit is used to represent the sign: this bit is 0 for positive integers and 1 for negative integers.
For positive integers, the remaining bits are identical to the binary representation of the integer. Thus using 4 bits, the One’s complement representation of 5 is 0101.
For negative integers, the remaining bits are found by first finding the binary representation of the absolute value of the integer and then taking the complement of each of these bits in the binary representation. (Thus 1’s and 0’s get switched)
Thus for example using 4 bits, the One’s complement representation of -5 is 1010, where the initial 1 is to indicate the sign, and the subsequent 010 is the complement of 101 the binary representation of 5.
Find the One’s Complement representations, using bit strings of length six, of the following integers. (Your answers must be bit strings of length 6):
(a) 31
(b) 24
(c) −18
(d) −20

14. (1 pt) setDiscrete6Integers/ur_dis_f_14.pg
A Cantor expansion is a sum of the form
$$a_n n! + a_{n-1} (n-1)! + \ldots + a_2 2! + a_1 1!$$
where $0 \leq a_i \leq i$ are integers for $i = 1, 2, \ldots, n$.
For example, the Cantor expansion of 17 is $17 = 2(3!) + 2(2!) + 1(1!)$. Note that $17 = 1(3!) + 5(2!) + 1(1!)$ is correct but is not a valid Cantor expansion of 17 since we insist the coefficient of $n!$ is no more than n. Thus the coefficient of 2! should be a 0,1 or 2 for example.
Complete the following Cantor expansions of:
(a) 4
(b) 331
(c) 38

$\sum_{i=1}^{n} a_i i!$
The goal of this exercise is to practice finding the inverse modulo $m$ of some (relatively prime) integer $n$. We will find the inverse of 21 modulo 151, i.e., an integer $c$ such that $21c \equiv 1 \pmod{151}$.

First we perform the Euclidean algorithm on 21 and 151:

\[
151 = 7 \times 21 + 4 \\
21 = 5 \times 4 + 1 \\
4 = 4 \times 1 + 0
\]

[Note your answers on the second row should match the ones on the first row.]

Thus gcd(21, 151) = 1, i.e., 21 and 151 are relatively prime. Now we run the Euclidean algorithm backwards to write 1 = 151s + 21t for suitable integers s, t.

\[
s = \underline{\hspace{2cm}} \\
t = \underline{\hspace{2cm}}
\]

when we look at the equation 151s + 21t ≡ 1 (mod 151), the multiple of 151 becomes zero and so we get 21t ≡ 1 (mod 151). Hence the multiplicative inverse of 21 modulo 151 is

\[
\hat{y}_2 = \underline{\hspace{2cm}}
\]

Next we find the $\hat{y}_k$ which are given by solving

\[
\hat{y}_k\hat{m}_k \equiv 1 \pmod{m_k}
\]

Thus for example, to find $\hat{y}_1$ we need to solve

\[
8789\hat{y}_1 \equiv 1 \pmod{3}
\]

Since we know 8789 ≡ 2 mod 3, this simplifies to

\[
2\hat{y}_1 \equiv 1 \pmod{3}
\]

Solve this either by trial and error or by using the Euclidean algorithm and enter the value of $\hat{y}_1$ below: (Use the canonical representative modulo 3.)

\[
\hat{y}_1 = \underline{\hspace{2cm}}
\]

Similarly, to find $\hat{y}_2$ we need to solve

\[
1551\hat{y}_2 \equiv 1 \pmod{17}
\]

Since we know 1551 ≡ 4 mod 17, this simplifies to

\[
4\hat{y}_2 \equiv 1 \pmod{17}
\]

Solve this either by trial and error or by using the Euclidean algorithm and enter the value of $\hat{y}_2$ below: (Use the canonical representative modulo 17.)

\[
\hat{y}_2 = \underline{\hspace{2cm}}
\]

Similarly, to find $\hat{y}_3$ we need to solve

\[
2397\hat{y}_3 \equiv 1 \pmod{11}
\]

Since we know 2397 ≡ 10 mod 11, this simplifies to

\[
10\hat{y}_3 \equiv 1 \pmod{11}
\]

Solve this either by trial and error or by using the Euclidean algorithm and enter the value of $\hat{y}_3$ below: (Use the canonical representative modulo 11.)

\[
\hat{y}_3 = \underline{\hspace{2cm}}
\]

Similarly, to find $\hat{y}_4$ we need to solve

\[
561\hat{y}_4 \equiv 1 \pmod{47}
\]

Since we know 561 ≡ 44 mod 47, this simplifies to

\[
44\hat{y}_4 \equiv 1 \pmod{47}
\]

Solve this either by trial and error or by using the Euclidean algorithm and enter the value of $\hat{y}_4$ below: (Use the canonical representative modulo 47.)

\[
\hat{y}_4 = \underline{\hspace{2cm}}
\]

Now that we have all the $a_k$, $\hat{m}_k$ and $\hat{y}_k$, use the formula

\[
x = \sum_{k=1}^{4} a_k\hat{m}_k\hat{y}_k
\]

to find an integer solution $x$ to the original system. The Chinese remainder theorem says that this $x$ and any integer congruent modulo $m$ to it, will solve the original system. Enter the SMALLEST positive integer solution to the original system here:
5. Use Fermat’s Little theorem to compute the following remainders for \(2^{482}\) (Always use canonical representatives.)

\[\begin{align*}
2^{482} & \equiv 5 \mod 5 \\
2^{482} & \equiv 7 \mod 7 \\
2^{482} & \equiv 11 \mod 11 \\
\end{align*}\]

Use your answers above to find the canonical representative of \(2^{482} \mod 385\) by using the Chinese Remainder Theorem.

[Note 385 = 5 \cdot 7 \cdot 11 and that Fermat’s Little Theorem cannot be used to directly find \(2^{482} \mod 385\) as 385 is not a prime and also since it is larger than the exponent.]

\(2^{482} \mod 385\) is \_\_\_

6. Fill in the blanks in the table with the unique integers \(a\) in the range \(0 \leq a \leq 27\) with the given remainders.

Hint: It is probably easiest to just make a table with the numbers between 0 and 27 and their remainders and use that to find the answers. However one can also use the Chinese Remainder Formula

\[x = a_1\hat{m}_1\hat{y}_1 + a_2\hat{m}_2\hat{y}_2\]

by finding the \(\hat{m}_k,\hat{y}_k\) once and then plugging in the various remainders for the \(a_k\) to get the various answers.

<table>
<thead>
<tr>
<th>(a)</th>
<th>(a \mod 4)</th>
<th>(a \mod 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

7. (Modification of exercise 36 in section 2.5 of Rosen.)

The goal of this exercise is to work thru the RSA system in a simple case:

We will use primes \(p = 47, q = 71\) and form \(n = 47 \cdot 71 = 3337\).

[This is typical of the RSA system which chooses two large primes at random generally, and multiplies them to find \(n\). The public will know \(n\) but \(p\) and \(q\) will be kept private.]

Now we choose our public key \(e = 17\). This will work since \(\gcd(17, (p - 1)(q - 1)) = \gcd(17, 3220) = 1\). [In general as long as we choose an 'e' with \(\gcd(e,(p-1)(q-1))=1\), the system will work.]

Next we encode letters of the alphabet numerically say via the usual:

(A=0, B=1, C=2, D=3, E=4, F=5, G=6, H=7, I=8, J=9, K=10, L=11, M=12, N=13, O=14, P=15, Q=16, R=17, S=18, T=19, U=20, V=21, W=22, X=23, Y=24, Z=25)

We will practice the RSA encryption on the single integer 15.

(which is the numerical representation for the letter "P"). In the language of the book, M=15 is our original message.

The coded integer is formed via \(c = M^e \mod n\).

Thus we need to calculate \(15^{17} \mod 3337\).

This is not as easy as it seems and you might consider using fast modular multiplication.

The canonical representative of \(15^{17} \mod 3337\) is \_\_\_
1. (1 pt) setDiscrete8Reasoning/ur_dis8_1.png

Which rule of inference is used in each of the following arguments? Check the correct answers.

1. If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
   - A. Addition.
   - B. Disjunctive syllogism.
   - C. Modus tollens.
   - D. Modus ponens.
   - E. Disjunction.
   - F. Simplification.
   - G. Hypothetical syllogism.

2. If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.
   - A. Hypothetical syllogism.
   - B. Conjunction.
   - C. Disjunctive syllogism.
   - D. Simplification.
   - E. Addition.
   - F. Modus ponens.
   - G. Modus tollens.

3. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
   - A. Modus ponens.
   - B. Conjunction.
   - C. Disjunctive syllogism.
   - D. Simplification.
   - E. Addition.
   - F. Modus ponens.
   - G. Modus tollens.

4. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
   - A. Conjuntion.
   - B. Addition.
   - C. Modus tollens.
   - D. Disjunctive syllogism.
   - E. Simplification.
   - F. Modus ponens.
   - G. Hypothetical syllogism.

2. (1 pt) setDiscrete8Reasoning/ur_dis8_2.png

On a $8 \times 8$ chessboard, the squares are colored alternately white and black. Thus there are ______ white squares and ______ black squares. Each row/column of the chessboard has ______ squares. It is thus possible to tile this chessboard with dominoes (1 x 2 pieces) by laying say 4 dominoes per column. (tile means lay the dominoes, so that they cover the chessboard, no two dominoes overlapping.)

Now suppose we remove two squares from the chessboard, from DIAGONALLY opposite corners. Suppose one of the squares we remove is white. Now there are ______ white squares left and ______ black squares left.

Q: Is it possible to cover the modified chessboard (with the two diagonally opposite corners removed) with dominoes? Why?
   - A. No. Since every time we lay down a domino it covers one white square and one black square. Thus since the number of white squares is not equal to the number of black squares on the modified chessboard, it is impossible.
   - B. Yes. It is possible to tile the modified chessboard by placing dominoes, alternating between horizontal and vertical placements in a suitable way.
   - C. Yes. Since there are an equal number of white and black squares remaining on the modified chessboard, one can tile the modified chessboard with dominoes each covering one white and one black square.
   - D. No. Since the total number of remaining squares on the chessboard is odd and every domino covers 2 squares and hence can only be used to tile a region with an even number of squares.

3. (1 pt) setDiscrete8Reasoning/ur_dis8_3.png

For $n$ a nonnegative integer, either $n \equiv 0 \mod 3$ or $n \equiv 1 \mod 3$ or $n \equiv 2 \mod 3$. In each case, fill out the following table with the canonical representatives modulo 3 of the expressions given:

<table>
<thead>
<tr>
<th>$n \mod 3$</th>
<th>$n^3 \mod 3$</th>
<th>$2n \mod 3$</th>
<th>$n^3 + 2n \mod 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From this, we can conclude:
   - A. Since $n^3 + 2n \equiv 0 \mod 3$ for all $n$, we conclude that 3 divides $n^3 + 2n$ for any nonnegative integer $n$.
   - B. Since $n^3 + 2n \not\equiv 0 \mod 3$ for all $n$, we conclude that 3 does not necessarily divide $n^3 + 2n$ for all nonnegative integers $n$.

4. (1 pt) setDiscrete8Reasoning/ur_dis8_4.png

Find $f(1)$, $f(2)$, $f(3)$ and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 4$ and for $n = 0, 1, 2, \ldots$ by:

(a) $f(n + 1) = -1 \cdot f(n)$

\[
\begin{align*}
  f(1) &= 4 \\
  f(2) &= -4 \\
  f(3) &= 16 \\
  f(4) &= -64
\end{align*}
\]

(b) $f(n + 1) = 4f(n) + 4$

\[
\begin{align*}
  f(1) &= 4 \\
  f(2) &= 20 \\
  f(3) &= 92 \\
  f(4) &= 376
\end{align*}
\]

(a) $f(n + 1) = -1f(n)$

\[
\begin{align*}
  f(1) &= -4 \\
  f(2) &= 4 \\
  f(3) &= -16 \\
  f(4) &= 64
\end{align*}
\]

(b) $f(n + 1) = f(n)^2 - 1f(n) - 1$
Consider the following inductive definition of a version of Ackermann’s function:

\[
A(m, n) =
\begin{cases}
2n & \text{if } m = 0 \\
0 & \text{if } m \geq 1 \text{ and } n = 0 \\
2 & \text{if } m \geq 1 \text{ and } n = 1 \\
A(m-1, A(m, n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \\
\end{cases}
\]

Find the following values of the Ackermann’s function:

\[
\begin{align*}
A(3, 2) &= \quad A(1, 3) = \quad A(2, 2) = \\ A(1, 1) &= \quad A(3, 3) = \\
\end{align*}
\]
1. (1 pt) setDiscrete9Counting/ur_dis_9_1.pg
(a) A particular brand of shirt comes in 13 colors, has a male version and a female version, and comes in 4 sizes for each sex. How many different types of this shirt are made?

(b) How many bit strings of length 7 are there?

(c) How many bit strings of length 7 or less are there? (Count the empty string of length zero also.)

(d) How many strings of 4 lower case English letters are there that have the letter x in them somewhere? Here strings may use the same letter more than once. (Hint: It might be easier to first count the strings that don’t have an x in them.)

2. (1 pt) setDiscrete9Counting/ur_dis_9_2.pg
Find how many positive integers with exactly four decimal digits, that is, positive integers between 1000 and 9999 inclusive, have the following properties:
(a) are divisible by 7.

(b) are divisible by 5.

(c) are not divisible by either 5 or 7.

(d) are divisible by 5 and by 7.

3. (1 pt) setDiscrete9Counting/ur_dis_9_3.pg
How many strings of four decimal digits (Note there are 10 possible digits and a string can be of the form 0014 etc., i.e., can start with zeros.)
(a) begin with an odd digit? (can repeat digits.)

(b) end with an even digit? (Can repeat digits.)

4. (1 pt) setDiscrete9Counting/ur_dis_9_4.pg
How many strings of five uppercase English letters are there that start and end with the letters BO (in that order), if letters can be repeated?

(b) if no letter can be repeated?

(c) that start with an X, if letters can be repeated?

(d) if letters can be repeated?

5. (1 pt) setDiscrete9Counting/ur_dis_9_5.pg
Solve the following two “union” type questions:
(a) How many bit strings of length 10 either begin with 1 0s or end with 1 1s? (inclusive or)

(b) Every student in a discrete math class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 32 computer science majors (including joint majors), 22 math majors (including joint majors) and 8 joint majors?

6. (1 pt) setDiscrete9Counting/ur_dis_9_6.pg
A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
(a) How many balls must she select (minimum) to be sure of having at least three blue balls?

(b) How many balls must she select (minimum) to be sure of having at least three balls of the same color?

7. (1 pt) setDiscrete9Counting/ur_dis_9_7.pg
This question concerns bit strings of length six. These bit strings can be divided up into four types depending on their initial and terminal bit. Thus the types are: 0XXXX0, 0XXXX1, 1XXXX0, 1XXXX1.

How many bit strings of length six must you select before you are sure to have at least 4 that are of the same type? (Assume that when you select bit strings you always select different ones from ones you have already selected.)

8. (1 pt) setDiscrete9Counting/ur_dis_9_8.pg
Find the value of each of the following quantities:
\[ C(8, 5) = \]
\[ C(6, 6) = \]
\[ C(6, 3) = \]
\[ C(9, 5) = \]
\[ C(10, 7) = \]
\[ C(7, 4) = \]

9. (1 pt) setDiscrete9Counting/ur_dis_9_9.pg
There are 4 different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

10. (1 pt) setDiscrete9Counting/ur_dis_9_10.pg
How many bit strings of length 6 have:
(a) Exactly three 0s?

(b) The same number of 0s as 1s?

(d) At least three 1s?

11. (1 pt) setDiscrete9Counting/ur_dis_9_11.pg
17 players for a softball team show up for a game:
(a) How many ways are there to choose 10 players to take the field?

(b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
(c) Of the 17 people who show up, 7 are women. How many ways are there to choose 10 players to take the field if at least one of these players must be women?

12. (1 pt) setDiscrete9Counting/ur_dis_9_12.pg
Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with 6 members if it must have strictly more women than men?

13. (1 pt) setDiscrete9Counting/ur_dis_9_13.pg
How many ways are there to select 8 countries in the United Nations to serve on a council if 3 is selected from a block of 58, 2 are selected from a block of 61 and 3 are selected from the remaining 70 countries?

Find the coefficient of \(x^8\) in \((1 + x)^{13}\).

15. (1 pt) setDiscrete9Counting/ur_dis_9_15.pg
What is the coefficient of \(x^5y^{15}\) in the expansion of \((3x + 3y)^{20}\)?

16. (1 pt) setDiscrete9Counting/ur_dis_9_16.pg
Two six-sided dice are rolled (one red one and one green one). Some possibilities are (Red=1, Green=5) or (Red=2, Green=2) etc.
(a) How many total possibilities are there?

For the rest of the questions, we will assume that the dice are fair and that all of the possibilities in (a) are equally likely.
(b) What is the probability that the sum on the two dice comes out to be 11? (Remember the answer will be a ratio, the denominator of which will be your answer in (a).)

(c) What is the probability that the sum on the two dice comes out to be 12?

(d) What is the probability that the numbers on the two dice are equal?

A card is selected at random from a standard 52-card deck.
(a) What is the probability that it is an ace?_____
(b) What is the probability that it is a heart?_____
(c) What is the probability that it is an ace or a heart?_____

18. (1 pt) setDiscrete9Counting/ur_dis_9_18.pg
A five-card poker hand is dealt at random from a standard 52-card deck.
Note the total number of possible hands is \(C(52,5)=2,598,960\). Find the probabilities of the following scenarios:
(a) What is the probability that the hand contains exactly one ace? Answer= \(\frac{\alpha}{C(52,5)}\), where \(\alpha = \) _______
(b) What is the probability that the hand is a flush? (That is all the cards are of the same suit: hearts, clubs, spades or diamonds.) Answer= \(\frac{\beta}{C(52,5)}\), where \(\beta = \) _______
(c) What is the probability that the hand is a straight flush? Answer= \(\frac{\gamma}{C(52,5)}\), where \(\gamma = \) _______
What is the probability that a positive integer \(m\) in the range \(1 \leq m \leq 100\), which is selected randomly, is divisible by 9?
For example if \( \{a_n\} \) is the sequence 1, 3, 5, 7, 9, \ldots then \( \nabla a_n \) is the sequence \( -2, 2, 2, 2, \ldots \). Notice the sequence \( \nabla a_n \) always starts with 0 and the subsequent entries keep track of the differences in the original sequence \( \{a_n\} \). Fill in the blanks below:

- \( a_n: 1, 3, 9, 27, 81, \ldots \)
- \( \nabla a_n: 0, 2, 6, 18, 54, \ldots \)
- \( b_n: 1, 2, 5, 10, 17, 26, \ldots \)
- \( \nabla b_n: 0, 3, 5, 7, 9, \ldots \)
- \( c_n: 1, 2, 4, 8, 16, 32, \ldots \)
- \( \nabla c_n: 0, \ldots \)

Similarly one defines \( \nabla^2 a_n \) by:

\[
\nabla^2 a_n = \begin{cases} 
  a_n - a_{n-1} & \text{if } n > 1 \\
  0 & \text{if } n = 1 
\end{cases}
\]

So for example:

- \( a_n: 1, 2, 3, 5, 8, 13, 21, \ldots \)
- \( \nabla a_n: 0, 1, 1, 2, 3, 5, 8, \ldots \)
- \( \nabla^2 a_n: 0, 1, 0, 1, 2, 3, \ldots \)

Fill in the following blanks:

- \( b_n: 0, 1, 3, 6, 10, 17, 26, \ldots \)  
- \( \nabla b_n: 0, \ldots \)
- \( \nabla^2 b_n: 0, \ldots \)

The first step in any problem like this is to find the characteristic equation by trying a solution of the "geometric" format \( a_n = r^n \). (We assume also \( r \neq 0 \).) In this case we get:

\[ r^n = -2r^{n-1} + 3r^{n-2} \]

Since we are assuming \( r \neq 0 \) we can divide by the smallest power of \( r \), i.e., \( r^{n-2} \) to get the characteristic equation:

\[ r^2 = -2r + 3 \]

(Notice since our lhcc recurrence was degree 2, the characteristic equation is degree 2.)

Find the two roots of the characteristic equation \( r_1 \) and \( r_2 \). When entering your answers use \( r_1 \leq r_2 \):

\[ r_1 = \ldots, r_2 = \ldots \]

Since the roots are distinct, the general theory (Theorem 1 in section 5.2 of Rosen) tells us that the general solution to our lhcc recurrence looks like:

\[ a_n = \alpha_1(r_1)^n + \alpha_2(r_2)^n \]

for suitable constants \( \alpha_1, \alpha_2 \).

To find the values of these constants we have to use the initial conditions \( a_0 = 1, a_1 = 8 \). These yield by using \( n=0 \) and \( n=1 \) in the formula above:

\[ 1 = \alpha_1(r_1)^0 + \alpha_2(r_2)^0 \]

and

\[ 8 = \alpha_1(r_1)^1 + \alpha_2(r_2)^1 \]

By plugging in your previously found numerical values for \( r_1 \) and \( r_2 \) and doing some algebra, find \( \alpha_1, \alpha_2 \):

[Be careful to note that \((-x)^n \neq -(x^n)\) when \( n \) is even, for example \((-3)^2 \neq -(3^2).\)]

\[ \alpha_1 = \ldots, \alpha_2 = \ldots \]

Note the final solution of the recurrence is:

\[ a_n = \alpha_1(r_1)^n + \alpha_2(r_2)^n \]

where the numbers \( r_1, \alpha_1 \) have been found by your work. This gives an explicit numerical formula in terms of \( n \) for the \( a_n \).

Find the solution to the following lhcc recurrence:

\[ a_n = 1a_{n-1} + 30a_{n-2} \quad \text{for } n \geq 2 \]

with initial conditions \( a_0 = 1, a_1 = 5 \).

The solution of the form:

\[ a_n = \alpha_1(r_1)^n + \alpha_2(r_2)^n \]

for suitable constants \( \alpha_1, \alpha_2 \), with \( r_1 \leq r_2 \). Find these constants.

\[ r_1 = \ldots, r_2 = \ldots, \alpha_1 = \ldots, \alpha_2 = \ldots \]
Find the solution to the following lhcc recurrence:
\[ a_n = 4a_{n-2} \text{ for } n \geq 2 \] with initial conditions \( a_0 = 2, a_1 = 4 \).

The solution is of the form:
\[ a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n \]
for suitable constants \( \alpha_1, \alpha_2, r_1, r_2 \) with \( r_1 \leq r_2 \). Find these constants.

\[ r_1 = \quad \quad r_2 = \quad \quad \alpha_1 = \quad \quad \alpha_2 = \]

We will find the solution to the following lhcc recurrence:
\[ a_n = -6a_{n-1} - 9a_{n-2} \text{ for } n \geq 2 \] with initial conditions \( a_0 = 4, a_1 = 5 \).

The first step as usual is to find the characteristic equation by trying a solution of the “geometric” format \( a_n = r^n \). (We assume also \( r \neq 0 \).) In this case we get:
\[ r^n = -6r^{n-1} - 9r^{n-2} \]

Since we are assuming \( r \neq 0 \) we can divide by the smallest power of \( r \), i.e., \( r^{n-2} \) to get the characteristic equation:
\[ r^2 = -6r - 9 \]
(Notice since our lhcc recurrence was degree 2, the characteristic equation is degree 2.)

This characteristic equation has a single root \( r \). (We say the root has multiplicity 2). Find \( r \).
\[ r = \]

Since the root is repeated, the general theory (Theorem 2 in section 5.2 of Rosen) tells us that the general solution to our lhcc recurrence looks like:
\[ a_n = \alpha_1 (r_1)^n + \alpha_2 n (r_1)^n \]
for suitable constants \( \alpha_1, \alpha_2 \).

To find the values of these constants we have to use the initial conditions \( a_0 = 4, a_1 = 5 \). These yield by using \( n=0, n=1 \) in the formula above:
\[ 4 = \alpha_1 (r_1)^0 + \alpha_2 0 (r_1)^0 \]
and
\[ 5 = \alpha_1 (r_1)^1 + \alpha_2 1 (r_1)^1 \]

By plugging in your previously found numerical value for \( r_1, r_2 \) and doing some algebra, find \( \alpha_1, \alpha_2 \):
\[ \alpha_1 = \quad \quad \alpha_2 = \]

Note the final solution of the recurrence is:
\[ a_n = \alpha_1 (r_1)^n + \alpha_2 n (r_1)^n \]
where the numbers \( r, \alpha_1 \) have been found by your work. This gives an explicit numerical formula in terms of \( n \) for the \( a_n \).

We will find the solution to the following lhcc recurrence:
\[ a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3} \text{ for } n \geq 3 \] with initial conditions \( a_0 = 5, a_1 = 8, a_2 = 18 \).

The first step as usual is to find the characteristic equation by trying a solution of the "geometric" format \( a_n = r^n \). (We assume also \( r \neq 0 \).) In this case we get:
\[ r^n = 6r^{n-1} - 11r^{n-2} + 6r^{n-3} \]

Since we are assuming \( r \neq 0 \), we can divide by the smallest power of \( r \), i.e., \( r^{n-3} \) to get the characteristic equation:
\[ r^3 = 6r^2 - 11r + 6 \]
(Notice since our lhcc recurrence was degree 3, the characteristic equation is degree 3.)

Find the three roots of the characteristic equation \( r_1, r_2 \) and \( r_3 \). When entering your answers use \( r_1 \leq r_2 \leq r_3 \):
\[ r_1 = \quad r_2 = \quad r_3 = \]

Since the roots are distinct, the general theory (Theorem 3 in section 5.2 of Rosen) tells us that the general solution to our lhcc recurrence looks like:
\[ a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n + \alpha_3 (r_3)^n \]
for suitable constants \( \alpha_1, \alpha_2, \alpha_3 \).

To find the values of these constants we have to use the initial conditions \( a_0 = 5, a_1 = 8, a_2 = 18 \). These yield by using \( n=0, n=1 \) and \( n=2 \) in the formula above:
\[ 5 = \alpha_1 (r_1)^0 + \alpha_2 (r_2)^0 + \alpha_3 (r_3)^0 \]
and
\[ 8 = \alpha_1 (r_1)^1 + \alpha_2 (r_2)^1 + \alpha_3 (r_3)^1 \]
and
\[ 18 = \alpha_1 (r_1)^2 + \alpha_2 (r_2)^2 + \alpha_3 (r_3)^2 \]

By plugging in your previously found numerical values for \( r_1, r_2 \) and \( r_3 \) and doing some algebra, find \( \alpha_1, \alpha_2, \alpha_3 \):

Notation: Ad hoc substitution should work to find the \( \alpha_i \) but for those who know linear algebra, note the system of equations above can be written in matrix form as:
\[
\begin{pmatrix}
(r_1)^0 & (r_2)^0 & (r_3)^0 \\
(r_1)^1 & (r_2)^1 & (r_3)^1 \\
(r_1)^2 & (r_2)^2 & (r_3)^2
\end{pmatrix}
\begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{pmatrix}
= \begin{pmatrix}
5 \\
8 \\
18
\end{pmatrix}
\]

\[ \alpha_1 = \quad \quad \alpha_2 = \quad \quad \alpha_3 = \]

Note the final solution of the recurrence is:
\[ a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n + \alpha_3 (r_3)^n \]
where the numbers \( r_i, \alpha_i \) have been found by your work. This gives an explicit numerical formula in terms of \( n \) for the \( a_n \).
1. (1 pt) setDiscrete11InclusionEx/ur_dis_11_1.pg
There are 359 students in a college who have taken a course in calculus, 222 who have taken a course in discrete mathematics, and 173 who have taken a course in both calculus and discrete mathematics. How many students at this college have taken a course in either calculus or discrete mathematics?

2. (1 pt) setDiscrete11InclusionEx/ur_dis_11_2.pg
Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 103 elements in $A_1$, 1003 elements in $A_2$ and 9946 elements in $A_3$ in each of the following situations:
(a) The sets are pairwise disjoint.
(b) $A_1 \subseteq A_2 \subseteq A_3$.
(c) There are 18 elements common to each pair of sets and 3 elements in all three sets.

3. (1 pt) setDiscrete11InclusionEx/ur_dis_11_3.pg
In a survey of 285 college students, it is found that 64 like brussels sprouts, 99 like broccoli, 57 like cauliflower, 30 like both brussels sprouts and broccoli, 22 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower and 11 of the students like all three vegetables. How many of the 285 college students do not like any of these three vegetables?

4. (1 pt) setDiscrete11InclusionEx/ur_dis_11_4.pg
How many elements are in the union of four sets if each of the sets has 98 elements, each pair of sets share 55 elements, each triple of sets shares 27 elements and there are 7 elements in all four sets.
1. (1 pt) setSetTheory1/ur_st_1_1.pg
(a) If \( n(A) = 16, n(B) = 37 \) and \( n(A \cap B) = 15 \), then \( n(A \cup B) = \) __________
(b) If \( n(A \cup B) = 43, n(A \cap B) = 5 \) and \( n(A) = n(B) \), then \( n(A) = \) __________

2. (1 pt) setSetTheory1/ur_st_1_2.pg
Let \( A = \{2, 3, 4, 7, 8\} \), \( B = \{0, 2, 3, 8\} \), \( C = \{0, 1, 4, 5, 6, 8\} \). List the elements of the following sets in the increasing order:
- \( A \cap B = \{ \) __________ \\
- \( A \cup B = \{ \) __________ \\
- \( (B \cup C) \cap A = \{ \) __________ \\
- \( B \cup (C \cap A) = \{ \) __________ \\

3. (1 pt) setSetTheory1/ur_st_1_3.pg
Let \( U = \) Universal set = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \( A = \{1, 3, 4, 5, 6, 9\} \), \( B = \{1, 2, 4, 6\} \).
List the elements of the following sets in the increasing order:
- \( A' = \{ \) __________ \\
- \( A \cup B' = \{ \) __________ \\
- \( A \cap B' = \{ \) __________ \\
- \( A \cup B' = \{ \) __________ \\

4. (1 pt) setSetTheory1/ur_st_1_4.pg
There are a total of 119 foreign language students in a high school where they offer Spanish, French, and German. There are 27 students who take at least 2 languages at once. If there are 51 Spanish students, 49 French students, and 43 German students, how many students take all three languages at once?
Answer: __________

5. (1 pt) setSetTheory1/ur_st_1_5.pg
Suppose that \( A = \{2, 4, 6\}, B = \{2, 6\}, C = \{4, 6\} \) and \( D = \{4, 6, 8\} \). Determine which of these sets are subsets of which other of these sets.
Check ALL correct answers below.
- A. \( C \subseteq A \)
- B. \( B \subseteq D \)
- C. \( B \subseteq A \)
- D. \( C \subseteq D \)
- E. \( D \subseteq A \)
- F. \( A \subseteq D \)
- G. \( A \subseteq C \)
- H. \( D \subseteq C \)
- I. \( B \subseteq C \)
- J. \( D \subseteq B \)
- K. \( A \subseteq B \)

6. (1 pt) setSetTheory1/ur_st_1_6.pg
What is the cardinality of each of the following sets?
(a) \( \emptyset \) __________
(b) \( \{\emptyset\} \) __________
(c) \( \{\emptyset, \{\emptyset\}\} \) __________
(d) \( \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \) __________

7. (1 pt) setSetTheory1/ur_st_1_7.pg
\( A = \{1, 3, 5\} \), \( B = \{2, 3\} \)
Check ALL of the following Cartesian products to which the following elements belong:
(a) \( 1, 2 \) __________
- A. \( A \times A \)
- B. \( B \times B \)
- C. \( A \times B \)
- D. \( B \times A \)
(b) \( 3, 1 \) __________
- A. \( A \times B \)
- B. \( B \times A \)
- C. \( B \times B \)
- D. \( A \times A \)
(c) \( 1, 1 \) __________
- A. \( A \times A \)
- B. \( A \times B \)
8. (1 pt) setSetTheory1/ur_st_1_8.pg
A = \{1, 3, 5\}, B = \{2, 3\}
Check ALL elements of the following sets:
(a) A \cap B
- A. 3
- B. 1
- C. 5
- D. 2
- E. 4
(b) A \cup B
- A. 2
- B. 5
- C. 4
- D. 3
- E. 1
(c) A \setminus B
- A. 2
- B. 5
- C. 3
- D. 1
- E. 4

9. (1 pt) setSetTheory1/ur_st_1_9.pg
Complete the following membership table by filling in the blanks with 1 or 0 as appropriate.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>B \setminus A</th>
<th>A \cup B</th>
<th>A \cap (B - A)</th>
<th>A \cup (B - A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Use the membership table above to answer the following questions.
For each part, check the answer that most completely describes the general situation.
(1) A \setminus B
- A. A
- B. A \subseteq B
- C. A = A \setminus B
- D. A \subseteq A
(2) A \cap (B - A)
- A. \emptyset
- B. A \cap B
- C. A
- D. B

10. (1 pt) setSetTheory1/ur_st_1_10.pg
Complete the following membership table by filling in the blanks with 1 or 0 as appropriate.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A \cap B</th>
<th>A \cup B</th>
<th>(A \cap B) \cup (A \cap B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Check the statement above that MOST COMPLETELY describes the relationship between the two sets:
- A. A \subseteq (A \cap B) \cup (A \cap B)
- B. (A \cap B) \cup (A \cap B) \subseteq A
- C. A \subseteq (A \cap B) \cup (A \cap B)
- D. (A \cap B) \cup (A \cap B) \subseteq A
- E. (A \cap B) \cup (A \cap B) = A

11. (1 pt) setSetTheory1/ur_st_1_11.pg
Suppose that the universal set is U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.
Express each of the following subsets with bit strings (of length 10) where the ith bit (from left to right) is 1 if i is in the subset and zero otherwise.
(a) 3, 4, 5 _____
(b) 1, 3, 6, 10 _____
(c) 2, 3, 4, 7, 8, 9 _____

12. (1 pt) setSetTheory1/ur_st_1_12.pg
A = \{1, 3, 5\}, B = \{2, 3\}
Check ALL elements of the following sets:
(a) A \cap B
- A. 3
- B. 5
- C. 4
- D. 1
- E. 2
(b) A \cup B
- A. 4
- B. 3
- C. 2
- D. 5
- E. 1
(c) A \setminus B
- A. 3
- B. 4
- C. 5
- D. 2
- E. 1
(d) The Symmetric difference of $A$ and $B$, denoted by $A \oplus B$, is the set containing those elements in either $A$ or $B$, but NOT in both. 
Check all elements below that are in $A \oplus B$.
- A. 3
- B. 2
- C. 4
- D. 5
- E. 1

13. (1 pt) setSetTheory1/ar_dis_11-1.pg
There are 370 students in a college who have taken a course in calculus, 216 who have take a course in discrete mathematics, and 162 who have taken a course in both calculus and discrete mathematics. How many students at this college have taken a course in either calculus or discrete mathematics?

14. (1 pt) setSetTheory1/ar_dis_11-2.pg
Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 105 elements in $A_1$, 1007 elements in $A_2$ and 9989 elements in $A_3$ in each of the following situations:

(a) The sets are pairwise disjoint.
(b) $A_1 \subseteq A_2 \subseteq A_3$.
(c) There are 13 elements common to each pair of sets and 2 elements in all three sets.

15. (1 pt) setSetTheory1/ar_dis_11-3.pg
In a survey of 268 college students, it is found that 61 like brussels sprouts, 99 like broccoli, 58 like cauliflower, 25 like both brussels sprouts and broccoli, 24 like both brussels sprouts and cauliflower, 25 like both broccoli and cauliflower and 14 of the students like all three vegetables. How many of the 268 college students do not like any of these three vegetables?

16. (1 pt) setSetTheory1/ar_dis_11-4.pg
How many elements are in the union of four sets if each of the sets has 96 elements, each pair of sets share 45 elements, each triple of sets shares 20 elements and there are 6 elements in all four sets.
1. (1 pt) setSetTheory2Fuzzy/ur_dis_1_6.pg

Fuzzy Logic is used in artificial intelligence. In fuzzy logic, a proposition has a truth value that is a number between 0 and 1 inclusive. A proposition with a truth value of 0 is false and one with truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.75 can be assigned to the statement “Fred is happy,” since Fred is happy most of the time, and the truth value 0.45 can be assigned to the statement “John is happy,” since John is happy slightly less than half the time.

The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. The truth value of a conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions.

What are the truth value of the statements:
(a) “Fred and John are happy.”
(b) “Neither Fred nor John is happy.”

2. (1 pt) setSetTheory2Fuzzy/ur_st_2_1.pg

Fuzzy sets are used in artificial intelligence. Each element in the universal set $U$ has a degree of membership, which is a real number between 0 and 1 (including 0 and 1 as possibilities), in a fuzzy set $S$. The fuzzy set $S$ is denoted by listing the elements with their degrees of membership (elements with 0 degree of membership are not listed).

For example, we write $F = \{0.75 Alice, 0.85 Brian, 0.55 Rita, 0.05 Oscar\}$ for the (fuzzy) set $F$ of famous people to indicate that Alice has a 0.75 degree membership to $F$, that Brian has a 0.85 membership to $F$ and so on. (For example Brian is the most famous of these people while Oscar is the least famous.)

Also suppose that $R$ is the (fuzzy) set of rich people given by $R = \{0.3 Alice, 0.55 Brian, 0.15 Rita, 0.55 Oscar, 0.05 Fred\}$. The complement of a fuzzy set $S$ is the fuzzy set $\overline{S}$, where the degree of membership of an element in $\overline{S}$ is 1 minus the degree of membership of that element in $S$.

Thus for example we have:

$\overline{F} = Alice, \overline{Brian}, \overline{Rita}, \overline{Oscar}, \overline{Fred}$.

The intersection of two fuzzy sets $S$ and $T$ is the fuzzy set $S \cap T$, where the degree of membership of an element in $S \cap T$ is the minimum of the degrees of membership of this element in $S$ and in $T$. Thus the fuzzy set $F \cap R$ of the rich and famous people is:

$F \cap R = Alice, \overline{Brian}, \overline{Rita}, \overline{Oscar}, \overline{Fred}$. 
1. (1 pt) setTrigonometry1/ur_tr_1_1.pg
For each of the following angles, find the degree measure of the angle with the given radian measure:

\[
\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, 2\pi
\]

2. (1 pt) setTrigonometry1/ur_tr_1_1a.pg
For each of the following angles, find the degree measure of the angle with the given radian measure:

\[
-\frac{3\pi}{8}, -\frac{3\pi}{4}, -\frac{3\pi}{2}, 0\pi, 0\pi
\]

3. (1 pt) setTrigonometry1/ur_tr_1_2.pg
For each of the following angles, find the radian measure of the angle with the given degree measure (you can enter \(\pi\) as 'pi' in your answers):

\[
-290, -170, 280, -400, -320
\]

4. (1 pt) setTrigonometry1/ur_tr_1_3.pg
For each of the following angles (in radian measure), find the sine of the angle (your answer cannot contain trig functions, it must be an arithmetic expression or number):

\[
\pi
\]

5. (1 pt) setTrigonometry1/ur_tr_1_3a.pg
For each of the following angles (in radian measure), find the cosine of the angle (your answer cannot contain trig functions, it must be an arithmetic expression or number):

\[
\pi
\]

6. (1 pt) setTrigonometry1/ur_tr_1_4.pg
Convert \(\frac{17\pi}{6}\) to degrees:

Convert 889° to radians:

\(\pi\)
\[
\tan(\theta) = \ldots \\
\sec(\theta) = \ldots
\]

15. (1 pt) setTrigonometry1/tr_J_7a.pg
If \( \cos(\theta) = \frac{2}{\sqrt{5}}, \) \( 0 \leq \theta \leq \pi/2, \) then
- \( \sin(\theta) = \ldots \)
- \( \tan(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)

16. (1 pt) setTrigonometry1/tr_J_7b.pg
If \( \tan(\theta) = \frac{3}{2}, \) \( 0 \leq \theta \leq \pi/2, \) then
- \( \sin(\theta) = \ldots \)
- \( \cos(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)

17. (1 pt) setTrigonometry1/tr_J_7c.pg
If \( \sec(\theta) = 5, \) \( 0 \leq \theta \leq \pi/2, \) then
- \( \sin(\theta) = \ldots \)
- \( \cos(\theta) = \ldots \)
- \( \tan(\theta) = \ldots \)
- \( \cot(\theta) = \ldots \)

18. (1 pt) setTrigonometry1/tr_J_8.pg
If \( \sin(\theta) = -\frac{2}{\sqrt{5}}, \) and \( \theta \) is in quadrant IV, then find
- \( \cos(\theta) = \ldots \)
- \( \tan(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)
- \( \csc(\theta) = \ldots \)
- \( \cot(\theta) = \ldots \)

19. (1 pt) setTrigonometry1/tr_J_9.pg
If \( \tan(\theta) = -\frac{3}{2} \) and \( \sin(\theta) > 0, \) then find
- \( \sin(\theta) = \ldots \)
- \( \cos(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)
- \( \csc(\theta) = \ldots \)
- \( \cot(\theta) = \ldots \)

20. (1 pt) setTrigonometry1/tr_J_10.pg
For each angle below, determine the quadrant in which the terminal side of the angle is found and find the corresponding reference angle.

NOTE 1: Enter ‘1’ for quadrant I, ‘2’ for quadrant II, ‘3’ for quadrant III, and ‘4’ for quadrant IV.

NOTE 2: You can enter \( \pi \) as ‘pi’ in your answers.

\( \theta = \frac{3\pi}{4}, \) \( \hat{\theta} = \ldots \)
- \( \theta = \frac{4\pi}{3}, \) \( \hat{\theta} = \ldots \)
- \( \theta = \frac{5\pi}{4}, \) \( \hat{\theta} = \ldots \)

21. (1 pt) setTrigonometry1/tr_J_11.pg
If \( \cos(\theta) = -\frac{6}{11}, \) \( \theta \) is in quadrant III, then find
- \( \tan(\theta) = \ldots \)
- \( \csc(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)

22. (1 pt) setTrigonometry1/tr_J_12.pg
For \( 0 < \theta < \pi/2, \) find the values of the trigonometric functions based on the given one (give your answers with THREE DECIMAL PLACES or as expressions, e.g. you can enter 3/5).

- \( \cos(\theta) = \frac{5}{3}, \) then
  - \( \sin(\theta) = \ldots \)
  - \( \sec(\theta) = \ldots \)
  - \( \csc(\theta) = \ldots \)
  - \( \tan(\theta) = \ldots \)
  - \( \cot(\theta) = \ldots \)

23. (1 pt) setTrigonometry1/tr_J_12a.pg
For \( 0 < \theta < \pi/2, \) find the values of the trigonometric functions based on the given one (give your answers with THREE DECIMAL PLACES or as fractions, e.g. you can enter 3/5).

\( \csc(\theta) = \frac{5}{3}, \) then
- \( \cos(\theta) = \ldots \)
- \( \sin(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)
- \( \tan(\theta) = \ldots \)
- \( \cot(\theta) = \ldots \)

24. (1 pt) setTrigonometry1/tr_J_12b.pg
For \( 0 < \theta < \pi/2, \) find the values of the trigonometric functions based on the given one (give your answers with THREE DECIMAL PLACES or as fractions, e.g. you can enter 3/5).

\( \tan(\theta) = \frac{5}{3}, \) then
- \( \cot(\theta) = \ldots \)
- \( \cos(\theta) = \ldots \)
- \( \sin(\theta) = \ldots \)
- \( \sec(\theta) = \ldots \)
- \( \csc(\theta) = \ldots \)

25. (1 pt) setTrigonometry1/tr_J_13.pg
Find an angle between \( 0 \) and \( 2\pi \) that is coterminal with the given angle. (Note: You can enter \( \pi \) as ‘pi’ in your answers.)

- \( \frac{7\pi}{6}, \) \( \hat{\theta} = \ldots \)
- \( \frac{13\pi}{6}, \) \( \hat{\theta} = \ldots \)
- \( \frac{15\pi}{6}, \) \( \hat{\theta} = \ldots \)
- \( \frac{17\pi}{6}, \) \( \hat{\theta} = \ldots \)
1. (1 pt) setTrigonometry2Waves/mec_4_4.pg

Let \( y = -\sqrt{24} \sin(5\pi x + 8e^{-4}) \).
What is the amplitude? _______
What is the period? _______
What is the phase shift? _______
[NOTE: If needed, you can enter \( \pi \) as 'pi' in your answers.]

2. (1 pt) setTrigonometry2Waves/mec_4_6.pg

Let \( y = 4\cos[6(x - \frac{\pi}{2})] \).

What is the amplitude? _______
What is the period? _______
What is the phase shift? _______
[NOTE: If needed, you can enter \( \pi \) as 'pi' in your answers.]

3. (1 pt) setTrigonometry2Waves/mec_4_7.pg

Let \( y = -3 \sin(5x - 5) \).
What is the amplitude? _______
What is the period? _______
What is the phase shift? _______
[NOTE: If needed, you can enter \( \pi \) as 'pi' in your answers.]
1. (1 pt) setTrigonometry3WordProblems/srw6_2_41.pg
The angle of elevation to the top of a building is found to be 11° from the ground at a distance of 5000 feet from the base of the building. Find the height of the building.

2. (1 pt) setTrigonometry3WordProblems/srw6_2_42.pg
A plane is flying at an elevation of 22000 feet. It is within sight of the airport and the pilot finds that the angle of depression to the airport is 14°.
Find the distance between the plane and the airport.
Find the distance between a point on the ground directly below the plane and the airport.

3. (1 pt) setTrigonometry3WordProblems/srw6_2_44.pg
The captain of a ship at sea sights a lighthouse which is 180 feet tall. The captain measures the the angle of elevation to the top of the lighthouse to be 22°. How far is the ship from the base of the lighthouse?

4. (1 pt) setTrigonometry3WordProblems/srw6_2_54.pg
A hot-air balloon is floating above a straight road. To calculate their height above the ground, the balloonists simultaneously measure the angle of depression to two consecutive mileposts on the road on the same side of the balloon. The angles of depression are found to be 14° and 17°. How high (in feet) is the balloon?

5. (1 pt) setTrigonometry3WordProblems/srw6_2_54a.pg
A hot-air balloon is floating above a straight road. To calculate their height above the ground, the balloonists simultaneously measure the angle of depression to two consecutive mileposts on the road on the same side of the balloon. The angles of depression are found to be 16° and 20°. How high (in feet) is the balloon? [NOTE: 1 mile = 5280 feet]

6. (1 pt) setTrigonometry3WordProblems/srw6_2_55.pg
A survey team is trying to estimate the height of a mountain above a level plain. From one point on the plain, they observe that the angle of elevation to the top of the mountain is 29°. From a point 1500 feet closer to the mountain along the plain, they find that the angle of elevation is 31°. How high (in feet) is the mountain?

7. (1 pt) setTrigonometry3WordProblems/srw6_2_55-sol.pg
A survey team is trying to estimate the height of a mountain above a level plain. From one point on the plain, they observe that the angle of elevation to the top of the mountain is 31°. From a point 2000 feet closer to the mountain along the plain, they find that the angle of elevation is 34°. How high (in feet) is the mountain?

8. (1 pt) setTrigonometry3WordProblems/ur_tr_3_1.pg
A 25 -ft ladder leans against a building so the the angle between the ground and the ladder is 77°. How high does the ladder reach on the building?

9. (1 pt) setTrigonometry3WordProblems/ur_tr_3_4.pg
A circular arc of length 15 feet subtends a central angle of 85 degrees. Find the radius of the circle in feet. (Note: You can enter π as 'pi' in your answer.)

10. (1 pt) setTrigonometry3WordProblems/ur_tr_3_2.pg
Find the equation of the tangent line to the curve $y = 6 \sin x$ at the point $(\pi/6, 3)$. The equation of this tangent line can be written in the form $y = mx + b$ where $m = \ldots$ and $b = \ldots$

11. (1 pt) setTrigonometry3WordProblems/ur_tr_3_3.pg
Find the equation of the tangent line to the curve $y = 2x \cos x$ at the point $(\pi, -2\pi)$. The equation of this tangent line can be written in the form $y = mx + b$ where $m = \ldots$ and $b = \ldots$
1. (1 pt) setTrigonometry4Inverse/srw7_6_1-8a.pg
Evaluate the following expressions. Your answer must be an angle $-\pi/2 \leq \theta \leq \pi$ in radians.
- $\sin^{-1}(\frac{\sqrt{2}}{2})$
- $\sin^{-1}(\frac{1}{2})$
- $\cos^{-1}(\frac{\sqrt{2}}{2})$
- $\cos^{-1}(\frac{-\sqrt{2}}{2})$

2. (1 pt) setTrigonometry4Inverse/srw7_6_1-8b.pg
Evaluate the following expressions. Your answer must be an angle in radians and in the interval $[-\pi, \pi].$
- (a) $\sin^{-1}(1)$
- (b) $\sin^{-1}(\frac{-1}{2})$
- (c) $\sin^{-1}(\frac{\sqrt{3}}{2})$

3. (1 pt) setTrigonometry4Inverse/srw7_6_1-8c.pg
Evaluate the following expressions. Your answer must be in radians.
- (a) $\tan^{-1}(\sqrt{3})$
- (b) $\tan^{-1}(-1)$
- (c) $\tan^{-1}(0)$

4. (1 pt) setTrigonometry4Inverse/srw7_6_11.pg
Evaluate the following expressions.
- $\sin(\sin^{-1}(\frac{\sqrt{3}}{2}))$
- $\cos(\cos^{-1}(\frac{\sqrt{3}}{2}))$
- $\tan(\tan^{-1}(\frac{\sqrt{3}}{2}))$

5. (1 pt) setTrigonometry4Inverse/srw7_6_11a.pg
Evaluate the following expressions.
- (a) $\sin(\sin^{-1}(1))$
- (b) $\tan(\tan^{-1}(\frac{-\sqrt{3}}{2}))$

6. (1 pt) setTrigonometry4Inverse/srw7_6_12-16a.pg
Evaluate the following expressions.
- $\cos(\sin^{-1}(\frac{\sqrt{3}}{2}))$
- $\tan(\sin^{-1}(\frac{1}{2}))$

7. (1 pt) setTrigonometry4Inverse/srw7_6_12-16b.pg
Evaluate the following expressions.
- $\sin(\cos^{-1}(\frac{1}{2}))$
- $\tan(\cos^{-1}(\frac{\sqrt{3}}{2}))$

8. (1 pt) setTrigonometry4Inverse/srw7_6_12-16c.pg
Evaluate the following expressions.
- $\sin(\arctan(\frac{1}{2}))$
- $\cos(\arctan(\frac{\sqrt{3}}{2}))$

9. (1 pt) setTrigonometry4Inverse/srw7_6_25.pg
Evaluate the following expressions.
- $\sin(\sin^{-1}(\frac{1}{2}))$
- $\tan(\sin^{-1}(\frac{\sqrt{3}}{2}))$

10. (1 pt) setTrigonometry4Inverse/tr4_1.pg
Match each of the trigonometric expressions below with the equivalent non-trigonometric function from the following list. Enter the appropriate letter (A,B,C,D, or E) in each blank.
- A. $\tan(\arcsin(x/7))$
- B. $\cos(\arcsin(x/7))$
- C. $(1/2)\sin(2\arcsin(x/7))$
- D. $\sin(\arctan(x/7))$
- E. $\cos(\arctan(x/7))$

11. (1 pt) setTrigonometry4Inverse/osu_tr4_2.pg
Simplify the expression

$\tan\left(2\cos^{-1}(x/4)\right)$

answer = ____________
The next three problems deal with two new functions. The first is called the HYPERBOLIC SINE FUNCTION and is denoted as sinh(x).

The second is called the HYPERBOLIC COSINE FUNCTION and is denoted as cosh(x).

These two functions are both defined using either the difference or sum of exponential functions and then dividing by 2:

\[
\begin{align*}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2}
\end{align*}
\]

1. (1 pt) setTrigonometry5Hyperbolic/srw4_1_33.pg

\[
\sinh(0) = \underline{\phantom{0.00}}
\]

2. (1 pt) setTrigonometry5Hyperbolic/srw4_1_35.pg

\[
\begin{align*}
(a) \sinh(5) &= \underline{\phantom{0.00}} \\
(b) \sinh(-5) &= \underline{\phantom{0.00}} \\
(c) \cosh(5) &= \underline{\phantom{0.00}} \\
(d) \cosh(-5) &= \underline{\phantom{0.00}}
\end{align*}
\]

3. (1 pt) setTrigonometry5Hyperbolic/srw4_1_37.pg

\[
\begin{align*}
(a) \cosh^2(-10) - \sinh^2(-10) &= \underline{\phantom{0.00}} \\
(b) \cosh^2(-1) - \sinh^2(-1) &= \underline{\phantom{0.00}} \\
(c) \cosh^2(2) - \sinh^2(2) &= \underline{\phantom{0.00}} \\
(d) \sinh^2(9) - \cosh^2(9) &= \underline{\phantom{0.00}}
\end{align*}
\]

[Note: sinh^2(x) is defined as (sinh(x))^2 and cosh^2(x) is defined as (cosh(x))^2]
Given any Cartesian coordinates, \((x, y)\), there are polar coordinates \((r, \theta)\) with \(-\pi < \theta \leq \pi\),

Find polar coordinates with \(-\pi < \theta \leq \frac{\pi}{2}\) for the following Cartesian coordinates:

(a) If \((x, y) = (5, 0)\) then \((r, \theta) = (\ldots, \ldots)\).
(b) If \((x, y) = (8, 0)\) then \((r, \theta) = (\ldots, \ldots)\).
(c) If \((x, y) = (-2, -6)\) then \((r, \theta) = (\ldots, \ldots)\).
(d) If \((x, y) = (5, -10)\) then \((r, \theta) = (\ldots, \ldots)\).
(e) If \((x, y) = (-2, 6)\) then \((r, \theta) = (\ldots, \ldots)\).
(f) If \((x, y) = (0, -8)\) then \((r, \theta) = (\ldots, \ldots)\).

For each set of Polar coordinates \((r, \theta)\), match the equivalent Cartesian coordinates \((x, y)\).

Don’t use a calculator.

1. \((-2, \frac{5\pi}{4})\)
2. \((6, \frac{\pi}{2})\)
3. \((2, \frac{7\pi}{4})\)
4. \((2, \frac{9\pi}{4})\)
5. \((5, -\frac{\pi}{2})\)
6. \((2, \frac{\pi}{4})\)
   - A. \((2.5, -2.5\sqrt{3})\)
   - B. \((-1\sqrt{3}, -1)\)
   - C. \((1\sqrt{2}, 1\sqrt{2})\)
   - D. \((1\sqrt{2}, 1\sqrt{2})\)
   - E. \((1\sqrt{3}, -1)\)
   - F. \((3, 3\sqrt{3})\)

Decide if the points given in polar coordinates are the same. If so, enter T. If not, enter F.

1. \((2, \frac{\pi}{4}), (-2, -\frac{\pi}{4})\)
2. \((2, \frac{3\pi}{4}), (2, -\frac{\pi}{4})\)
3. \((0, 2\pi), (0, \frac{4\pi}{3})\)
4. \((1, \frac{4\pi}{3}), (-1, \frac{2\pi}{3})\)
5. \((2, \frac{18\pi}{7}), (-2, -\frac{\pi}{7})\)
6. \((2, 2\pi), (-2, 2\pi)\)

For each set of Cartesian coordinates \((x, y)\), match the equivalent set of Polar coordinates \((r, \theta)\), with \(-\pi \leq \theta \leq \pi\)

1. \((7.7, 1.3)\)
2. \((-3.6, 7.1)\)
3. \((-7.5, 8.7)\)
4. \((1.3, 1.9)\)
A. \((7.9605, -1.1015)\)
B. \((7.8090, 0.1673)\)
C. \((2.3022, 0.9707)\)
D. \((11.4865, -0.8593)\)

For each set of Polar coordinates, match the equivalent Cartesian coordinates.

1. \((-6.6287, -0.1974)\)
2. \((1.9209, 0.6747)\)
3. \((7.5802, 0.1456)\)
4. \((-4.0817, -0.5404)\)
5. \((9.6047, 0.6747)\)
6. \((7.1197, 0.1836)\)
A. \((7.1, 1.3)\)
B. \((-6.5, 1.3)\)
C. \((7.5, 1.1)\)
D. \((7.5, 6)\)
E. \((1.5, 1.2)\)
F. \((-3.5, 2.1)\)
1. (1 pt) setPolarCoord2Curves/ur_pc_2_1.png
A curve in polar coordinates is given by: \( r = 9 + 5 \cos \theta \).
Point \( P \) is at \( \theta = \frac{3\pi}{18} \).
(1) Find polar coordinate \( r \) for \( P \), with \( r > 0 \) and \( \pi < \theta < \frac{3\pi}{2} \).
(2) Find cartesian coordinates for point \( P \).
\( x = \text{ } \) \( y = \text{ } \)
(3) How many times does the curve pass through the origin when \( 0 < \theta < 2\pi \)?

2. (1 pt) setPolarCoord2Curves/ur_pc_2_2.png
A curve with polar equation
\[ r = \frac{17}{9 \sin \theta + 64 \cos \theta} \]
represents a line. This line has a Cartesian equation of the form
\( y = mx + b \), where \( m \) and \( b \) are constants. Give the formula for \( y \) in terms of \( x \).
For example, if the line had equation \( y = 2x + 3 \) then the answer would be \( 2x + 3 \).

3. (1 pt) setPolarCoord2Curves/ur_pc_2_3.png
A circle \( C \) has center at the origin and radius 4. Another circle \( K \) has a diameter with one end at the origin and the other end at the point \((0, 15)\). The circles \( C \) and \( K \) intersect in two points. Let \( P \) be the point of intersection of \( C \) and \( K \) which lies in the first quadrant. Let \((r, \theta)\) be the polar coordinates of \( P \), chosen so that \( r \) is positive and \( 0 \leq \theta \leq 2 \). Find \( r \) and \( \theta \).
\( r = \text{ } \) \( \theta = \text{ } \)

4. (1 pt) setPolarCoord2Curves/ur_pc_2_4.png
Match each polar equation below to the best description. Possible answers are C, E, F, H, L, P, and S.

DESCRIPTIONS
C. Circle,
E. Ellipse,
F. Figure eight,
H. Hyperbola,
L. Line,
P. Parabola,
S. Spiral

POLAR EQUATIONS
\[ 1. \quad r = \frac{1}{8 \sin \theta + 16 \cos \theta} \]
\[ 2. \quad r = \frac{1}{16 + 8 \cos \theta} \]
\[ 3. \quad r = \frac{1}{8 \sin \theta + 16 \cos \theta} \]
\[ 4. \quad r = \frac{1}{8 + 16 \cos \theta} \]
\[ 5. \quad r = \frac{1}{8 + 8 \cos \theta} \]

5. (1 pt) setPolarCoord2Curves/ur_pc_2_5.png
Match each polar equation below to the best description. Each answer should be C, F, I, L, M, O, or T.

DESCRIPTIONS
C. Cardioid,
F. Rose with four petals,
I. Inwardly spiraling spiral,
L. Lemacon,
M. Lemniscate,
O. Outwardly spiraling spiral,
T. Rose with three petals

POLAR EQUATIONS
\[ 1. \quad r = 17 \sin 2 \theta \]
\[ 2. \quad r = \frac{8}{r}, r > 0 \]
\[ 3. \quad r^2 = 16 \cos 2 \theta \]
\[ 4. \quad r = 8 \cos 3 \theta \]
\[ 5. \quad r = 8 + 16 \cos \theta \]
\[ 6. \quad r = 80, r > 0 \]
\[ 7. \quad r = 8 - 8 \sin \theta \]

6. (1 pt) setPolarCoord2Curves/ur_pc_2_6.png
Match each polar equation below to the best description. Each answer should be C, E, F, H, L, O, P, R, S, T, or W.

DESCRIPTIONS
C. Cardioid,
E. Ellipse,
F. Lemniscate,
H. Hyperbola,
L. Line,
O. Oval,
P. Parabola,
R. Rose with four petals,
S. Spiral,
T. Three-petaled rose

POLAR EQUATIONS

1. \( r^2 = 11 \sin 2\theta \)
2. \( r = \sin 3\theta \)
3. \( 1 = \tan \theta \)
4. \( r = \sin 2\theta \)
5. \( r^2 = \csc 2\theta \)
6. \( r = 4 - 4 \sin \theta \)
7. \( r = \theta, r > 0 \)

8. (1 pt) setPolarCoord2Curves/ur_pc_2_11.png
Match each polar equation below to the best description. Each answer should be C, F, I, L, M, O, or T.

DESCRIPTIONS
C. Cardioid,
F. Rose with four petals,
I. Inwardly spiraling spiral,
L. Lemacon,
M. Lemniscate,
O. Outwardly spiraling spiral,
T. Rose with three petals

POLAR EQUATIONS

1. \( r = 2\theta, r > 0 \)
2. \( r^2 = 4 \cos 2\theta \)
3. \( r = 15 \sin 2\theta \)
4. \( r = 2 - 2 \sin \theta \)
5. \( r = 2 \cos 3\theta \)
6. \( r = 15 + 15 \cos \theta \)

9. (1 pt) setPolarCoord2Curves/ur_pc_2_4.png
Find the length of the spiraling polar curve

\[ r = -5e^{2\theta} \]

from 0 to 2\( \pi \).

Length =

10. (1 pt) setPolarCoord2Curves/ur_pc_2_13.png
Find the area inside one leaf of the rose:

\[ r = 3 \sin (5\theta) \]

The area is

11. (1 pt) setPolarCoord2Curves/ur_pc_2_12.png
Find the area of the region which is bounded by the polar curves

\[ \theta = \pi \]
and

\[ r = 1\theta \]
for \( 0 \leq \theta \leq 1.5\pi \)

The area is

12. (1 pt) setPolarCoord2Curves/ur_pc_2_9.png
Find the area of the region which is inside the polar curve

\[ r = 3 \cos (\theta) \]
and outside the curve

\[ r = 2 - 1 \cos (\theta) \]

The area is

13. (1 pt) setPolarCoord2Curves/ur_pc_2_10.png
Find the length of the spiraling polar curve

\[ r = 4e^{2\theta} \]
From 0 to 2\( \pi \).

The length is
1. (1 pt) setParametric1Curves/ur_pa_1_1.pg
Assume time $t$ runs from zero to $2\pi$ and that the unit circle has been labeled as a clock.
Match each of the pairs of parametric equations with the best description of the curve from the following list. Enter the appropriate letter (A, B, C, D, E, F) in each blank.
A. Starts at 12 o'clock and moves clockwise one time around.
B. Starts at 6 o'clock and moves clockwise one time around.
C. Starts at 3 o'clock and moves clockwise one time around.
D. Starts at 9 o'clock and moves clockwise one time around.
E. Starts at 3 o'clock and moves counterclockwise two times around.
F. Starts at 3 o'clock and moves counterclockwise to 9 o'clock.

2. (1 pt) setParametric1Curves/ur_pa_1_2.pg
Suppose parametric equations for the line segment between $(3, 9)$ and $(1, -2)$ have the form:

$$x = a + bt$$
$$y = c + dt$$

If the parametric curve starts at $(3, 9)$ when $t = 0$ and ends at $(1, -2)$ at $t = 1$, then find $a$, $b$, $c$, and $d$.

3. (1 pt) setParametric1Curves/ur_pa_1_3.pg
Eliminate the parameter $t$ to find a Cartesian equation for

$$x = -3 - t$$
$$y = 17 - 3t$$

The Cartesian equation has the form

$$y = mx + b$$

where $m = \ldots$ and $b = \ldots$

4. (1 pt) setParametric1Curves/ur_pa_1_4.pg
Find the equation for the line tangent to the parametric curve:

$$x = t^3 - 25t$$
$$y = 25t^2 - t^4$$

at the points where $t = 5$ and $t = -5$.
For $t = 5$, the tangent line (in form $y = mx + b$) is $y = \ldots$
For $t = -5$, the tangent line is $y = \ldots$

5. (1 pt) setParametric1Curves/ur_pa_1_5.pg
Find $\frac{d^2y}{dx^2}$ as a function of $t$, for the given parametric equations:

$$x = 2 - 6\cos(t)$$
$$y = 4 + \cos^3(t)$$

$\frac{d^2y}{dx^2} = \ldots$

6. (1 pt) setParametric1Curves/ur_pa_1_6.pg
Notice that the curve given by the parametric equations

$$x = 16 - t^2$$
$$y = t^3 - 9t$$
is symmetric about the $x$-axis. (If $t$ gives us the point $(x, y)$, then $-t$ will give $(x, -y)$).
At which $x$ value is the tangent to this curve horizontal? $x = \ldots$
At which $t$ value is the tangent to this curve vertical? $t = \ldots$
The curve makes a loop which lies along the $x$-axis. What is the total area inside the loop?
Area = \ldots

7. (1 pt) setParametric1Curves/ur_pa_1_7.pg
(a) Find $\frac{dy}{dx}$ as a function of $t$ for the given parametric equations.

$$x = t - t^2$$
$$y = -3 - 8t$$

$\frac{dy}{dx} = \ldots$

(b) Find $\frac{dy}{dx}$ as a function of $t$ for the given parametric equations.

$$x = 6t + 3$$
$$y = t^2 - 9t$$

$\frac{dy}{dx} = \ldots$

8. (1 pt) setParametric1Curves/ur_pa_1_8.pg
The following parametric equations trace out a loop.

$$x = 5 - \frac{4}{5}t^2$$
$$y = -\frac{4}{5}t^3 + 4t + 1$$

Find the $t$ values at which the curve intersects itself: $t = \pm \ldots$
What is the total area inside the loop?
Area = \ldots

9. (1 pt) setParametric1Curves/ur_pa_1_9.pg
(a) Find $\frac{dy}{dx}$ expressed as a function of $t$ for the given parametric equations:

$$x = \cos^9(t)$$
$$y = 6 \sin^5(t)$$

$\frac{dy}{dx} = \ldots$
\[ \frac{dy}{dx} = \] 

(b) Find \( \frac{d^2y}{dx^2} \) expressed as a function of \( t \). 

\[ \frac{d^2y}{dx^2} = \] 

(c) Except for at the points where \( \frac{dy}{dx} \) is undefined, is the curve concave up or concave down? (Enter 'up' or 'down'). Concave \( \underline{\quad} \) 

10. (1 pt) setParametricCurves/ur_pa_J_10.pg

The ellipse

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

can be drawn with parametric equations. If 

\[ x = r \cos(t) \]

then \( r = \underline{\quad} \) and \( y = \underline{\quad} \)

11. (1 pt) setParametricCurves/ur_pa_J_11.pg

A bicycle wheel has radius \( R \). Let \( P \) be a point on the spoke of a wheel at a distance \( d \) from the center of the wheel. The wheel begins to roll to the right along the the x-axis. The curve traced out by \( P \) is given by the following parametric equations:

\[ x = 17t - 10 \sin(t) \]
\[ y = 17 - 10 \cos(t) \]

What must we have for \( R \) and \( d \)?

\( R = \underline{\quad} \) and \( d = \underline{\quad} \)

12. (1 pt) setParametricCurves/ur_pa_J_12.pg

Eliminate the parameter \( t \) to find a Cartesian equation for:

\[ x = t^2 \]
\[ y = 9 + 1t \]

\[ x = Ay^2 + By + C \]

where

\( A = \underline{\quad} \) and \( B = \underline{\quad} \) and \( C = \underline{\quad} \)


Assume \( t \) is defined for all time. Enter the letter of the graph below which corresponds to the curve traced by the parametric equations. Think about the range of \( x \) and \( y \), and whether there is periodicity and or symmetry.

1. \( x = \frac{t^4}{4} - t + 1; y = \frac{t^2}{4} - 1 \)
2. \( x = t + \cos(10t); y = t^2 + \sin(t) \)
3. \( x = \lvert \cos(t) \rvert \cos(t); y = \lvert \sin(t) \rvert + \sin(t) \)
4. \( x = t + \sin(5t); y = t + \cos(5t) \)
5. \( x = \sin(t)(3 - 2 \sin(t)); y = \cos(t)(3 - 2 \sin(t)) \)

14. (1 pt) setParametricCurves/ur_pa_J_14.pg

Suppose a curve is traced by the parametric equations

\[ x = 2 \sin(t) \]
\[ y = 39 - 16 \cos^2(t) - 32 \sin(t) \]

At what point \((x,y)\) on this curve is the tangent line horizontal? \( x = \underline{\quad} \) \( y = \underline{\quad} \)

15. (1 pt) setParametricCurves/ur_pa_J_15.pg

Notice that the curve given by the parametric equations

\[ x = 36 - t^2 \]
\[ y = t^3 - 4t \]

is symmetric about the x-axis. (If \( t \) gives us the point \((x,y)\), then \(-t\) will give \((x,-y)\)).

At which \( x \) value is the tangent to this curve horizontal? At \( x = \underline{\quad} \)
At which \( t \) value is the tangent to this curve vertical? At \( t = \underline{\quad} \)

The curve makes a loop which lies along the x-axis. What is the total area inside the loop? Area = \underline{\quad}
### 1. (1 pt) setLimitsRates0Theory/c3s1p1.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

A good technique is to think of several examples, especially examples which might show that the statement is false!

For reference you can find some definitions [here](#).

You must get all of the answers correct to receive credit.

<table>
<thead>
<tr>
<th>Number</th>
<th>Statement</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>1.</em></td>
<td>The sequence of rational numbers 3.1, 3.14, 3.141, 3.14159, ... which approximates the ratio of the circumference of a circle and its diameter, has a limit point but it is not a rational number.</td>
<td>F</td>
</tr>
<tr>
<td><em>2.</em></td>
<td>The sequence 1, 2, 3, 4, ... has no finite limit.</td>
<td>F</td>
</tr>
<tr>
<td><em>3.</em></td>
<td>The sequence of rational numbers 3.1, 3.14, 3.141, 3.14159, ... which approximates the ratio of the circumference of a circle and its diameter, has a rational number as its limit point.</td>
<td>T</td>
</tr>
<tr>
<td><em>4.</em></td>
<td>The sequence 1, 2, 3, 4, ... has a finite accumulation point.</td>
<td>T</td>
</tr>
</tbody>
</table>

### 2. (1 pt) setLimitsRates0Theory/c3s1p2.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

A good technique is to think of several examples, especially examples which might show that the statement is false!

For reference you can find some definitions [here](#).

You must get all of the answers correct to receive credit.

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<td><em>1.</em></td>
<td>Every function on the interval $[-2, 1]$ must have both a maximum and a minimum.</td>
<td>T</td>
</tr>
<tr>
<td><em>2.</em></td>
<td>Every differentiable function on the interval $(1, 5]$ must have both a maximum and a minimum.</td>
<td>F</td>
</tr>
<tr>
<td><em>3.</em></td>
<td>Every differentiable function on the interval $[4, 5]$ must have a minimum.</td>
<td>T</td>
</tr>
<tr>
<td><em>4.</em></td>
<td>Every continuous function on the interval $[-4, 0]$ must have a maximum.</td>
<td>F</td>
</tr>
</tbody>
</table>

### 3. (1 pt) setLimitsRates0Theory/c3s1p3.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

A good technique is to think of several examples, especially examples which might show that the statement is false!

For reference you can find some definitions [here](#).

You must get all of the answers correct to receive credit.

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<td>Every differentiable function has a maximum value.</td>
<td>T</td>
</tr>
<tr>
<td><em>2.</em></td>
<td>If a continuous function $f(x)$ has a maximum value on an interval then the function $-f(x)$ has a minimum on that same interval.</td>
<td>T</td>
</tr>
</tbody>
</table>

### 4. (1 pt) setLimitsRates0Theory/c3s1p4.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

A good technique is to think of several examples, especially examples which might show that the statement is false!

For reference you can find some definitions [here](#).

You must get all of the answers correct to receive credit.

<table>
<thead>
<tr>
<th>Number</th>
<th>Statement</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>1.</em></td>
<td>Every continuous function whose domain is a bounded, closed interval and which has a maximum value also has a minimum value.</td>
<td>F</td>
</tr>
<tr>
<td><em>2.</em></td>
<td>If a continuous function $f(x)$ has a maximum value on an interval then the function $-f(x)$ has a minimum on that same interval.</td>
<td>T</td>
</tr>
<tr>
<td><em>3.</em></td>
<td>If the linear approximation of a differentiable function is decreasing at a point $a$ then the function could be constant near the point $a$.</td>
<td>T</td>
</tr>
<tr>
<td><em>4.</em></td>
<td>Every differentiable function whose domain is a bounded, closed interval has a maximum value.</td>
<td>F</td>
</tr>
</tbody>
</table>

### 5. (1 pt) setLimitsRates0Theory/c3s1p5.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

A good technique is to think of several examples, especially examples which might show that the statement is false!

For reference you can find some definitions [here](#).

You must get all of the answers correct to receive credit.

<table>
<thead>
<tr>
<th>Number</th>
<th>Statement</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>1.</em></td>
<td>If the linear approximation of a differentiable function is increasing at a point $a$ then the function is also increasing near the point $a$.</td>
<td>T</td>
</tr>
<tr>
<td><em>2.</em></td>
<td>If $f(x)$ is a continuous function and the sequence $a_1, a_2, a_3, ...$ converges to a finite limit, then the sequence $f(a_1), f(a_2), f(a_3), ...$ also converges to a limit.</td>
<td>F</td>
</tr>
<tr>
<td><em>3.</em></td>
<td>Every continuous function whose domain is a bounded, closed interval has a maximum value.</td>
<td>T</td>
</tr>
<tr>
<td><em>4.</em></td>
<td>If the linear approximation of a differentiable function is constant at a point $a$ then the function could be increasing near the point $a$.</td>
<td>F</td>
</tr>
<tr>
<td><em>5.</em></td>
<td>If the linear approximation of a differentiable function is constant at a point $a$ then the function could be decreasing near the point $a$.</td>
<td>T</td>
</tr>
<tr>
<td><em>6.</em></td>
<td>If a continuous function has a maximum value then it also has a minimum value.</td>
<td>F</td>
</tr>
<tr>
<td><em>7.</em></td>
<td>If a continuous function $f(x)$ has a maximum value on an interval then the function $-f(x)$ has a minimum on that same interval.</td>
<td>T</td>
</tr>
<tr>
<td><em>8.</em></td>
<td>If a differentiable function has a maximum value then it also has a minimum value.</td>
<td>T</td>
</tr>
</tbody>
</table>
1. If the tangent line to \( y = f(x) \) at \((6, 8)\) passes through the point \((1, 4)\), find
   A. \( f(6) = \) 
   B. \( f'(6) = \)

2. The point \( P(3, 17) \) lies on the curve \( y = x^2 + x + 5 \). If \( Q \) is the point \((x, x^2 + x + 5)\), find the slope of the secant line \( PQ \) for the following values of \( x \).
   If \( x = 3.1 \), the slope of \( PQ \) is: 
   and if \( x = 3.01 \), the slope of \( PQ \) is: 
   and if \( x = 2.9 \), the slope of \( PQ \) is: 
   and if \( x = 2.99 \), the slope of \( PQ \) is: 
   Based on the above results, guess the slope of the tangent line to the curve at \( P(3, 17) \).

3. The point \( P(16, 8) \) lies on the curve \( y = \sqrt{x} + 4 \). If \( Q \) is the point \((x, \sqrt{x} + 4)\), find the slope of the secant line \( PQ \) for the following values of \( x \).
   If \( x = 16.1 \), the slope of \( PQ \) is: 
   and if \( x = 16.01 \), the slope of \( PQ \) is: 
   and if \( x = 15.9 \), the slope of \( PQ \) is: 
   and if \( x = 15.99 \), the slope of \( PQ \) is: 
   Based on the above results, guess the slope of the tangent line to the curve at \( P(16, 8) \).

4. The point \( P(0.25, 8) \) lies on the curve \( y = \frac{2}{x} \). If \( Q \) is the point \((x, \frac{2}{x})\), find the slope of the secant line \( PQ \) for the following values of \( x \).
   If \( x = 0.35 \), the slope of \( PQ \) is: 
   and if \( x = 0.26 \), the slope of \( PQ \) is: 
   and if \( x = 0.15 \), the slope of \( PQ \) is: 
   and if \( x = 0.24 \), the slope of \( PQ \) is: 
   Based on the above results, guess the slope of the tangent line to the curve at \( P(0.25, 8) \).

5. If a ball is thrown straight up into the air with an initial velocity of 85 ft/s, it height in feet after \( t \) second is given by \( y = 85t - 16t^2 \). Find the average velocity for the time period beginning when \( t = 2 \) and lasting
   (i) 0.1 seconds
   (ii) 0.01 seconds
   (iii) 0.001 seconds
   Finally based on the above results, guess what the instantaneous velocity of the ball is when \( t = 2 \).

6. A ball is thrown into the air by a baby alien on a planet in the system of Alpha Centauri with a velocity of 41 ft/s. Its height in feet after \( t \) seconds is given by \( y = 41t - 28t^2 \).
   a. Find the average velocity for the time period beginning when \( t = 2 \) and lasting
      .01 s:
      .005 s:
      .002 s:
      .001 s:
   b. Estimate the instantaneous velocity when \( t = 2 \).

7. The experimental data in the table below define \( y \) as a function of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2.3</td>
<td>1</td>
<td>0.7</td>
<td>1.1</td>
<td>1.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

   a. Let \( P \) be the point \((2, 0.7)\). Find the slopes of the secant lines \( PQ \) when \( Q \) is the point of the graph with \( x \) coordinate \( x_1 \).

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Draw the graph of the function for yourself and estimate the slope of the tangent line at \( P \).

8. Below is an "oracle" function. An oracle function is a function presented interactively. When you type in an \( t \) value, and press the –f– button and the value \( f(t) \) appears in the right hand window. There are three lines, so you can easily calculate three different values of the function at one time.

   The function \( f(t) \) represents the height in feet of a ball thrown into the air, \( t \) seconds after it has been thrown.

   Calculate the velocity 0.1 seconds after the ball has been thrown.

   The velocity at 0.1 = __________ You can use a calculator

9. The position of a cat running from a dog down a dark alley is given by the values of the table.

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) (feet)</td>
<td>0</td>
<td>8</td>
<td>47</td>
<td>74</td>
<td>96</td>
<td>120</td>
</tr>
</tbody>
</table>
A. Find the average velocity of the cat (ft/sec) for the time period beginning when \( t=2 \) and lasting
a) 3 s
b) 2 s
c) 1 s

B. Draw the graph of the function for yourself and estimate the instantaneous velocity of the cat (ft/sec) when \( t=2 \)
1. (1 pt) setLimitsRates1_5Graphs/ur_jr_1-5_1.png

Let \( F \) be the function below.

Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist or is not defined.

a) \( \lim_{x \to -1^-} F(x) = \) 

b) \( \lim_{x \to 1^+} F(x) = \) 

c) \( F(-1) = \) 

d) \( F(1) = \) 

2. (1 pt) setLimitsRates1_5Graphs/ur_jr_1-5_2.png

Below is an "oracle" function. An oracle function is a function presented interactively. When you type in an \( x \) value, and press the \(-\rightarrow\) button and the value \( f(x) \) appears in the right hand window. There are three lines, so you can easily calculate three different values of the function at one time.

Determine the limits for the function \( f \) at 4.49.

\[
\begin{align*}
\lim_{x \to 4.49^-} f(4.49) &= \quad \\
\lim_{x \to 4.49^+} f(4.49) &= \\
\end{align*}
\]

Are all of these values the same?: (Y or N)

3. (1 pt) setLimitsRates1_5Graphs/ur_jr_1-5_3.png

The graphs of \( f \) and \( g \) are given above. Use them to evaluate each quantity below. Write 'DNE' if the limit or value does not exist (or if it's infinity).

\[
\begin{align*}
\lim_{x \to -4} f(x) &= \\
\lim_{x \to -4^+} f(x) &= \\
\lim_{x \to -4} g(x) &= \\
\end{align*}
\]
Evaluate the limit
\[
\lim_{x \to 3} \frac{2x^3 - 54}{x - 3}
\]
By trying values of \(x\) near 3, find the slope of the tangent line.

Evaluate the limit
\[
\lim_{x \to -7} (4x^2 + 3)(6x + 3)
\]
Evaluate the limit
\[
\lim_{x \to 2} \frac{x - 7}{6x^2 - 5x + 5}
\]
Evaluate the limit
\[
\lim_{x \to 4} 8(8x + 3)^3
\]
Evaluate the limit
\[
\lim_{x \to -3} \frac{x - 7}{6x^2 - 5x + 5}
\]
Evaluate the limit
\[
\lim_{x \to 3} \frac{7x^2 - 5x + 7}{x - 8}
\]
Evaluate the limit
\[
\lim_{y \to 3} \frac{3(y^2 - 1)}{7y^2(y - 1)^3}
\]
Evaluate the limit
\[
\lim_{x \to 1} \frac{x^2 + 9x + 8}{x + 1}
\]
Evaluate the limit
\[
\lim_{x \to 9} \frac{x^2 + 17x + 72}{x + 9}
\]
Evaluate the limit
\[
\lim_{x \to -3} \frac{x - 3}{x^2 + 5x - 24}
\]
Evaluate the limit
\[
\lim_{y \to 1} \frac{y^3 - y}{y^2 - 1}
\]
Evaluate the limit
\[
\lim_{x \to -4} \frac{64 - t}{t - 64}
\]
Evaluate the limit
\[
\lim_{y \to -6} \frac{|y + 6|}{y + 6}
\]
Evaluate the limits. If a limit does not exist, enter "DNE".
\[
\lim_{x \to 9^+} \frac{|x + 9|}{x + 9} = \frac{|x + 9|}{x + 9}; \quad \lim_{x \to 9^-} \frac{|x + 9|}{x + 9} = \frac{|x + 9|}{x + 9}; \quad \lim_{x \to 9} \frac{|x + 9|}{x + 9} = \frac{|x + 9|}{x + 9}
\]
Let
\[
f(x) = \begin{cases} 
  x + 5 & \text{if } x \leq -2 \\
  5 & \text{if } x > -2 
\end{cases}
\]
Sketch the graph of this function for yourself and find following limits if they exist (if not, enter DNE).

Let
\[
f(x) = \begin{cases} 
  7 & \text{if } x > 4 \\
  -2 & \text{if } x = 4 \\
  -x + 7 & \text{if } -9 \leq x < 4 \\
  16 & \text{if } x < -9 
\end{cases}
\]
Sketch the graph of this function and find following limits if they exist (if not, enter DNE).
Let \( \lim_{\substack{x \to 38 \\downarrow}} f(x) = \ldots \)

\( f(3.38) = \ldots \)

Is this function continuous at 3.38?: (Y or N) \( \ldots \)

Can this function be made continuous by changing its value at 3.38?: (Y or N) \( \ldots \)

Let \( \lim f(x) = -9 \), \( \lim h(x) = 0 \), \( \lim g(x) = 8 \).

Find following limits if they exist. If not, enter DNE (does not exist) as your answer.

1. \( \lim_{x \to a} (f(x) + h(x)) \)
2. \( \lim_{x \to a} (f(x) - h(x)) \)
3. \( \lim_{x \to a} (f(x) \cdot g(x)) \)
4. \( \lim_{x \to a} \frac{f(x)}{h(x)} \)
5. \( \lim_{x \to a} \frac{f(x)}{g(x)} \)
6. \( \lim_{x \to a} \frac{g(x)}{f(x)} \)
7. \( \lim_{x \to a} \sqrt{h(x)} \)
8. \( \lim_{x \to a} h(x) \)
9. \( \lim_{x \to a} \frac{1}{h(x) - g(x)} \)

The graphs of \( f \) and \( g \) are given above. Use them to evaluate each quantity below. Write 'DNE' if the limit or value does not exist (or if it’s infinity).

1. \( \lim_{x \to -3} \frac{[f(x)/g(x)]}{2} \)
2. \( \lim_{x \to 0} \frac{[f(g(x))]}{2} \)
3. \( \lim_{x \to 3} \frac{[f(x)]g(x)}{2} \)
4. \( \lim_{x \to -3} \frac{[f(x)]/g(x)}{2} \)

Using the table above calculate the limits below.

Enter 'DNE' if the limit doesn’t exist OR if limit can’t be determined from the information given.

1. \( f(2) \cdot g(2) \)
2. \( \lim_{x \to 0} \frac{[f(g(x))]}{2} \)
3. \( \lim_{x \to -2} \frac{[f(x)]}{2} \)
4. \( \lim_{x \to -2} \frac{[f(x)]/g(x)}{2} \)

Evaluate \( \lim_{x \to 2} (x + 1)^3 (5x^2) \).

Enter the letters corresponding to the Limit Laws that you used to find this limit:

**Limit Laws**
A. Power Law
B. Sum Law
C. Difference Law
D. Constant Multiple Law
E. Quotient Law
23. (1 pt) setLimitsRates2Limits/ur Jr_2.11.pg
If
\[ 9x - 22 \leq f(x) \leq x^2 + 5x - 18 \]
determine \( \lim_{x \to 2} f(x) = \ldots \)

What theorem did you use to arrive at your answer?

24. (1 pt) setLimitsRates2Limits/ns2.3.18.pg
Use factoring to calculate this limit
\[ \lim_{a \to 1} \frac{a^2 - s^2}{a^4 - s^4} \]

If you want a hint, try doing this numerically for a couple of values of \( a \) and \( s \).

25. (1 pt) setLimitsRates2Limits/ur Jr_2.6.pg
Enter the integer which is the apparent limit of the following sequences or enter N if the sequence does not appear to have a limit.

### 1.
the sequence generated by \( f(h) \) where \( h \) is a sequence of positive numbers approaching zero and \( f(x) = 3 \tan(x)/x \).

### 2.
the sequence generated by \( f(h) \) where \( h \) is a sequence of negative numbers approaching zero and \( f(x) = x^3 + 7 \) if \( x \) is greater than or equal to 0 and \( f(x) = -x^3 - 7 \) if \( x \) is less than zero.

### 3.
2.199505sin(1.8), 2.199505sin(2.199505sin(1.8)), 2.199505sin(2.199505sin(2.199505sin(1.8))), ...

### 4.
\( \sqrt{3}, \sqrt[3]{3}, \sqrt[5]{3}, ... \)

26. (1 pt) setLimitsRates2Limits/ur Jr_2.5.pg
What is the limit of the sequence \( f(k) \) generated by the sequence
\[ f(x) = \frac{(43.6x - 10.4)(9.7x + 33.3)}{38.5x^2 - 5.9} \]
when \( k = 1, 2, 3, 4, 5, ... \)?

27. (1 pt) setLimitsRates2Limits/ur Jr_2.4.pg
Find an integer which is the limit of
\( \tan(8x)/2x \)
as \( x \) goes to 0. (Enter I for infinity or DNE for does not exist.) You should also try using identities to transform the expressions algebraically so that you can identify the limits without using a calculator.

28. (1 pt) setLimitsRates2Limits/ur Jr_2.4.pg
Let \( f(x) = \begin{cases} 8 - x - x^2, & \text{if } x \leq 1 \\ 2x - 7, & \text{if } x > 1 \end{cases} \)
Calculate the following limits. Enter 1000 if the limit does not exist.

<table>
<thead>
<tr>
<th>( x \to 1^- )</th>
<th>( x \to 1^+ )</th>
<th>( x \to 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \ldots )</td>
<td>( f(x) = \ldots )</td>
<td>( f(x) = \ldots )</td>
</tr>
</tbody>
</table>

29. (1 pt) setLimitsRates2Limits/ur Jr_2.5.pg
Let \( f(x) = \begin{cases} \sqrt{2 - x^2}, & \text{if } x \leq -4 \\ 2, & \text{if } x = -4 \\ 4x + 19, & \text{if } x > -4 \end{cases} \)
Calculate the following limits. Enter 1000 if the limit does not exist.

<table>
<thead>
<tr>
<th>( x \to -4^- )</th>
<th>( x \to -4^+ )</th>
<th>( x \to -4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \ldots )</td>
<td>( f(x) = \ldots )</td>
<td>( f(x) = \ldots )</td>
</tr>
</tbody>
</table>

30. (1 pt) setLimitsRates2Limits/ur Jr_2.6.pg
Let \( f(x) = \begin{cases} -1/x^2, & \text{if } x < -5 \\ 3x + 15, & \text{if } x > 5 \end{cases} \)
Calculate the following limits. Enter 1000 if the limit does not exist.

<table>
<thead>
<tr>
<th>( x \to -5^- )</th>
<th>( x \to -5^+ )</th>
<th>( x \to -5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \ldots )</td>
<td>( f(x) = \ldots )</td>
<td>( f(x) = \ldots )</td>
</tr>
</tbody>
</table>

31. (1 pt) setLimitsRates2Limits/ur Jr_2.7.pg
Let \( f(x) = \frac{4x^2 - 10x + 24}{x^2 + 2x - 24} \)
Calculate \( \lim_{x \to 4} f(x) \) by first finding a continuous function which is equal to \( f \) everywhere except \( x = 4 \).

<table>
<thead>
<tr>
<th>( x \to 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \ldots )</td>
</tr>
</tbody>
</table>

32. (1 pt) setLimitsRates2Limits/ur Jr_2.8.pg
Let \( f(x) = \frac{7x + 1}{x^2 - 2x - 3} \)
Calculate \( \lim_{x \to -1} f(x) \) by first finding a continuous function which is equal to \( f \) everywhere except \( x = -1 \).

<table>
<thead>
<tr>
<th>( x \to -1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \ldots )</td>
</tr>
</tbody>
</table>

33. (1 pt) setLimitsRates2Limits/ur Jr_2.9.pg
Let \( f(x) = \frac{25 - x}{5 - 3x} \)
Calculate \( \lim_{x \to 25} f(x) \) by first finding a continuous function which is equal to \( f \) everywhere except \( x = 25 \).

<table>
<thead>
<tr>
<th>( x \to 25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \ldots )</td>
</tr>
</tbody>
</table>

34. (1 pt) setLimitsRates2Limits/ur Jr_2.12.pg
Let \( f(s) = \frac{s - 4}{s - 8} \)
Calculate \( \lim_{s \to 8} f(s) \) by first finding a continuous function which is equal to \( f \) everywhere except \( s = 8 \).

<table>
<thead>
<tr>
<th>( s \to 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(s) = \ldots )</td>
</tr>
</tbody>
</table>

35. (1 pt) setLimitsRates2Limits/ur Jr_2.13.pg
The main theorem of Ste 2.3 tells us that many functions are continuous so that their limits can be evaluated by direct substitution. Calculate the following limits by direct substitution, making use of this big theorem from Ste 2.3.
\[
\begin{align*}
\lim_{s \to 0} \sqrt{3(x^2 + 12)} &= \lim_{t \to 3} \frac{(1-t)(t+5)}{3t-7} = \lim_{a \to 10} \frac{(a+7)^4}{a+1} = \lim_{s \to 8} \frac{13-s}{s+12} = \lim_{y \to 3} \frac{y^2(5-3y^2)}{y^2-4} = \lim_{a \to 2} \frac{a^2 - 3a + 4}{a - 12} = 36.
\end{align*}
\]

Let \( f(a) = \frac{4}{a-2} - \frac{16}{a^2-4} \)

Calculate \( \lim_{a \to 2} f(a) \) by first finding a continuous function which is equal to \( f \) everywhere except \( a = 2 \).

\( \lim_{a \to 2} f(a) = \)
1. Evaluate the limit
\[
\lim_{x \to \infty} \frac{1 + 5x}{7 - 7x}
\]

2. Evaluate the limit
\[
\lim_{x \to \infty} \frac{2x + 8}{6x^2 - 7x + 10}
\]

3. Evaluate the limit
\[
\lim_{x \to \infty} \frac{4x^3 - 4x^2 - 7x}{6x^2 - 4x - 2x^3}
\]

4. Find the horizontal limit(s) of the following function:
\[
f(x) = \frac{4x^3 - 8x^2 - 4x}{10 - 3x - 7x^3}
\]

5. Evaluate the limit
\[
\lim_{x \to \infty} \frac{(8 - x)(11 + 5x)}{(3 - 3x)(9 + 5x)}
\]

6. Evaluate the limit
\[
\lim_{x \to \infty} \frac{\sqrt{2 + 11x^2}}{(9 + 2x)}
\]

7. Evaluate
\[
\lim_{x \to \infty} \frac{x^4 + 2x^3 - 6}{9x^2 + 3}
\]

8. Evaluate
\[
\lim_{t \to \infty} \frac{7t - 9}{\sqrt{t^2 - 8t + 5}}
\]

9. The horizontal asymptotes of the curve
\[
y = \frac{5x}{(x^4 + 1)^{\frac{1}{2}}}
\]
are given by
\[
y_1 = \quad \text{and} \quad y_2 =
\]

where \( y_1 > y_2 \).

The vertical asymptote of the curve
\[
y = \frac{2x^3}{x - 2}
\]
is given by \( x = \quad \).

10. Evaluate
\[
\lim_{x \to \infty} \frac{\sqrt{x^2 - 1x + 1} - x}{x}
\]

11. Determine the infinite limit of the following functions. Enter INF for \( \infty \) and MINF for \(-\infty\).

\[\begin{align*}
1. & \quad \lim_{x \to -3} \frac{2}{x - 3} = \\
2. & \quad \lim_{x \to -5} \frac{1}{(x - 5)^3} = \\
3. & \quad \lim_{x \to -7} \frac{1}{x^2(x + 7)} = \\
4. & \quad \lim_{x \to -3} \frac{2}{x - 3} = 
\end{align*}\]

12. Evaluate the following limits. If needed, enter INF for \( \infty \) and MINF for \(-\infty\).

(a) \[
\lim_{x \to 11} \frac{14x}{11 - 6x} = 
\]

(b) \[
\lim_{x \to 11} \frac{14x}{11 - 6x} = 
\]

13. Evaluate the following limits.

(a) \[
\lim_{x \to ^{e^t} + 1} = 
\]

(b) \[
\lim_{x \to ^{e^t} + 1} = 
\]

[NOTE: If needed, enter INF for \( \infty \) and MINF for \(-\infty\).]

[HINT: Look at where the exponential function is going in the fraction. If you need a reminder, look up infinite limits in Section 2.5 (in particular, see pg 138-139).]
Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to 7} \frac{1 + 8x}{7 - 7x} = \]

(b) \[ \lim_{x \to 7} \frac{1 + 8x}{7 - 7x} = \]

15. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_8.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to \infty} \frac{3x + 3}{3x^2 - 7x + 10} = \]

(b) \[ \lim_{x \to \infty} \frac{3x + 3}{3x^2 - 7x + 10} = \]

16. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_9.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to \infty} \frac{7x^3 - 4x^2 - 11x}{6 - 9x - 10x^3} = \]

(b) \[ \lim_{x \to \infty} \frac{7x^3 - 4x^2 - 11x}{6 - 9x - 10x^3} = \]

17. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_10.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to \infty} \frac{(7 - x)(4 + 6x)}{(3 - 6x)(7 + 4x)} = \]

(b) \[ \lim_{x \to \infty} \frac{(7 - x)(4 + 6x)}{(3 - 6x)(7 + 4x)} = \]

18. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_11.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to \infty} \frac{\sqrt{4 + 5x^2}}{2 + 4x} = \]

(b) \[ \lim_{x \to \infty} \frac{\sqrt{4 + 5x^2}}{2 + 4x} = \]

19. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_12.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to -\infty} \frac{\sqrt{x^4 - 8x^3 + 8}}{8x^2 + 9} = \]

(b) \[ \lim_{x \to -\infty} \frac{\sqrt{x^4 - 8x^3 + 8}}{8x^2 + 9} = \]

20. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_13.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to -\infty} \left( \sqrt{x^2 - 7x + 1 - x} \right) = \]

(b) \[ \lim_{x \to -\infty} \left( \sqrt{x^2 - 7x + 1 - x} \right) = \]

21. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_14.pg

Evaluate the following limits. If needed, enter INF for $\infty$ and MINF for $-\infty$.

(a) \[ \lim_{x \to -\infty} \left( -25x^2 + 13x \right) = \]

(b) \[ \lim_{x \to -\infty} \left( -25x^2 + 13x \right) = \]

22. \(1\) pt setLimitsRates3Infinite/ur_Jr_3_15.pg

A function is said to have a vertical asymptote wherever the limit on the left or right (or both) is either positive or negative infinity.

For example, the function \( f(x) = \frac{x^2 + 1}{(2x + 3)(x - 5)} \) has a vertical asymptote at \( x = 5 \).

For each of the following limits, enter either 'P' for positive infinity, 'N' for negative infinity, or 'D' when the limit simply does not exist.

\[ \lim_{x \to 5^-} \frac{x^2 + 1}{(2x + 3)(x - 5)} = \]

\[ \lim_{x \to 5^+} \frac{x^2 + 1}{(2x + 3)(x - 5)} = \]

\[ \lim_{x \to 5^-} \frac{x^2 + 1}{(2x + 3)(x - 5)} = \]

\[ \lim_{x \to 5^+} \frac{x^2 + 1}{(2x + 3)(x - 5)} = \]
23. (1 pt) setLimitsRates3Infinite/ur_Jr_3.16.png

A function is said to have a **vertical asymptote** wherever the limit on the left or right (or both) is either positive or negative infinity.

For example, the function \( f(x) = \frac{-3(x+2)}{x^2+4x+4} \) has a vertical asymptote at \( x = -2 \).

For each of the following limits, enter either 'P' for positive infinity, 'N' for negative infinity, or 'D' when the limit simply does not exist.

\[
\lim_{x \to -2} \frac{-3(x+2)}{x^2+4x+4} = \underline{D}
\]

\[
\lim_{x \to -2^+} \frac{-3(x+2)}{x^2+4x+4} = \underline{D}
\]

\[
\lim_{x \to -2^-} \frac{-3(x+2)}{x^2+4x+4} = \underline{D}
\]

**24. (1 pt) setLimitsRates3Infinite/ur_Jr_3.17.png**

A function is said to have a **vertical asymptote** wherever the limit on the left or right (or both) is either positive or negative infinity.

For example, the function \( f(x) = \frac{9x^2}{(x-1)^3(x-6)} \) has a vertical asymptote at \( x = 1 \).

For each of the following limits, enter either 'P' for positive infinity, 'N' for negative infinity, or 'D' when the limit simply does not exist.

\[
\lim_{x \to 1^-} \frac{9 - x^2}{(x-1)^3(x-6)} = \underline{D}
\]

\[
\lim_{x \to 1^+} \frac{9 - x^2}{(x-1)^3(x-6)} = \underline{D}
\]

**25. (1 pt) setLimitsRates3Infinite/ur_Jr_3.18.png**

A function is said to have a **horizontal asymptote** if either the limit at infinity exists or the limit at negative infinity exists.

Show that each of the following functions has a horizontal asymptote by calculating the given limit.

\[
\lim_{x \to \infty} \frac{-8x}{15 + 2x} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{2 - 11x^2}{x^3 + 7x - 5} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{15 - 10x}{\sqrt{x^2 + 12x}} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{15 - 10x}{\sqrt{x^2 + 12x}} = \underline{\text{_____}}
\]

**26. (1 pt) setLimitsRates3Infinite/ur_Jr_3.19.png**

A function is said to have a **horizontal asymptote** if either the limit at infinity exists or the limit at negative infinity exists.

Show that each of the following functions has a horizontal asymptote by calculating the given limit.

\[
\lim_{x \to \infty} \frac{7x}{x^2 - 13x + 6} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{8 - 5x}{15 + x} + \frac{14x^2 + 10}{(12x - 3)^2} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{5x + 15}{9x - 7} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{x - 3}{-x - 4} = \underline{\text{_____}}
\]

\[
\lim_{x \to \infty} \frac{\sqrt{x^2 + 6x - 3} - x}{x^2 - 6x + 3 + x} = \underline{\text{_____}}
\]
The function \( f(x) \) is given by the formula
\[
f(x) = \frac{6x^3 + 13x^2 + 12x + 20}{x + 2}
\]
when \( x < -2 \) and by the formula
\[
f(x) = -6x^2 + 1x + a
\]
when \(-2 \leq x\).

What value must be chosen for \( a \) in order to make this function continuous at \(-2\)?

\[
a = 
\]

A function \( f(x) \) is said to have a **removable** discontinuity at \( x = a \) if:

1. \( f \) is either not defined or not continuous at \( x = a \).
2. \( f(a) \) could either be defined or redefined so that the new function IS continuous at \( x = a \).

Let \( f(x) = \frac{2x^2 + 5x - 75}{x-5} \)

Show that \( f(x) \) has a removable discontinuity at \( x = 5 \) and determine what value for \( f(5) \) would make \( f(x) \) continuous at \( x = 5 \).

Must define \( f(5) = \)

A function \( f(x) \) is said to have a **removable** discontinuity at \( x = a \) if:

1. \( f \) is either not defined or not continuous at \( x = a \).
2. \( f(a) \) could either be defined or redefined so that the new function IS continuous at \( x = a \).

Let \( f(x) = \begin{cases} \frac{2}{3} + \frac{-6x^2}{5x-4} & \text{if } x \neq 0, 1 \\ 3 & \text{if } x = 0 \end{cases} \)

Show that \( f(x) \) has a removable discontinuity at \( x = 0 \) and determine what value for \( f(0) \) would make \( f(x) \) continuous at \( x = 0 \).

Must redefine \( f(0) = \)

Hint: Try combining the fractions and simplifying.

The discontinuity at \( x = 1 \) is actually NOT a removable discontinuity, just in case you were wondering.
Now for fun, try to graph $f(x)$. It’s just a couple of parabolas!

9. (1 pt) setLimitsRates5Continuity/ur_Jr_5_4.pg

A function $f(x)$ is said to have a jump discontinuity at $x = a$ if:
1. $\lim_{x \to a^-} f(x)$ exists.
2. $\lim_{x \to a^+} f(x)$ exists.
3. The left and right limits are not equal.

Let $f(x) = \begin{cases} 3x - 4, & \text{if } x < 3 \\ 3, & \text{if } x \geq 3 \end{cases}$
Show that $f(x)$ has a jump discontinuity at $x = 3$ by calculating the limits from the left and right at $x = 3$.
$\lim_{x \to 3^-} f(x) = \underline{\phantom{0}}$
$\lim_{x \to 3^+} f(x) = \underline{\phantom{0}}$
Now for fun, try to graph $f(x)$.

10. (1 pt) setLimitsRates5Continuity/ur_Jr_5_5.pg

A function $f(x)$ is said to have a jump discontinuity at $x = a$ if:
1. $\lim_{x \to a^-} f(x)$ exists.
2. $\lim_{x \to a^+} f(x)$ exists.
3. The left and right limits are not equal.

Let $f(x) = \begin{cases} x^2 + 5x + 6, & \text{if } x < 7 \\ 7, & \text{if } x = 7 \\ -7x + 1, & \text{if } x > 7 \end{cases}$
Show that $f(x)$ has a jump discontinuity at $x = 7$ by calculating the limits from the left and right at $x = 7$.
$\lim_{x \to 7^-} f(x) = \underline{\phantom{0}}$
$\lim_{x \to 7^+} f(x) = \underline{\phantom{0}}$
Now for fun, try to graph $f(x)$.

11. (1 pt) setLimitsRates5Continuity/ur_Jr_5_6.pg

Let $f(x) = \begin{cases} 2x - 2, & \text{if } x \leq 7 \\ -6x + b, & \text{if } x > 7 \end{cases}$
If $f(x)$ is a function which is continuous everywhere, then we must have $b = \underline{\phantom{0}}$
Now for fun, try to graph $f(x)$.

12. (1 pt) setLimitsRates5Continuity/ur_Jr_5_6b.pg

Let $f(x) = \begin{cases} mx - 14, & \text{if } x < -9 \\ x^2 + 9x - 5, & \text{if } x \geq -9 \end{cases}$
If $f(x)$ is a function which is continuous everywhere, then we must have $m = \underline{\phantom{0}}$
Now for fun, try to graph $f(x)$.

13. (1 pt) setLimitsRates5Continuity/ur_Jr_5_7.pg

Let $f(x) = \begin{cases} -2x + b, & \text{if } x < 2 \\ -24, & \text{if } x \geq 2 \end{cases}$
There are exactly two values for $b$ which make $f(x)$ a continuous function at $x = 2$. The one with the greater absolute value is $b = \underline{\phantom{0}}$
Now for fun, try to graph $f(x)$.

14. (1 pt) setLimitsRates5Continuity/ur_Jr_5_8.pg

Find $c$ such that the function $f(x) = \begin{cases} x^2 - 3, & \text{if } x \leq c \\ 10x - 28, & \text{if } x > c \end{cases}$ is continuous everywhere.
$c = \underline{\phantom{0}}$

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, © UR
1. The slope of the tangent line to the parabola $y = 2x^2 + 6x + 4$ at the point $(0, 4)$ is: $\frac{\text{m}}{\text{s}}$

2. The slope of the tangent line to the curve $y = 4x^3$ at the point $(-5, -500)$ is: $\frac{\text{m}}{\text{s}}$

3. The slope of the tangent line to the curve $y = 2\sqrt{x}$ at the point $(3, 3.4641)$ is: $\frac{\text{m}}{\text{s}}$

4. The slope of the tangent line to the curve $y = \frac{4}{x}$ at the point $(3, 1.333)$ is: $\frac{\text{m}}{\text{s}}$

5. The slope of the tangent line to the parabola $y = 3x^2 - 2x + 6$ at the point where $x = 1$ is: $\frac{\text{m}}{\text{s}}$

6. If a rock is thrown into the air with an upward velocity of $23$ m/s, its height (in meters) after $t$ seconds is given by $y = 23t - 4.9t^2$. Find the velocity of the rock when $t = 1$.

7. If an arrow is shot straight upward on the moon with a velocity of $68$ m/s, its height (in meters) after $t$ seconds is given by $s(t) = 68t - 0.83t^2$.

8. The displacement (in meters) of a particle moving in a straight line is given by $s = 3t^3$ where $t$ is measured in seconds. Find the average velocity of the particle over the time interval $[6, 8]$.

9. Let $p(x) = 4.9x^2 + 0000$. Use a calculator or a graphing program to find the slope of the tangent line to the point $(x, p(x))$ when $x = 0.7$. Give the answer to 3 places.

10. A rock is thrown off of a 100 foot cliff with an upward velocity of $40$ m/s. As a result its height after $t$ seconds is given by the formula:

11. The following chart shows "living wage" jobs in Rochester per 1000 working age adults over a 5 year period.

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jobs</td>
<td>640</td>
<td>700</td>
<td>750</td>
<td>785</td>
<td>805</td>
</tr>
</tbody>
</table>

What is the average rate of change in the number of living wage jobs from 1997 to 1999? $\frac{\text{jobs}}{\text{year}}$

What is the average rate of change in the number of living wage jobs from 1999 to 2001? $\frac{\text{jobs}}{\text{year}}$

Based on these two answers, should the mayor from the last two years be reelected? (These numbers are made up. Please do not actually hold the mayor accountable.)
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false. A statement is true only if it is true for all possibilities. You must get all of the answers correct to receive credit.

1. If \( \lim_{x \to 4} f(x) = 0 \) and \( \lim_{x \to 4} g(x) = 0 \), then \( \lim_{x \to 4} \frac{f(x)}{g(x)} \) does not exist.

2. If \( f(x) \) is differentiable at \( a \), then \( f(x) \) is continuous at \( a \).

3. If \( f'(3) \) exists, then the limit \( \lim_{x \to 3} f(x) \) is \( f(3) \).

4. If \( \lim_{x \to 4} f(x) = 2 \) and \( \lim_{x \to 4} g(x) = 0 \), then \( \lim_{x \to 4} \frac{f(x)}{g(x)} \) does not exist.

5. If \( p(x) \) is a polynomial, then the limit \( \lim_{x \to 3} p(x) \) is \( p(3) \).
1. (1 pt) setDerivatives1/s2_1_20.pg
If \( f(x) = 18 \), find \( f'(5) \).

2. (1 pt) setDerivatives1/s2_1_19.pg
If \( f(x) = 7x + 21 \), find \( f'(-3) \).

3. (1 pt) setDerivatives1/s2_1_7.pg
If \( f(x) = 2 + 7x - 3x^2 \), find \( f'(3) \).

4. (1 pt) setDerivatives1/s2_1_26.pg
If \( f(x) = \frac{2}{x} \), find \( f'(4) \).

5. (1 pt) setDerivatives1/s2_1_23.pg
Let \( f(x) = \sqrt{1 + 2x} \)
\[ f'(3) = \]

6. (1 pt) setDerivatives1/ur_dr_1_10.pg
Let \( f(x) = \frac{1}{x - 2} \)

7. (1 pt) setDerivatives1/osu_dr_1_11.pg
For each of the given functions \( f(x) \), find the derivative \((f^{-1})'(c)\) at the given point \( c \), using Theorem , first finding \( a = f^{-1}(c) \).
\[ f(x) = 6x + 10x^2; c = -16 \]
\[ a = \]
\[ (f^{-1})'(c) = \]
\[ f(x) = x^2 - 9x + 27 \text{ on the interval } [4.5, \infty); c = 9 \]
\[ a = \]
\[ (f^{-1})'(c) = \]

8. (1 pt) setDerivatives1/s2_1_8.pg
The position of a cat running from a dog down a dark alley is given by the values of the table.

\[
\begin{array}{c|ccccc}
\text{t(seconds)} & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{s(feets)} & 0 & 5 & 33 & 68 & 87 & 104 \\
\end{array}
\]

A. Find the average velocity for the time period beginning when \( t=2 \) and lasting

1. 3 s
2. 2 s
3. 1 s

B. Draw the graph of the function for yourself and estimate the instantaneous velocity when \( t=2 \)

9. (1 pt) setDerivatives1/cls5p8.pg
This problem tests calculating new functions from old ones:
From the table below calculate the quantities asked for:

\[
\begin{array}{c|ccccc}
x & -3 & 17 & 3 & 34 & 5 & -68 \\
\hline
f(x) & 17 & 257 & 5 & 1090 & 17 & 4762 \\
g(x) & 34 & -10386 & -68 & -80885 & -294 & 619549 \\
f'(x) & -8 & 32 & 4 & 66 & 8 & -138 \\
g'(x) & -41 & -1801 & -65 & -7071 & -169 & -27471 \\
\end{array}
\]

\[ f(-3)/(g(-3) + 5) \]
\[ (fg)'(-3) \]
If \( h(x) = f(f(x)) \), calculate \( h'(-3) \)
\[ (f/g)'(-3) \]
Constructing new functions from old ones and calculating the derivative of the new function from the derivatives of the old functions:

From the table below calculate the quantities asked for:

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-33</th>
<th>33</th>
<th>3</th>
<th>60</th>
<th>-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>60</td>
<td>71940</td>
<td>-71940</td>
<td>-60</td>
<td>-432120</td>
<td>432120</td>
</tr>
<tr>
<td>g(x)</td>
<td>-33</td>
<td>-36003</td>
<td>36003</td>
<td>33</td>
<td>216120</td>
<td>-216120</td>
</tr>
<tr>
<td>f'(x)</td>
<td>-56</td>
<td>-6536</td>
<td>-6536</td>
<td>-56</td>
<td>-21602</td>
<td>-21602</td>
</tr>
<tr>
<td>g'(x)</td>
<td>29</td>
<td>3269</td>
<td>3269</td>
<td>29</td>
<td>10802</td>
<td>10802</td>
</tr>
</tbody>
</table>

\[(f g)'(3)\]
\[(f + g)'(3)\]
\[(f - g)'(3)\]
\[f(3)/(g(3) + 5)\]

11. (1 pt) setDerivatives1/c1s5p9.pg

This problem tests calculating new functions from old ones:

From the table below calculate the quantities asked for:

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>72</th>
<th>26</th>
<th>-2</th>
<th>164</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>72</td>
<td>-373392</td>
<td>-17628</td>
<td>12</td>
<td>-4411272</td>
<td>-1752</td>
</tr>
<tr>
<td>g(x)</td>
<td>164</td>
<td>-736200</td>
<td>-33826</td>
<td>26</td>
<td>-8768260</td>
<td>-3180</td>
</tr>
<tr>
<td>f'(x)</td>
<td>-50</td>
<td>-15554</td>
<td>-2030</td>
<td>-14</td>
<td>-80690</td>
<td>-434</td>
</tr>
<tr>
<td>g'(x)</td>
<td>-113</td>
<td>-30817</td>
<td>-3953</td>
<td>-33</td>
<td>-160721</td>
<td>-817</td>
</tr>
</tbody>
</table>

If \(h(x) = f(f(x))\), calculate \(h'(-2)\)

\[f(f(-4))\]
\[(f g)'(-2)\]

If \(h(x) = f(g(x))\), calculate \(h'(-4)\)

\[f(-2)/(g(-2) + 5)\]

12. (1 pt) setDerivatives1/c1s5p9b.pg

This problem tests calculating new functions from old ones:

From the table below calculate the quantities asked for:

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>67</th>
<th>-52</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>67</td>
<td>1</td>
<td>-39</td>
<td>29</td>
<td>-592413</td>
<td>286521</td>
</tr>
<tr>
<td>g(x)</td>
<td>-52</td>
<td>-1</td>
<td>14</td>
<td>14</td>
<td>291918</td>
<td>-146121</td>
</tr>
<tr>
<td>f'(x)</td>
<td>-64</td>
<td>2</td>
<td>0</td>
<td>-40</td>
<td>-26664</td>
<td>-16430</td>
</tr>
<tr>
<td>g'(x)</td>
<td>41</td>
<td>2</td>
<td>17</td>
<td>17</td>
<td>13201</td>
<td>8322</td>
</tr>
</tbody>
</table>

\[(f + g)'(-3)\]
\[(f(-3))/(g(-3) + 5)\]
\[(f/g)'(-3)\]
\[f(f(1))\]
\[(f g)'(-3)\]

13. (1 pt) setDerivatives1/c1s6p1.pg

Given the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>0.0005</th>
<th>0.0054</th>
<th>0.0055</th>
<th>0.0056</th>
<th>0.0057</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>0.18265486</td>
<td>0.16798117</td>
<td>-0.38406237</td>
<td>0.47885736</td>
<td>-0.47114824</td>
</tr>
</tbody>
</table>

Given the following table:
Calculate the value of \( f'(0.0055) \) to two places of accuracy.

To obtain more precise information about the value of \( f \) near 0.0055 enter a new increment value for \( x \) here rule 1in.01in and then press the Submit Answer button.

How will you tell when your increment is small enough to give you a good answer for the problem?

Identify the graphs A (blue), B (red) and C (green) as the graphs of a function and its derivatives:

\[ \text{___} \text{is the graph of the function} \]
\[ \text{___} \text{is the graph of the function’s first derivative} \]
\[ \text{___} \text{is the graph of the function’s second derivative} \]

Identify the graphs A (blue), B (red) and C (green) as the graphs of a function and its derivatives:

\[ \text{___} \text{is the graph of the function} \]
\[ \text{___} \text{is the graph of the function’s first derivative} \]
\[ \text{___} \text{is the graph of the function’s second derivative} \]

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\[ \text{___} \text{is the graph of the function’s second derivative} \]

Let

\[ f(x) = 3x^3 + 7x + 1 \]

Use the limit definition of the derivative on page 163 to calculate the derivative of \( f \):

\[ f'(x) = \ldots \]

Use the same formula from above to calculate the derivative of this new function (i.e. the second derivative of \( f \)):

\[ f''(x) = \ldots \]

The oracle function \( f(x) \) is presented below. For each \( x \) value you enter the oracle will tell you the value \( f(x) \). Calculate the derivative of the function at \(-1\) using the Newton quotient definition.

\[ f'(x) \text{ at } -1 = \ldots \] You can use a calculator

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \rightarrow )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter x</td>
<td>( \rightarrow )</td>
<td>result: ( f(x) )</td>
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<td>( \rightarrow )</td>
<td>result: ( f(x) )</td>
</tr>
</tbody>
</table>

Remember the technique for finding instantaneous velocities from average velocities? This is the same thing.
Below is an "oracle" function. An oracle function is a function presented interactively. When you type in a \( t \) value, and press the \(-\rightarrow\) button the value \( f(t) \) appears in the right hand window. There are three lines, so you can calculate three different values of the function at one time.

The function \( f(t) \) represents the height in feet of a ball thrown into the air, \( t \) seconds after it has been thrown.

Calculate the average velocity 2.05 seconds after the ball has been thrown.

Average velocity at 2.05 = 

You can use a calculator

Remember this technique for finding velocities. Later we will use the same method to find the derivative of functions such as \( f(t) \).

Find \( a \) and \( b \) such that the function

\[
f(x) = \begin{cases} 
  x^2 + 6x + 11 & \text{if } x \leq -2 \\
  ax + b & \text{if } x > -2
\end{cases}
\]

is differentiable everywhere.

\( a = \) 
\( b = \) 

Let \( f(x) \) be the function \( 5x^2 - 5x + 8 \). Then the quotient \( \frac{f(9+h)-f(9)}{h} \) can be simplified to \( ah + b \) for:

\( a = \) 
and
\( b = \) 

Let \( f(x) \) be the function \( \frac{1}{x+1} \). Then the quotient \( \frac{f(6+h)-f(6)}{h} \) can be simplified to \( -\frac{1}{ab+1} \) for:

\( a = \) 
and
\( b = \) 

Let \( f(x) = x^3 - 4x \). Calculate the difference quotient \( \frac{f(3+h)-f(3)}{h} \) for:

\( h = .1 \) 
\( h = .01 \) 
\( h = -.01 \) 
\( h = -.1 \)

If someone now told you that the derivative (slope of the tangent line to the graph) of \( f(x) \) at \( x = 3 \) was \(-1/n^2\) for some integer \( n \) what would you expect \( n \) to be?

\( n = \) 

Let \( f(x) = \sqrt{x + 1} \). Calculate the difference quotient \( \frac{f(3+h)-f(3)}{h} \) for:

\( h = .1 \) 
\( h = .01 \) 
\( h = -.01 \) 
\( h = -.1 \)

If someone now told you that the derivative (slope of the tangent line to the graph) of \( f(x) \) at \( x = 3 \) was \( 1/n \) for some integer \( n \) what would you expect \( n \) to be?

\( n = \) 

Let \( f(x) = \frac{1}{x^2} \). Calculate the difference quotient \( \frac{f(5+h)-f(5)}{h} \) for:

\( h = .1 \)
Graphs A and B are approximate graphs of $f$ and $f'$ for $f(x) = x^2 - 8x - 11$.
So $f$ is decreasing (and $f'$ is negative) on the interval $(a, \infty)$ for $a = \underline{\phantom{0000}}$.

Graphs A and B are approximate graphs of $f$ and $f'$ for $f(x) = x^2 - 6x + 11$.
So $f$ is increasing (and $f'$ is positive) on the interval $(a, \infty)$ for $a = \underline{\phantom{0000}}$.

Graphs A and B are approximate graphs of $f$ and $f'$ for $f(x) = x^2(x - 15)$.
So $f$ is decreasing (and $f'$ is negative) on the interval $(0, a)$ for $a = \underline{\phantom{0000}}$.

Let $f(x) = \frac{3}{x - 7}$.
Then according to the definition of derivative $f'(x) = \lim_{t \to x} \underline{\phantom{0000}}$.
The expression inside the limit simplifies to a simple fraction with numerator and denominator. We can cancel the factor appearing in the denominator against a similar factor appearing in the numerator leaving a simpler fraction with numerator and denominator. Taking the limit of this fractional expression gives us $f'(x) = \text{[value]}$.

---

30. (1 pt) setDerivatives1/ur_dr_1_11.pg
Answer the following True-False quiz. (Enter "T" or "F").

1. $(f(x) + g(x))' = f'(x) + g'(x)$.  
   **[Answer: T]**

2. If $f'(c) = 0$ and $f''(c) > 0$, then $f(x)$ has a local minimum at $c$.  
   **[Answer: T]**

3. If $f'(c) = 0$, then $c$ is either a local maximum or a local minimum.  
   **[Answer: T]**

4. If a function has a local maximum at $c$, then $f'(c)$ exists and is equal to 0.  
   **[Answer: T]**

5. Continuous functions are always differentiable.  
   **[Answer: T]**

6. A continuous function on a closed interval always attains a maximum and a minimum value.  
   **[Answer: T]**

7. If $f(x)$ and $g(x)$ are increasing on an interval $I$, then $f(x)g(x)$ is increasing on $I$.  
   **[Answer: F]**

---

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, @UR
1. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_1.png
If \( f(x) = 4x^2 - 3x + 5 \), find \( f'(0) \).

Use this to find the equation of the tangent line to the parabola \( y = 4x^2 - 3x + 5 \) at the point \((0, 5)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

2. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_2.png
If \( h(x) = 2 - 3x^3 \), find \( h'(3) \).

Use this to find the equation of the tangent line to the curve \( y = 2 - 3x^3 \) at the point \((3, -79)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

3. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_3.png
If \( f(x) = \frac{4}{x} \), find \( f'(5) \).

Use this to find the equation of the tangent line to the hyperbola \( y = \frac{4}{x} \) at the point \((5, 0.8000)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

4. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_4.png
If \( f(x) = 2x + \frac{2}{x} \), find \( f'(5) \).

Use this to find the equation of the tangent line to the curve \( y = 2x + \frac{2}{x} \) at the point \((5, 10.4000)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

5. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_5.png
If \( f(x) = \frac{5}{\sqrt{x}} \), find \( f'(5) \).

Use this to find the equation of the tangent line to the curve \( y = \frac{5}{\sqrt{x}} \) at the point \((5, 1.66667)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

6. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_6.png
If \( f(x) = 4x + 3\sqrt{x} \), find \( f'(4) \).

Use this to find the equation of the tangent line to the curve \( y = 4x + 3\sqrt{x} \) at the point \((4, 22.0000)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

7. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_7.png
If \( f(x) = \frac{5x}{1 + x^2} \)

find \( f'(4) \).

Use this to find the equation of the tangent line to the curve \( y = \frac{5x}{1 + x^2} \) at the point \((4, 1.17647)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \).

8. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_8.png
The parabola \( y = x^2 + 3 \) has two tangents which pass through the point \((0, -4)\). One is tangent to the to the parabola at \((A, A^2 + 3)\) and the other at \((-A, A^2 + 3)\). Find the (positive number) \( A \).

9. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_9a.png
On a separate piece of paper, sketch the graph of the parabola \( y = x^2 + 9 \). On the same graph, plot the point \((0, -2)\). Note that there are two tangent lines of \( y = x^2 + 9 \) that pass through the point \((0, -2)\).

Specifically, the tangent line of the parabola \( y = x^2 + 9 \) at the point \((a, a^2 + 9)\) passes through the point \((0, -2)\) where \( a > 0 \). The other tangent line that passes through the point \((0, -2)\) occurs at the point \((-a, a^2 + 9)\).

Find the number \( a \).

10. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_10.png
The graph of \( f(x) = 2x^3 + 6x^2 - 90x + 8 \) has two horizontal tangents. One occurs at a negative value of \( x \) and the other at a positive value of \( x \). What is the negative value of \( x \) where a horizontal tangent occurs?

What is the positive value of \( x \) where a horizontal tangent occurs?

11. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_10a.png
For what values of \( x \) does the graph of

\[ f(x) = 10x^3 - 30x^2 - 240x - 30 \]

have a horizontal tangent? Enter the \( x \) values in order, smallest first, to 4 places of accuracy:

\( x_1 = \ldots \leq x_2 = \ldots \)

12. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_11.png
For what values of \( x \) does the graph of

\[ f(x) = 9.7x^3 - 2.1825x^2 - 107.67x - 85.845 \]

have a horizontal tangent? Enter the \( x \) values in order, smallest first, to 4 places of accuracy:

\( x_1 = \ldots \leq x_2 = \ldots \)

13. (1 pt) setDerivatives1_5Tangents/ur_dr_1_5_12.png
For what values of \( x \) is the tangent line of the graph of

\[ f(x) = 6x^3 - 9x^2 - 108x - 36 \]

parallel to the line \( y = 0x - 1.1 \)? Enter the \( x \) values in order, smallest first, to 4 places of accuracy:

\( x_1 = \ldots \leq x_2 = \ldots \)
14. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_13.png
For what values of \( x \) is the tangent line of the graph of \( f(x) = 2.6x^3 - 15.21x^2 + 20.62x - 24.96 \) parallel to the line \( y = -2x + 1.6 \)? Enter the \( x \) values in order, smallest first, to 4 places of accuracy:
\( x_1 = \quad \leq \quad x_2 = \quad \)

15. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_14.png
Given
\[ f(x) = x + \sqrt{x} \]
Calculate the tangent line at the point \((64, 72)\)
For similar problems see p120:36–39.

16. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_15.png
At what point does the normal to \( y = 3 + 5x + 4x^2 \) at \((1, 12)\) intersect the parabola a second time?
\( \quad , \quad \)
The normal line is perpendicular to the tangent line. If two lines are perpendicular their slopes are negative reciprocals – i.e. if the slope of the first line is \( m \) then the slope of the second line is \(-1/m\)

17. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_16.png
For what values of \( a \) and \( b \) is the line \(-3x + y = b\) tangent to the curve \( y = ax^3 \) when \( x = -4\)?
\( a = \quad b = \quad \)

18. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_17.png
Let \( f(x) = 4x^2 - 6x + 11 \)
The slope of the tangent line to the graph of \( f(x) \) at the point \((2, 15)\) is \( \quad \)
The equation of the tangent line to the graph of \( f(x) \) at \((2, 15)\) is \( y = mx + b \) for
\( m = \quad \) and
\( b = \quad \)
Hint: the slope is given by the derivative at \( x = 2 \), ie.
\[ \lim_{x \to 2} \frac{f(2+h) - f(2)}{h} \]

19. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_17a.png
Let \( f(x) = 22 - x^2 \)
The slope of the tangent line to the graph of \( f(x) \) at the point \((-4, 6)\) is \( \quad \)
The equation of the tangent line to the graph of \( f(x) \) at \((-4, 6)\) is \( y = mx + b \) for
\( m = \quad \) and
\( b = \quad \)
Hint: the slope is given by the derivative at \( x = -4 \), ie.
\[ \lim_{x \to -4} \frac{f(-4+h) - f(-4)}{h} \]

20. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_18.png
Let \( f(x) = \sqrt{41 - x} \)
The slope of the tangent line to the graph of \( f(x) \) at the point \((5, 6)\) is \( \quad \)
The equation of the tangent line to the graph of \( f(x) \) at \((5, 6)\) is \( y = mx + b \) for
\( m = \quad \) and
\( b = \quad \)
Hint: the slope is given by the derivative at \( x = 5 \), ie.
\[ \lim_{x \to 5} \frac{f(5+h) - f(5)}{h} \]

21. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_19.png
Let \( f(x) = \frac{13}{x} \)
The slope of the tangent line to the graph of \( f(x) \) at the point \((-3, -\frac{13}{3})\) is \( \quad \)
The equation of the tangent line to the graph of \( f(x) \) at \((-3, -\frac{13}{3})\) is \( y = mx + b \) for
\( m = \quad \) and
\( b = \quad \)
Hint: the slope is given by the derivative at \( x = -3 \), ie.
\[ \lim_{x \to -3} \frac{f(-3+h) - f(-3)}{h} \]

22. (1 pt) setDerivatives1_5Tangents/ur_dr_1.5_20.png
Find \( a, b, c, d \) such that the cubic function \( f(x) = ax^3 + bx^2 + cx + d \) has horizontal tangent lines at \((-1, -15)\) and \((5, 93)\).
\( a = \quad b = \quad c = \quad d = \quad \)
1. (1 pt) setDerivatives2Formulas/ur_dr_2_11.pg
   If \( f(x) = 3x + 6 \), find \( f'(x) \).

2. (1 pt) setDerivatives2Formulas/s2_2_1.pg
   If \( f(x) = 5x^3 - 12x - 20 \), find \( f'(x) \).

   Find \( f'(4) \).

3. (1 pt) setDerivatives2Formulas/s2_2_1a.pg
   If \( f(x) = 6x^2 - 6x - 34 \), find \( f'(x) \).

4. (1 pt) setDerivatives2Formulas/s2_2_4f.pg
   If \( f(x) = 6x^8 - 6x^3 - 2x^3 + 3x \), find \( f'(x) \).

5. (1 pt) setDerivatives2Formulas/s2_2_4.pg
   If \( f(x) = 2x^8 - 8x^5 - 3x^2 + 6x \), find \( f'(x) \).

   Find \( f'(5) \).

6. (1 pt) setDerivatives2Formulas/s2_2_6f.pg
   If \( f(x) = (6x^2 - 6)(2x + 5) \), find \( f'(x) \).

7. (1 pt) setDerivatives2Formulas/s2_2_6.pg
   If \( f(x) = (7x^2 - 2)(7x + 6) \), find \( f'(x) \).

   Find \( f'(5) \).

8. (1 pt) setDerivatives2Formulas/s2_2_6a.pg
   If \( f(x) = (2x^3 - 4)(4x + 3) \), find \( f'(x) \).

9. (1 pt) setDerivatives2Formulas/s2_2_13.pg
   If \( f(x) = (t^2 + 5t + 6)(6t^2 + 6) \), find \( f'(t) \).

   Find \( f'(1) \).

10. (1 pt) setDerivatives2Formulas/s2_2_13a.pg
    Let \( f(t) = (t^2 + 5t + 6)(6t^2 + 6) \).
    (a) \( f'(t) = \) ____________________________
    (b) \( f'(3) = \) ____________________________
    [NOTE: Your answer to part (a) should be a function in terms of the variable 't' and not a number! Your answer to part (b) should be a number.]

11. (1 pt) setDerivatives2Formulas/s2_2_7.pg
    If \( f(x) = 7t^{-3} \), find \( f'(t) \).

    Find \( f'(1) \).

12. (1 pt) setDerivatives2Formulas/s2_2_8f.pg
    If \( f(t) = \frac{\sqrt{6}}{t^6} \), find \( f'(t) \).

13. (1 pt) setDerivatives2Formulas/s2_2_8a.pg
    If \( f(t) = \frac{\sqrt{5}}{t^3} \), find \( f'(t) \).

14. (1 pt) setDerivatives2Formulas/s2_2_8a.pg
    Let \( f(x) = \frac{4x^3 + 4x + 7}{\sqrt{x}} \), find \( f'(x) \).

15. (1 pt) setDerivatives2Formulas/s2_2_11a.pg
    Let \( f(x) = \frac{7}{3x + 4} \)
    \( f'(x) = \) ____________________________

16. (1 pt) setDerivatives2Formulas/s2_2_11a.pg
    Let \( f(x) = \frac{5 - x^2}{6 + x^2} \)
    find \( f'(x) \).

17. (1 pt) setDerivatives2Formulas/s2_2_12f.pg
    If \( f(x) = \frac{5 - x^2}{3 + x^2} \)
    find \( f'(x) \).

18. (1 pt) setDerivatives2Formulas/s2_2_12a.pg
    If \( f(x) = \frac{5 - x^2}{3 + x^2} \)
    find \( f'(x) \).

19. (1 pt) setDerivatives2Formulas/s2_2_15a.pg
    If \( f(x) = \frac{4x^3 + 4x + 7}{\sqrt{x}} \), find \( f'(x) \).
Find $f'(1)$.

20. (1 pt) setDerivatives2Formulas/s2_2_15a.pg
If $f(x) = \frac{2x^2 + 3x + 4}{\sqrt{x}}$, find $f'(4)$.

21. (1 pt) setDerivatives2Formulas/s2_2_16.pg
If $f(x) = \frac{\sqrt{x} - 3}{\sqrt{x} + 3}$
find $f'(x)$.

Find $f'(5)$.

22. (1 pt) setDerivatives2Formulas/s2_2_16a.pg
Let $f(x) = \frac{x - 3}{x + 3}$.
$f'(9) = $

23. (1 pt) setDerivatives2Formulas/s2_2_17.pg
If $f(x) = \sqrt[3]{x^3}$, find $f'(x)$.

Find $f'(4)$.

24. (1 pt) setDerivatives2Formulas/s2_2_17a.pg
If $f(x) = \sqrt[7]{x}$, find $f'(x)$.

25. (1 pt) setDerivatives2Formulas/s2_2_22.b.js
If $f(x) = \sqrt[4]{x^4} + 10$, find $f'(x)$.

26. (1 pt) setDerivatives2Formulas/s2_2_22.pg
If $f(x) = 4 + \frac{2}{x} + \frac{6}{x^2}$, find $f'(x)$.

Find $f'(3)$.

27. (1 pt) setDerivatives2Formulas/s2_2_22b.pg
If $f(x) = 6 + \frac{2}{x} + \frac{3}{x^2}$, find $f'(x)$.

28. (1 pt) setDerivatives2Formulas/s2_2_11b.pg
Let $f(x) = 5x^5 \sqrt[3]{x} + \frac{3}{x^2 \sqrt[4]{x}}$

$f'(x) = $

29. (1 pt) setDerivatives2Formulas/s2_2_29.pg
If $f(x) = 7x \sqrt[3]{x} + \frac{1}{x \sqrt[4]{x}}$, find $f'(x)$.

Find $f'(3)$.

30. (1 pt) setDerivatives2Formulas/s2_2_29a.pg
If $f(x) = 5x \sqrt[3]{x} + \frac{1}{x \sqrt[4]{x}}$, find $f'(9)$.

31. (1 pt) setDerivatives2Formulas/s2_2_33.pg
If $f(x) = 7x^3 - \frac{2}{x^4}$, find $f'(x)$.

Find $f'(1)$.

32. (1 pt) setDerivatives2Formulas/s2_2_33a.pg
If $f(x) = \frac{7x^4 - 4}{x^2}$, find $f'(x)$.

33. (1 pt) setDerivatives2Formulas/s2_2_33b.pg
If $f(x) = -5x^3 + 2x^3 + 5x^3$, find $f'(x)$.

34. (1 pt) setDerivatives2Formulas/s2_2_34.pg
If $f(x) = 5\sqrt[3]{x^3} - 3\sqrt[5]{x} + 6$, find $f'(x)$.

Find $f'(4)$.

35. (1 pt) setDerivatives2Formulas/s2_2_34a.pg
If $f(x) = \sqrt[3]{x^3} - 2\sqrt[5]{x} + 5$, find $f'(4)$.

36. (1 pt) setDerivatives2Formulas/c2s5p1.pg
Calculate $G'(2)$ to 3 significant figures where
$G(x) = (4x - 2)^{10}(1x^2 + 4x + 4)^{12}$

37. (1 pt) setDerivatives2Formulas/c2s5p3.pg
Calculate $f'(-4)$ to 3 significant figures where
$f(t) = (2t^2 - 3t - 3)^{-8}$

Tip: You can enter an answer such as 3.14e-1 for 0.314.

38. (1 pt) setDerivatives2Formulas/c2s5p4.pg
Find the y-intercept of the tangent line to
$y = \frac{-1.8}{\sqrt{3} + 1x}$
at $(1.5, -0.848528137423857)$.

39. (1 pt) setDerivatives2Formulas/d2.pg
Let $f(x) = 4e^{x^2} + e^x$.

$f'(0) = $

[NOTE: A small algebraic manipulation is needed first to get $f(x)$ into a form so that the derivative can be taken.]

40. (1 pt) setDerivatives2Formulas/d3.pg
Given that
$f(x) = x^9h(x)$
$h(-1) = 4$
$h'(-1) = 7$,

Calculate $f'(-1)$.  

41. (1 pt) setDerivatives2Formulas/ns3_2_4.pg
Find the derivative of the function
\[ g(x) = (2x^2 + 1x + 4)e^x \]
\[ g'(x) = \]

42. (1 pt) setDerivatives2Formulas/ns3_2_5.pg
Find the derivative of the function
\[ g(x) = \frac{e^x}{1 - 4x} \]
\[ g'(x) = \]

43. (1 pt) setDerivatives2Formulas/ur_dr_2_1.pg
Given
\[ f(x) = \frac{x}{x + \frac{1}{3}} \]
The derivative function is given by
\[ f'(x) = \]

44. (1 pt) setDerivatives2Formulas/ur_dr_2_2.pg
If \( f(x) = 3e^x - 7x^4 + 16 \), find \( f'(x) \).

45. (1 pt) setDerivatives2Formulas/ur_dr_2_3.pg
Let \( f(x) = 1 \).
Then \( f'(8) = \)
And after simplifying \( f'(x) = \)

46. (1 pt) setDerivatives2Formulas/ur_dr_2_4.pg
Let \( f(x) = -2x + 15 \).
Then \( f'(-15) = \)
And after simplifying \( f'(x) = \)

47. (1 pt) setDerivatives2Formulas/ur_dr_2_5.pg
Let \( f(x) = x^2 + 9x - 13 \).
Then \( f'(5) = \)
And after simplifying \( f'(x) = \)

48. (1 pt) setDerivatives2Formulas/ur_dr_2_6.pg
Let \( f(x) = -6x(x - 2) \).
Then \( f'(1) = \)
And after simplifying \( f'(x) = \)
Hint: You may want to expand and simplify the expression for \( f(x) \) first.

49. (1 pt) setDerivatives2Formulas/ur_dr_2_7.pg
Let \( f(x) = 2x^3 + 5x - 8 \).
Then \( f'(3) = \)
And after simplifying \( f'(x) = \)

50. (1 pt) setDerivatives2Formulas/ur_dr_2_8.pg
Let \( f(x) = \frac{1}{x + 8} \).
Then \( f'(-3) = \)
And after simplifying \( f'(x) = \)

51. (1 pt) setDerivatives2Formulas/ur_dr_2_9.pg
Let \( f(x) = \frac{6x}{x - 3} \).
Then \( f'(-4) = \)
And after simplifying \( f'(x) = \)

52. (1 pt) setDerivatives2Formulas/ur_dr_2_10.pg
Let \( f(x) = \sqrt{11 + x} \).
Then \( f'(53) = \)
And after simplifying \( f'(x) = \)

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
Find the slope of the tangent line to the curve 
\[ 2(x^2 + y^2) = 25(x^2 - y^2) \]
at the point (3, 1).

Find the equation of the tangent line to the curve (a lemniscate)
\[ 2(x^2 + y^2) = 25(x^2 - y^2) \]
at the point (3, 1). The equation of this tangent line can be written in the form 
\[ y = mx + b \]
where \( m = \) _______ and where \( b = \) _______

Use implicit differentiation to find the slope of the tangent line to the curve
\[ \frac{y}{x + 9y} = x^2 + 4 \]
at the point \( (1, \frac{5}{9}) \).

Find \( y' \) by implicit differentiation. Match the expressions defining \( y' \) implicitly with the letters labeling the expressions for \( y' \).

1. \( 6 \sin(x - y) = 3y \sin x \)
2. \( 6 \cos(x - y) = 3y \cos x \)
3. \( 6 \sin(x - y) = 3y \cos x \)
4. \( 6 \cos(x - y) = 3y \sin x \)

Use implicit differentiation to find the equation of the tangent line to the curve \( xy^3 + xy = 8 \) at the point \( (4, 1) \). The equation of this tangent line can be written in the form 
\[ y = mx + b \]
where \( m = \) _______ and where \( b = \) _______.

Find the slope of the tangent line to the curve
\[ \sqrt{2x + 2y} + \sqrt{4xy} = 10.9 \]
1. (1 pt) setDerivatives3WordProblems/s2_3_1.pg
A particle moves along a straight line and its position at time $t$ is given by $s(t) = 2t^3 - 21t^2 + 36t$ where $s$ is measured in feet and $t$ in seconds.
Find the velocity (in ft/sec) of the particle at time $t = 0$:

The particle stops moving (i.e. is in a rest) twice, once when $t = A$ and again when $t = B$ where $A < B$. $A$ is ________ and $B$ is ________.

What is the position of the particle at time $t = 0$?

Finally, what is the TOTAL distance the particle travels between time 0 and time 14?

2. (1 pt) setDerivatives3WordProblems/s2_3_8.pg
If a ball is thrown vertically upward from the roof of 48 foot building with a velocity of 64 ft/sec, its height after $t$ seconds is $s(t) = 48 + 64t - 16t^2$. What is the maximum height the ball reaches?

What is the velocity of the ball when it hits the ground (height 0)?

3. (1 pt) setDerivatives3WordProblems/s2_3_10.pg
The area of a square with side $s$ is $A(s) = s^2$. What is the rate of change of the area of a square with respect to its side length when $s = 3$?

4. (1 pt) setDerivatives3WordProblems/s2_3_24.pg
The population of a slowly growing bacterial colony after $t$ hours is given by $p(t) = 5t^2 + 20t + 100$. Find the growth rate after 4 hours.

5. (1 pt) setDerivatives3WordProblems/s2_3_27.pg
The cost of producing $x$ units of stuffed alligator toys is $c(x) = 0.004x^2 + 8x + 4000$. Find the marginal cost at the production level of 1000 units.

6. (1 pt) setDerivatives3WordProblems/s2_7_41.pg
A mass attached to a vertical spring has position function given by $s(t) = 5\sin(3t)$ where $t$ is measured in seconds and $s$ in inches.
Find the velocity at time $t = 1$.
Find the acceleration at time $t = 1$.

7. (1 pt) setDerivatives3WordProblems/c2s3p1.pg
The mass of the part of a rod that lies between its left end and a point $x$ meters to the right is $4x^5$ kg. The linear density of the rod at 1 meters is ________ kg/meter and at 3 meters the density is ________ kg/meter.

8. (1 pt) setDerivatives3WordProblems/c2s3p2.pg
If $f$ is the focal length of a convex lens and an object is placed at a distance $p$ from the lens, then its image will be at a distance $q$ from the lens, where $f$, $p$, and $q$ are related by the lens equation $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$.

What is the rate of change of $p$ with respect to $q$ if $q = 8$ and $f = 6$? (Make sure you have the correct sign for the rate.)

9. (1 pt) setDerivatives3WordProblems/c2s7p2.pg
A particle moves along a straight line with equation of motion $s = t^4 - 3t^3$. Find the value of $t$ (other than 0) at which the acceleration is equal to zero.

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, © UR
1. Evaluate the limit \( \lim_{x \to 0} \frac{\sin 3x}{7x} \)

2. Evaluate the limit \( \lim_{x \to 0} \frac{\sin 6x}{x} \)

3. Evaluate the limit \( \lim_{x \to 0} \frac{\tan 8x}{\sin 2x} \)

4. Evaluate the limit \( \lim_{x \to 0} \frac{\tan x}{4x} \)

5. If \( f(x) = \cos x - 5 \tan x \), then \( f'(x) = \)

6. If \( f(x) = \cos x - 4 \tan x \), then \( f'(x) = \)

7. Let \( f(x) = 3 \cos x + 6 \tan x \)

\[ f'(x) = \]
\[ f'(\frac{\pi}{4}) = \]

8. If \( f(x) = 3 \sin x + 7 \cos x \), then \( f'(x) = \)

9. Let \( f(x) = 6 \sin x + 8 \cos x \)

\[ f'(x) = \]
\[ f'(\frac{\pi}{4}) = \]

[Note: When entering trigonometric functions into Webwork, you must include parentheses around the argument. I.e. “\sin x” would not be accepted but “\sin(x)” would.]

10. If \( f(x) = \frac{4 \sin x}{2 + \cos x} \)
then \( f'(x) = \)

11. If \( f(x) = \frac{3 \sin x}{3 + \cos x} \)
find \( f'(x) \).

12. If \( f(x) = \frac{4 \tan x}{x} \), find \( f'(x) \).

13. If \( f(x) = \tan x - \frac{5}{\sec x} \)
find \( f'(x) \).

14. Let \( f(x) = -\frac{4 \tan x + 5}{\sec x} \)

\[ f'(x) = \]
\[ f'(\frac{\pi}{4}) = \]

15. If \( f(x) = 4x(\sin x + \cos x) \), find \( f'(x) \).

16. Let \( f(x) = -10x(\sin x + \cos x) \)

\[ f'(x) = \]
\[ f'(\frac{\pi}{4}) = \]

17. Let \( f(x) = \frac{8x}{\sin x + \cos x} \)

\[ f'(2\pi) = \]
18. (1 pt) setDerivatives4Trig/s2_A_30f.pg
If \( f(x) = 2\sin x \cos x \), then \( f'(x) = \) __________

19. (1 pt) setDerivatives4Trig/s2_A_30.pg
If \( f(x) = 5\sin x \cos x \), find \( f'(x) \).

Find \( f'(4) \).

20. (1 pt) setDerivatives4Trig/s2_A_30a.pg
Let \( f(x) = -3\sin x \cos x \).
\[ f'(-\frac{\pi}{2}) = \] __________

21. (1 pt) setDerivatives4Trig/s2_A_31.pg
If \( f(x) = \frac{2\tan x}{\sec x} \), find \( f'(x) \).

Find \( f'(1) \).

22. (1 pt) setDerivatives4Trig/s2_A_32.pg
Find the equation of the tangent line to the curve \( y = 4\sin x \) at the point \((\pi/6, 2)\).

The equation of this tangent line can be written in the form \( y = mx + b \) where
\( m = \) __________
and \( b = \) __________

23. (1 pt) setDerivatives4Trig/s2_A_33.pg
Find the equation of the tangent line to the curve \( y = 4\tan x \) at the point \((\pi/4, 4)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m \) is: __________
and where \( b \) is: __________

24. (1 pt) setDerivatives4Trig/s2_A_34_pg
Find the equation of the tangent line to the curve \( y = 3\sec x - 6\cos x \) at the point \((\pi/3, 3)\). The equation of this tangent line can be written in the form \( y = mx + b \) where \( m \) is: __________
and where \( b \) is: __________

25. (1 pt) setDerivatives4Trig/s2_A_35.pg
Find the equation of the tangent line to the curve \( y = 2\cos x \) at the point \((\pi, -2\pi)\).

The equation of this tangent line can be written in the form \( y = mx + b \) where
\( m = \) __________
and \( b = \) __________

26. (1 pt) setDerivatives4Trig/ur_dr_A_1.pg
Let \( f(x) = -\frac{6\sin x}{4\sin x + 2\cos x} \).

Then \( f'(x) = \) __________

The equation of the tangent line to \( y = f(x) \) at \( a = \pi/6 \) can be written in the form \( y = mx + b \) where
\( m = \) __________ and \( b = \) __________

27. (1 pt) setDerivatives4Trig/c2s5p2.pg
Match the functions and their derivatives:

1. \( y = \cos(\tan(x)) \)
2. \( y = \cos^3(x) \)
3. \( y = \tan(x) \)
4. \( y = \sin(x) \tan(x) \)

A. \( y' = \sin(x) + \tan(x) \sec(x) \)
B. \( y' = -3\cos^3(x) \tan(x) \)
C. \( y' = 1 + \tan^2(x) \)
D. \( y' = -\sin(\tan(x))/\cos^2(x) \)

28. (1 pt) setDerivatives4Trig/s2_A_32.pg
Find the 69th derivative of \( \sin(x) \) by finding the first few derivatives and observing the pattern that occurs.

\( (\sin(x))^{(69)} = \) __________

29. (1 pt) setDerivatives4Trig/s2_A_33.pg
Let \( h(t) = \tan(2t + 6) \). Then \( h'(1) \) is __________
and \( h''(1) \) is __________
<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
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<tbody>
<tr>
<td>1.</td>
<td>Find ( f'(5) ). ( f(x) = (x^2 + 4x + 8)^4 ) find ( f'(x) ).</td>
</tr>
<tr>
<td>2.</td>
<td>Let ( f(x) = (x^3 + 5x + 3)^3 ). Find ( f'(1) ).</td>
</tr>
<tr>
<td>3.</td>
<td>Let ( f(x) = (x^3 + 5x + 4)^3 ). Find ( f'(4) ).</td>
</tr>
<tr>
<td>4.</td>
<td>If ( f(x) = (3x + 5)^{-2} ), find ( f'(x) ).</td>
</tr>
<tr>
<td>5.</td>
<td>If ( f(x) = \sqrt{2x + 7} ), find ( f'(x) ).</td>
</tr>
<tr>
<td>6.</td>
<td>Let ( f(x) = \sqrt{2x^2 + 3x + 3} ). Find ( f'(2) ).</td>
</tr>
<tr>
<td>7.</td>
<td>If ( f(x) = \sin(x^4) ), find ( f'(x) ).</td>
</tr>
<tr>
<td>8.</td>
<td>If ( f(x) = \sin^4 x ), find ( f'(x) ).</td>
</tr>
<tr>
<td>9.</td>
<td>Let ( f(x) = 7 \sin^3 x ). Find ( f'(5) ).</td>
</tr>
<tr>
<td>10.</td>
<td>If ( f(x) = \tan 2x ), find ( f'(x) ).</td>
</tr>
<tr>
<td>11.</td>
<td>Let ( f(x) = \cos(6x + 9) ). Find ( f'(1) ).</td>
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<tr>
<td>12.</td>
<td>If ( f(x) = \cos(5x + 6) ), find ( f'(x) ).</td>
</tr>
<tr>
<td>13.</td>
<td>Let ( f(x) = 5 \csc(6x) ). Find ( f'(5) ).</td>
</tr>
<tr>
<td>14.</td>
<td>If ( f(x) = 3 \sec(3x) ), find ( f'(x) ).</td>
</tr>
<tr>
<td>15.</td>
<td>Let ( f(x) = 7 \sin(\cos x) ). Find ( f'(1) ).</td>
</tr>
<tr>
<td>16.</td>
<td>Let ( F(x) = f(x^4) ) and ( G(x) = (f(x))^4 ). Find ( F'(a) = 12 ) and ( G'(a) = 5 ).</td>
</tr>
<tr>
<td>17.</td>
<td>Let ( F(x) = f(f(x)) ) and ( G(x) = (F(x))^2 ). Find ( F'(5) = 9 ) and ( G'(5) = 9 ).</td>
</tr>
<tr>
<td>18.</td>
<td>Let ( F(x) = f(f(x)) ) and ( G(x) = (F(x))^2 ). Find ( F'(5) = 9 ) and ( G'(5) = 9 ).</td>
</tr>
<tr>
<td>19.</td>
<td>If ( \frac{d}{dx}(2x^4) = 5x^2 ), calculate ( f'(x) ).</td>
</tr>
<tr>
<td>20.</td>
<td>Let ( f(x) = -5e^{3x} ). Find ( f'(x) ).</td>
</tr>
</tbody>
</table>
21. Let \( f(x) = (3x^2 + 4)^6(5x^2 + 3)^{11} \)
\[ f'(x) = \]

22. Let \( f(x) = 3\cos(\sin(x^2)) \)
\[ f'(x) = \]

23. If \( f(x) = \cos(\sin(x^2)) \), find \( f'(x) \).
\[ f'(1) = \]

24. Let \( f(x) = \sqrt{\sin(e^{x^2}\sin(x))} \)
\[ f'(x) = \]
1. (1 pt) setDerivatives6InverseTrig/sc3_6_25.pg
If \( f(x) = 5 \arcsin(x^3) \), find \( f'(x) \).

2. (1 pt) setDerivatives6InverseTrig/sc3_6_25a.pg
Let
\[
 f(x) = 2 \sin^{-1}(x^3)
\]
\( f'(x) = \quad \) 

3. (1 pt) setDerivatives6InverseTrig/ur_dr_6_1.pg
If \( f(x) = 7x \arcsin(x) \), find \( f'(x) \).

Find \( f'(0.3) \).

4. (1 pt) setDerivatives6InverseTrig/sc3_6_26.pg
If \( f(x) = 4x^2 \arctan(2x^4) \), find \( f'(x) \).

5. (1 pt) setDerivatives6InverseTrig/sc3_6_26a.pg
Let
\[
 f(x) = x^4 \tan^{-1}(4x)
\]
\( f'(x) = \quad \)

\( \text{Note: The WeBWorK system will accept } \arctan(x) \text{ but not } \tan^{-1}(x) \text{ as the inverse of } \tan(x). \)

6. (1 pt) setDerivatives6InverseTrig/ur_dr_6_2.pg
If \( f(x) = 5 \arctan(8x) \), find \( f'(x) \).

Find \( f'(5) \).

7. (1 pt) setDerivatives6InverseTrig/sc3_6_27.pg
If \( f(x) = 6 \arctan(6e^x) \), find \( f'(x) \).

8. (1 pt) setDerivatives6InverseTrig/sc3_6_27a.pg
Let
\[
 f(x) = \tan^{-1}(9x)
\]
\( f'(x) = \quad \)

9. (1 pt) setDerivatives6InverseTrig/sc3_6_32.pg
If \( f(x) = 5 \sin(4x) \arcsin(x) \), find \( f'(x) \).

10. (1 pt) setDerivatives6InverseTrig/sc3_6_32a.pg
Let
\[
 f(x) = 4 \cos(x) \sin^{-1}(x)
\]
\( f'(x) = \quad \)

\( \text{Note: The webwork system will accept } \arcsin(x) \text{ and not } \sin^{-1}(x) \text{ as the inverse of } \sin(x). \)

11. (1 pt) setDerivatives6InverseTrig/sc3_6_33a.pg
Let
\[
 f(x) = \tan^{-1}(6x)
\]
\( f'(x) = \quad \)

12. (1 pt) setDerivatives6InverseTrig/sc3_6_33.pg
If \( f(x) = 2 \arctan(3 \sin(2x)) \), find \( f'(x) \).

13. (1 pt) setDerivatives6InverseTrig/osu_dr_6_3.pg
Let
\[
 y = \tan^{-1}\left(\sqrt{6x^2 - 1}\right)
\]
Then \( \frac{dy}{dx} = \quad \)
1. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_2f.pg}\) 
If \(f(x) = 3 \ln(3 + x)\), find \(f'(x)\).

2. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_2f.pg}\) 
If \(f(x) = 5 \ln(8 + x)\), find \(f'(x)\).

Find \(f''(4)\).

3. \((1 \text{ pt}) \text{ setDerivatives7Log/mec1.pg}\) 
Let 
\[f(x) = -5 \ln(2x)\]

\(f'(x) = \) 
\(f''(2) = \) 

4. \((1 \text{ pt}) \text{ setDerivatives7Log/mec6.pg}\) 
Let 
\[f(x) = \ln(x^7)\]

\(f'(x) = \) 
\(f''(e^2) = \) 

5. \((1 \text{ pt}) \text{ setDerivatives7Log/mec4.pg}\) 
Let 
\[f(x) = [\ln(x)]^4\]

\(f'(x) = \) 
\(f''(e^5) = \) 

6. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_1l.pg}\) 
If \(f(x) = 2\sqrt{x}\ln(x)\), find \(f'(x)\).

Find \(f''(1)\).

7. \((1 \text{ pt}) \text{ setDerivatives7Log/mec3.pg}\) 
Let 
\[f(x) = -2x^4 \ln x\]

\(f'(x) = \) 
\(f''(e^3) = \) 

8. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_4f.pg}\) 
If \(f(x) = 4 \cos(3\ln(x))\), find \(f'(x)\).

9. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_4f.pg}\) 
If \(f(x) = 5 \cos(3\ln(x))\), find \(f'(x)\).

Find \(f''(3)\).

10. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_16.pg}\) 
If \(f(x) = 3 \ln(3x + 7 \ln(x))\), find \(f'(x)\).

Find \(f''(3)\).

11. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_17.pg}\) 
If \(f(x) = 7 \log_3(x)\), find \(f'(4)\).

12. \((1 \text{ pt}) \text{ setDerivatives7Log/mec5.pg}\) 
Let 
\[f''(x) = -3 \log_3(x)\]

13. \((1 \text{ pt}) \text{ setDerivatives7Log/mec12.pg}\) 
Let 
\[f''(x) = 3^x \log_9(x)\]

14. \((1 \text{ pt}) \text{ setDerivatives7Log/osu_dr_7_2.pg}\) 
Find the indicated derivatives.
(a) \[\frac{d}{dx} \left( e^{x^5 + \log_3(\pi)} \right) = \]  
(b) \[\frac{d}{dx} \left( (\sqrt[3]{x})^\ln(x) \right) = \]

15. \((1 \text{ pt}) \text{ setDerivatives7Log/mec9.pg}\) 
Let 
\[f'(x) = \ln[x^3(x + 7)^7(x^2 + 6)^4]\]

16. \((1 \text{ pt}) \text{ setDerivatives7Log/mec10.pg}\) 
Let 
\[f(x) = \frac{x^3(x - 6)^4}{(x^2 + 6)^8}\]

Use logarithmic differentiation to determine the derivative.
\(f'(6) = \)

17. \((1 \text{ pt}) \text{ setDerivatives7Log/sc3.7_25.pg}\) 
If \(f(x) = (7x - 2)^4 + (6x^2 + 2)^4\), find \(f'(4)\).

18. \((1 \text{ pt}) \text{ setDerivatives7Log/mec8.pg}\) 
Let 
\[f(x) = \ln \sqrt{\frac{8x - 5}{5x + 5}}\]

19. \((1 \text{ pt}) \text{ setDerivatives7Log/mec7.pg}\) 
Let 
\[f(x) = x^{8x}\]

Use logarithmic differentiation to determine the derivative.
\(f'(x) = \)

20. \((1 \text{ pt}) \text{ setDerivatives7Log/mec7.pg}\) 
Let 
\[f(x) = x^{8x}\]

Use logarithmic differentiation to determine the derivative.
\(f'(1) = \)
21. If \( f(x) = 2x^3 \), find \( f'(3) \).

22. If \( f(x) = 3 \sin(x) + 2x^3 \), find \( f''(2) \).

23. If \( f(x) = 10(\sin(x))^x \), find \( f'(1) \).

24. If \( f(x) = 4x\ln(x) \), find \( f'(3) \).

25. Let \( y = x^{\log(x)} \).

Then

\[ dy \]
\[ dx \]

**Note.** You must express your answer in terms of natural logs, as Webwork doesn’t understand how to evaluate logarithms to other bases.

26. Find \( \frac{dy}{dx} \) for each of the following functions:

- \( y = \ln \left( \frac{6x - 8}{x\sqrt{x^2 + 1}} \right) \)
- \( y = x^{\cos(x)} \)

27. If \( f(x) = e^9 + \ln(8) \), then \( f'(x) = \) ____.
1. Let \( xy = 5 \)
and let \( \frac{dy}{dt} = 3 \)
Find \( \frac{dx}{dt} \) when \( x = 2 \).

2. Let \( A \) be the area of a circle with radius \( r \). If \( \frac{dr}{dt} = 5 \), find \( \frac{dA}{dt} \) when \( r = 1 \).

3. A spherical snowball is melting in such a way that its diameter is decreasing at a rate of 0.2 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 15 cm? (Note the answer is a positive number).

4. The altitude of a triangle is increasing at a rate of 2.5 centimeters/minute while the area of the triangle is increasing at a rate of 5.000 square centimeters/minute. At what rate is the base of the triangle changing when the altitude is 9.000 centimeters and the area is 97.000 square centimeters?

5. The altitude of a triangle is increasing at a rate of 2.000 centimeters/minute while the area of the triangle is increasing at a rate of 2.500 square centimeters/minute. At what rate is the base of the triangle changing when the altitude is 10.500 centimeters and the area is 87.000 square centimeters?

6. When air expands adiabatically (without gaining or losing heat), its pressure \( P \) and volume \( V \) are related by the equation \( PV^{1.4} = C \) where \( C \) is a constant. Suppose that at a certain instant the volume is 590 cubic centimeters and the pressure is 77 kPa and is decreasing at a rate of 14 kPa/minute. At what rate in cubic centimeters per minute is the volume increasing at this instant?

(Pa stands for Pascal – it is equivalent to one Newton/(meter squared); kPa is a kiloPascal or 1000 Pascals.)

7. At noon, ship A is 40 nautical miles due west of ship B. Ship A is sailing west at 25 knots and ship B is sailing north at 20 knots.

8. At noon, ship A is 40 nautical miles due west of ship B. Ship A is sailing west at 22 knots and ship B is sailing north at 22 knots. How fast (in knots) is the distance between the ships changing at 3 PM? (Note: 1 knot is a speed of 1 nautical mile per hour.)

9. Gravel is being dumped from a conveyor belt at a rate of 20 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter and height are always the same. How fast is the height of the pile increasing when the pile is 23 feet high? Recall that the volume of a right circular cone with height \( h \) and radius of the base \( r \) is given by \( V = \frac{1}{3}\pi r^2 h \).

10. Gravel is being dumped from a conveyor belt at a rate of 10 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter and height are always the same. How fast is the height of the pile increasing when the pile is 18 feet high? Recall that the volume of a right circular cone with height \( h \) and radius of the base \( r \) is given by \( V = \frac{1}{3}\pi r^2 h \).

11. A street light is at the top of a 16 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 4 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 30 ft from the base of the pole?

12. A street light is at the top of a 20 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 5 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 50 ft from the base of the pole?

Note: You should draw a picture of a right triangle with the vertical side representing the pole, and the other end of the hypotenuse representing the tip of the woman’s shadow. Where
does the woman fit into this picture? Label her position as a variable, and label the tip of her shadow as another variable. You might like to use similar triangles to find a relationship between these two variables.

13. (1 pt) setDerivatives8RelatedRates/c2s8p5.pg
A plane flying with a constant speed of 24 km/min passes over a ground radar station at an altitude of 14 km and climbs at an angle of 50 degrees. At what rate, in km/min is the distance from the plane to the radar station increasing 2 minutes later?

14. (1 pt) setDerivatives8RelatedRates/c2s8p3.pg
Water is leaking out of an inverted conical tank at a rate of 11900.0 cubic centimeters per min at the same time that water is being pumped into the tank at a constant rate. The tank has height 14.000 meters and the diameter at the top is 6.000 meters. If the water level is rising at a rate of 23.000 centimeters per minute when the height of the water is 1.500 meters, find the rate at which water is being pumped into the tank in cubic centimeters per minute.

15. (1 pt) setDerivatives8RelatedRates/SRM_c2s8p3.pg
Water is leaking out of an inverted conical tank at a rate of 7500.0 cubic centimeters per min at the same time that water is being pumped into the tank at a constant rate. The tank has height 15.0 meters and the diameter at the top is 7.0 meters. If the water level is rising at a rate of 15.0 centimeters per minute when the height of the water is 4.0 meters, find the rate at which water is being pumped into the tank in cubic centimeters per minute. 

Note: Let "R" be the unknown rate at which water is being pumped in. Then you know that if $\frac{dV}{dt} = R - 7500.0$. Use geometry (similar triangles?) to find the relationship between the height of the water and the volume of the water at any given time. Recall that the volume of a cone with base radius r and height h is given by $\frac{1}{3}\pi r^2 h$.

16. (1 pt) setDerivatives8RelatedRates/ur_dr_81.pg
A particle is moving along the curve $y = 4\sqrt[3]{3x + 1}$. As the particle passes through the point $(5, 16)$, its x-coordinate increases at a rate of 2 units per second. Find the rate of change of the distance from the particle to the origin at this instant.
1. (1 pt) setDerivatives9Approximations/s2_9_13.png
Let \( y = 5x^2 \).
Find the differential \( dy \) when \( x = 3 \) and \( dx = 0.3 \).

2. (1 pt) setDerivatives9Approximations/s2_9_14.png
Let \( y = 5\sqrt{x} \).
Find the differential \( dy \) when \( x = 5 \) and \( dx = 0.3 \).

3. (1 pt) setDerivatives9Approximations/s2_9_7.png
Let \( y = 2x^2 + 7x + 3 \).
Find the differential \( dy \) when \( x = 1 \) and \( dx = 0.1 \).

4. (1 pt) setDerivatives9Approximations/s2_9_12.png
Let \( y = \tan(2x + 8) \).
Find the differential \( dy \) when \( x = 4 \) and \( dx = 0.4 \).

5. (1 pt) setDerivatives9Approximations/s2_9_35.png
The linear approximation at \( x = 0 \) to \( \sqrt{2 + 2x} \) is \( A + Bx \) where \( A = \) and where \( B = \) .

6. (1 pt) setDerivatives9Approximations/s2_9_36.png
The linear approximation at \( x = 0 \) to \( \sin(7x) \) is \( A + Bx \) where \( A = \) and where \( B = \) .

7. (1 pt) setDerivatives9Approximations/s2_9_38.png
The linear approximation at \( x = 0 \) to \( \frac{1}{\sqrt{1-x}} \) is \( A + Bx \) where \( A = \) and where \( B = \) .

8. (1 pt) setDerivatives9Approximations/s2_9_19.png
Use linear approximation, i.e. the tangent line, to approximate \( \sqrt{49.4} \) as follows:
Let \( f(x) = \sqrt{x} \). The equation of the tangent line to \( f(x) \) at \( x = 49 \) can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \) .
Using this, we find our approximation for \( \sqrt{49.4} \).

9. (1 pt) setDerivatives9Approximations/s2_9_20.png
Use linear approximation, i.e. the tangent line, to approximate \( \sqrt{125.4} \) as follows:
Let \( f(x) = \sqrt{x} \). The equation of the tangent line to \( f(x) \) at \( x = 125 \) can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \) .
Using this, we find our approximation for \( \sqrt{125.4} \).

10. (1 pt) setDerivatives9Approximations/s2_9_7.png
Use linear approximation, i.e. the tangent line, to approximate \( 18.2^2 \) as follows:
Let \( f(x) = x^2 \) and find the equation of the tangent line to \( f(x) \) at \( x = 18 \).

Using this, find your approximation for \( 18.2^2 \).

11. (1 pt) setDerivatives9Approximations/s2_9_22.png
Use linear approximation, i.e. the tangent line, to approximate \( 2.6^2 \) as follows:
Let \( f(x) = x^2 \). The equation of the tangent line to \( f(x) \) at \( x = 3 \) can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \) .
Using this, we find our approximation for \( 2.6^2 \).

12. (1 pt) setDerivatives9Approximations/s2_9_20.png
Use linear approximation, i.e. the tangent line, to approximate \( 4.9^4 \) as follows:
Let \( f(x) = x^4 \). The equation of the tangent line to \( f(x) \) at \( x = 5 \) can be written in the form \( y = mx + b \) where \( m = \) and where \( b = \) .
Using this, we find our approximation for \( 4.9^4 \).

13. (1 pt) setDerivatives9Approximations/s2_9_A.png
Use linear approximation, i.e. the tangent line, to approximate \( \frac{1}{0.204} \) as follows:
Let \( f(x) = \frac{1}{x} \) and find the equation of the tangent line to \( f(x) \) at a "nice" point near 0.204. Then use this to approximate \( \frac{1}{0.204} \).

14. (1 pt) setDerivatives9Approximations/ur_dr_9_1.png
Find the linear approximation of \( f(x) = \ln x \) at \( x = 1 \) and use it to estimate \( \ln 1.44 \).
\( L(x) = \) \( \ln 1.44 \approx \)

15. (1 pt) setDerivatives9Approximations/c2s9p8.png
Suppose that you can calculate the derivative of a function using the formula \( f'(x) = 2f(x) + 4x \). If the output value of the function at \( x = 2 \) is 3 estimate the value of the function at 2.012.

16. (1 pt) setDerivatives9Approximations/c2s9p6.png
The circumference of a sphere was measured to be 82,000 cm with a possible error of 0.50000 cm. Use linear approximation to estimate the maximum error in the calculated surface area.

Estimate the relative error in the calculated surface area.

17. (1 pt) setDerivatives9Approximations/c2s9p7.png
Use linear approximation to estimate the amount of paint in cubic centimeters needed to apply a coat of paint 0.03000 cm thick to a hemispherical dome with a diameter of 45.0000 meters.

18. (1 pt) setDerivatives9Approximations/c2s9p10.png
Let \( f(t) \) be the weight (in grams) of a solid sitting in a beaker of water. Suppose that the solid dissolves in such a way that the
rate of change (in grams/minute) of the weight of the solid at any time $t$ can be determined from the weight using the formula:

$$f'(t) = -5f(t)(1 + f(t))$$

If there is 5 grams of solid at time $t = 2$ estimate the amount of solid 1 second later.

19. (1 pt) setDerivatives9Approximations/nsc2s9p11.pg
Suppose you have a function $f(x)$ and all you know is that $f(2) = 8$ and the graph of its derivative is:

Use linear approximation to estimate $f(2.2)$: 
Is your answer a little too big or a little too small? (Enter TB or TS): 

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
1. (1 pt) setDerivatives10MaxMin/ur_dr_10.2.pg
The function \( f(x) = (6x - 6)e^{-5x} \) has one critical number. Find it.

2. (1 pt) setDerivatives10MaxMin/s3_1_25.pg
The function \( f(x) = 2x^3 - 24x^2 + 42x - 1 \) has two critical numbers. The smaller one equals _____ and the larger one equals _____.

3. (1 pt) setDerivatives10MaxMin/s3_1_11.pg
Consider the function \( f(x) = 5 - 3x^2 \), \(-5 \leq x \leq 2\). The absolute maximum value is ________
and this occurs at \( x = _____ \). The absolute minimum value is ________
and this occurs at \( x = _____ \).

4. (1 pt) setDerivatives10MaxMin/s3_1_18.pg
The function \( f(x) = 10 - 7x^4 \) has an absolute maximum value of _____ and this occurs at \( x = _____ \).

5. (1 pt) setDerivatives10MaxMin/s3_1_40.pg
Consider the function \( f(x) = -3x^2 + 6x - 2 \). The absolute maximum of \( f(x) \) is ________

6. (1 pt) setDerivatives10MaxMin/s3_1_42.pg
The function \( f(x) = 2x^3 - 36x^2 + 210x + 3 \) has one local minimum and one local maximum. This function has a local minimum at \( x = _____ \) with value ________ and a local maximum at \( x = _____ \) with value ________

7. (1 pt) setDerivatives10MaxMin/s3_1_43.pg
The function \( f(x) = -2x^3 + 36x^2 - 210x + 7 \) has one local minimum and one local maximum. This function has a local minimum at \( x = _____ \) with value ________ and a local maximum at \( x = _____ \) with value ________

8. (1 pt) setDerivatives10MaxMin/ur_dr_10.1.pg
The function \( f(x) = 2x + 4x^{-1} \) has one local minimum and one local maximum. This function has a local maximum at \( x = _____ \) with value ________ and a local minimum at \( x = _____ \) with value ________

9. (1 pt) setDerivatives10MaxMin/s3_3_3.pg
Consider the function \( f(x) = -2x^2 + 2x - 2 \). \( f(x) \) is increasing on the interval \((-\infty, A)\) and decreasing on the interval \([A, \infty)\) where \( A \) is the critical number.
Find \( A _____ \).
At \( x = A \), does \( f(x) \) have a local min, a local max, or neither? Type in your answer as LMIN, LMAX, or NEITHER. _____

10. (1 pt) setDerivatives10MaxMin/s3_3_10.pg
Consider the function \( f(x) = 12x^5 + 60x^4 - 100x^3 + 2 \). For this function there are four important intervals: \((-\infty, A), [A, B), [B, C], \text{ and } [C, \infty)\) where \( A, B, \) and \( C \) are the critical numbers.
Find \( A _____ \) and \( B _____ \) and \( C _____ \).
At each critical number \( A, B, \) and \( C \) does \( f(x) \) have a local min, a local max, or neither? Type in your answer as LMIN, LMAX, or NEITHER. _____
At \( A _____ \) At \( B _____ \) At \( C _____ \)

11. (1 pt) setDerivatives10MaxMin/sc4_2_53.pg
A University of Rochester student decided to depart from Earth after his graduation to find work on Mars. Before building a shuttle, he conducted careful calculations. A model for the velocity of the shuttle, from liftoff at \( t = 0 \) s until the solid rocket boosters were jettisoned at \( t = 55.3 \) s, is given by
\[
v(t) = 0.001161833t^3 - 0.080875t^2 + 34.58t + 14.6\]
(in feet per second). Using this model, estimate the absolute maximum value ________
and absolute minimum value ________
of the ACCELERATION of the shuttle between liftoff and the jettisoning of the boosters.

12. (1 pt) setDerivatives10MaxMin/s3_1_39.pg
Consider the function \( f(x) = 5x^3 - 8x + 4 \), \( 0 \leq x \leq 7 \). The absolute maximum of \( f(x) \) (on the given interval) is _____ and the absolute minimum of \( f(x) \) (on the given interval) is _____

13. (1 pt) setDerivatives10MaxMin/s3_1_44.pg
Consider the function \( f(x) = 2x^3 + 24x^2 - 54x + 3 \), \( -9 \leq x \leq 2 \). This function has an absolute minimum value equal to ________
and an absolute maximum value equal to ________

14. (1 pt) setDerivatives10MaxMin/s3_1_45.pg
Consider the function \( f(x) = x^4 - 50x^2 + 1 \), \( -4 \leq x \leq 11 \). This function has an absolute minimum value equal to ________
and an absolute maximum value equal to ________

15. (1 pt) setDerivatives10MaxMin/c3s3p1.pg
The function
\[
f(x) = 4x^3 + 12x^2 - 96x + 2
\]
is decreasing on the interval ________.
It is increasing on the interval ________ and the interval ________
The function has a local maximum at ________

16. (1 pt) setDerivatives10MaxMin/c3s3p2.pg
The function
\[
f(x) = -2x^3 + 2.97x^2 + 294.6336x - 0.3599999999999999
\]
is increasing on the interval (____, ____).
   It is decreasing on the interval (−∞, ____ ) and the interval (____, ∞).

The function has a local maximum at ____

17. (1 pt) setDerivatives10MaxMin/c3s3p3.pg
For $x \in [-15, 13]$ the function $f$ is defined by

$$f(x) = x^5(x + 5)^6$$

On which two intervals is the function increasing (enter intervals in ascending order)?

____ to ____

and ____ to ____

Find the region in which the function is positive: ____ to ____

Where does the function achieve its minimum? ____

18. (1 pt) setDerivatives10MaxMin/c3s3p4.pg
For $x \in [-13, 15]$ the function $f$ is defined by

$$f(x) = x^4(x - 2)^5$$

On which two intervals is the function increasing?

____ to ____

and ____ to ____

Find the region in which the function is positive: ____ to ____

Where does the function achieve its minimum? ____

19. (1 pt) setDerivatives10MaxMin/s3_3_6.pg
Consider the function $f(x) = -2x^3 + 36x^2 - 192x + 8$. For this function there are three important intervals: $(-∞, A], [A, B], \text{ and } [B, ∞)$ where $A$ and $B$ are the critical numbers.

Find $A$ __________

and $B$ __________

For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).

$(-∞, A]:$ __________

$[A, B]:$ __________

$[B, ∞):$ __________

20. (1 pt) setDerivatives10MaxMin/s3_3_6a.pg
Consider the function $f(x) = 4x + 9x^{-1}$. For this function there are four important intervals: $(-∞, A], [A, B), (B, C), \text{ and } [C, ∞)$ where $A$, $B$, and $C$ are the critical numbers and the function is not defined at $B$.

Find $A$ __________

and $B$ __________

and $C$ __________

For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).

$(-∞, A]:$ __________

$[A, B):$ __________

$(B, C]:$ __________

$[C, ∞):$ __________

21. (1 pt) setDerivatives10MaxMin/sc4_3_10.pg
Consider the function $f(x) = x^2 e^{4x}.$

For this function there are three important intervals: $(-∞, A], [A, B), \text{ and } [B, ∞)$ where $A$ and $B$ are the critical numbers.

Find $A$ __________

and $B$ __________

For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).

$(-∞, A]:$ __________

$[A, B]:$ __________

$[B, ∞):$ __________

22. (1 pt) setDerivatives10MaxMin/c3s4p1.pg
Answer the following questions for the function

$$f(x) = x \sqrt{x^2 + 1}$$

defined on the interval $[-7, 5]$.

A. $f(x)$ is concave down on the region _____ to _____

B. $f(x)$ is concave up on the region _____ to _____

C. The inflection point for this function is at _____

D. The minimum for this function occurs at _____

E. The maximum for this function occurs at _____

23. (1 pt) setDerivatives10MaxMin/c3s4p2.pg
Answer the following questions for the function

$$f(x) = x \sqrt{x^2 - 8x + 32 - 4\sqrt{x^2 - 8x + 32}}$$

defined on the interval $[-3, 9]$.

A. $f(x)$ is concave down on the region _____ to _____

B. $f(x)$ is concave up on the region _____ to _____

C. The inflection point for this function is at _____

D. The minimum for this function occurs at _____

E. The maximum for this function occurs at _____

24. (1 pt) setDerivatives10MaxMin/c3s4p2a.pg
Answer the following questions for the function

$$f(x) = x \sqrt{x^2 + 4x + 40 + 2\sqrt{x^2 + 4x + 40}}$$

defined on the interval $[-7, 3]$.

A. $f(x)$ is concave down on the region _____ to _____

B. $f(x)$ is concave up on the region _____ to _____

C. The inflection point for this function is at _____

D. The minimum for this function occurs at _____

E. The maximum for this function occurs at _____

25. (1 pt) setDerivatives10MaxMin/c3s4p3.pg
Answer the following questions for the function

$$f(x) = \frac{x^3}{x^4 - 36}$$

defined on the interval $[-16, 18]$.

Enter points, such as inflection points in ascending order, i.e. smallest x values first. Enter intervals in ascending order also.

The function $f(x)$ has vertical asymptotes at _____ and _____

$f(x)$ is concave up on the region _____ to _____ and _____ to
The inflection point for this function is

26. (1 pt) setDerivatives10MaxMin/c3s4p3graph.png
Answer the following questions for the function

\[ f(x) = \frac{x^3}{x^2 - 36}. \]

Enter points, such as inflection points in ascending order, i.e. smallest \( x \) values first. Enter "INF" for \( -\infty \) and "MINF" for \( -\infty \).

Enter intervals in ascending order also.

A. The function \( f(x) \) has two vertical asymptotes:
   \[ x = \quad \text{and} \quad x = \quad \]

B. \( f(x) \) has one local maximum and one local minimum:
   \[ \text{max} = \quad \text{and} \quad \text{min} = \quad \]

C. For each interval, tell whether \( f(x) \) is increasing (type in INC) or decreasing (type in DEC).
   \[ (-\infty, \text{max}) \quad (\text{max}, -6) \quad (-6, 0) \quad (0, 6) \quad (6, \text{min}) \quad (\text{min}, +\infty) \]

D. \( f(x) \) is concave up on the interval (_____ _____) and on the interval (_____ _____)

E. The inflection point for this function is _____

F. Sketch the graph of \( f(x) \) and bring it to class.

27. (1 pt) setDerivatives10MaxMin/c3s4p4.png
Answer the following questions for the function

\[ f(x) = \frac{x^3 + 6x^2 + 12x + 8}{x^2 + 4x + 3} \]

defined on the interval \([-21, 14]\).

Enter points, such as inflection points in ascending order, i.e. smallest \( x \) values first.

A. The function \( f(x) \) has vertical asymptotes at _____ and _____

B. \( f(x) \) is concave down on the region _____ to _____ and _____ to _____

28. (1 pt) setDerivatives10MaxMin/c3s4p5.png
Answer the following questions for the function

\[ f(x) = \sin^2 \left( \frac{x}{2} \right) \]

defined on the interval \([-5.3831852, 0.9707963]\).

Enter points, such as inflection points in ascending order, i.e. smallest \( x \) values first.

Remember that you can enter "pi" for \( \pi \) as part of your answer.

A. \( f(x) \) is concave down on the region _____ to _____

B. A global minimum for this function occurs at _____

C. A local maximum for this function which is not a global maximum occurs at _____

D. The function is increasing on _____ to _____ and on _____ to _____.

29. (1 pt) setDerivatives10MaxMin/c3s4p4.png
Consider the function \( f(x) = 12x^2 + 60x^4 - 240x^3 + 6 \).

\( f(x) \) has inflection points at (reading from left to right) \( x = D, E, \) and \( F \)

where \( D \) is ________

and \( E \) is ________

and \( F \) is ________

For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).

\[ (-\infty, D): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ [D, E]: \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ [E, F]: \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ [F, \infty): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

30. (1 pt) setDerivatives10MaxMin/c3s4p4.png
Consider the function \( f(x) = \frac{2x^2 + 7}{6x + 3} \).

For this function there are two important intervals: \((-\infty, A)\) and \((A, \infty)\) where the function is not defined at \( A \).

Find \( A \)

For each of the following intervals, tell whether \( f(x) \) is increasing (type in INC) or decreasing (type in DEC).

\[ (-\infty, A): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ (A, \infty): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

Note that this function has no inflection points, but we can still consider its concavity. For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).

\[ (-\infty, A): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ (A, \infty): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

Sketch the graph of \( f(x) \) and bring it to class.

31. (1 pt) setDerivatives10MaxMin/c3s4p4graph.png
Consider the function \( f(x) = \frac{2x + 7}{6x + 3} \).

For this function there are two important intervals: \((-\infty, A)\) and \((A, \infty)\) where the function is not defined at \( A \).

Find \( A \)

Find the horizontal asymptote of \( f(x) \):

\[ y = \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

Find the vertical asymptote of \( f(x) \):

\[ x = \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

For each of the following intervals, tell whether \( f(x) \) is increasing (type in INC) or decreasing (type in DEC).

\[ (-\infty, A): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ (A, \infty): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

Note that this function has no inflection points, but we can still consider its concavity. For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).

\[ (-\infty, A): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

\[ (A, \infty): \quad \quad \quad \quad \quad \quad \quad \quad \quad \]

Sketch the graph of \( f(x) \) and bring it to class.

32. (1 pt) setDerivatives10MaxMin/c3s4p5.png
Consider the function \( f(x) = 7(x - 2)^{2/3} \).

For this function there
are two important intervals: \((-\infty, A)\) and \((A, \infty)\) where A is a critical number.

Find A ______

For each of the following intervals, tell whether \(f(x)\) is increasing (type in INC) or decreasing (type in DEC).
\((-\infty, A): \) ______
\((A, \infty): \) ______

For each of the following intervals, tell whether \(f(x)\) is concave up (type in CU) or concave down (type in CD).
\((-\infty, A): \) ______
\((A, \infty): \) ______

33. (1 pt) setDerivatives10MaxMin/s3_4_6.pg
Consider the function \(f(x) = -2x^3 + 33x^2 - 168x + 6\). For this function there are three important intervals: \((-\infty, A], [A, B], \) and \([B, \infty)\) where A and B are the critical numbers.

Find A ______
and B ______

For each of the following intervals, tell whether \(f(x)\) is increasing (type in INC) or decreasing (type in DEC).
\((-\infty, A): \) ______
\([A, B]: \) ______
\([B, \infty] \) ______

\(f(x)\) has an inflection point at \(x = C\)
where C is ______

Finally for each of the following intervals, tell whether \(f(x)\) is concave up (type in CU) or concave down (type in CD).
\((-\infty, C): \) ______
\([C, \infty): \) ______

34. (1 pt) setDerivatives10MaxMin/s3_4_6a.pg
Consider the function \(f(x) = 5x + 4x^{-1}\). For this function there are four important intervals: \((-\infty, A], [A, B], [B, C], \) and \([C, \infty)\) where A, and C are the critical numbers and the function is not defined at B.

Find A ______
and B ______
and C ______

For each of the following intervals, tell whether \(f(x)\) is increasing (type in INC) or decreasing (type in DEC).
\((-\infty, A): \) ______
\([A, B]: \) ______
\([B, C]: \) ______
\([C, \infty) \) ______

35. (1 pt) setDerivatives10MaxMin/sc4_3_10a.png
Consider the function \(f(x) = x^2 e^{x^3}\).

\(f(x)\) has two inflection points at \(x = C\) and \(x = D\) with \(C \leq D\) where C is ______
and D is ______

Finally for each of the following intervals, tell whether \(f(x)\) is concave up (type in CU) or concave down (type in CD).
\((-\infty, C]: \) ______
\([C, D]: \) ______
\([D, \infty): \) ______

36. (1 pt) setDerivatives10MaxMin/osu_dr_10_1.png
Consider the function
\[ f(x) = \frac{e^x}{7 + e^x} \]
Then \(f'(x) = \) ______

The following questions ask for endpoints of intervals of increase or decrease for the function \(f(x)\).

Write INF for \(\infty\), MINF for \(-\infty\), and NA (ie. not applicable) if there are no intervals of that type.

The interval of increase for \(f(x)\) is from ______
to ______

The interval of decrease for \(f(x)\) is from ______
to ______

\(f(x)\) has a local minimum at ______ (Put NA if none.)
\(f(x)\) has a local maximum at ______ (Put NA if none.)
Then \(f''(x) = \) ______

The following questions ask for endpoints of intervals of upward and downward concavity for the function \(f(x)\).

Write INF for \(\infty\), MINF for \(-\infty\), and put NA if there are no intervals of that type.

The interval of upward concavity for \(f(x)\) is from ______
to ______

The interval of downward concavity for \(f(x)\) is from ______
to ______

\(f(x)\) has a point of inflection at ______ (Put NA if none.)
1. (1 pt) setDerivatives10_5Optim/c3s8p1.png
Find the point on the line $4x + 4y - 5 = 0$ which is closest to the point $(5, -5)$.

(______ ______)

2. (1 pt) setDerivatives10_5Optim/c3s8p2.png
A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 6 - x^2$. What are the dimensions of such a rectangle with the greatest possible area?

Width = ______
Height = ______

3. (1 pt) setDerivatives10_5Optim/c3s8p3.png
A cylinder is inscribed in a right circular cone of height 2 and radius (at the base) equal to 7.5. What are the dimensions of such a cylinder which has maximum volume?

Radius = ______
Height = ______

4. (1 pt) setDerivatives10_5Optim/nsc4_6_2.png
If 1800 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Volume = ____________ cubic centimeters.

5. (1 pt) setDerivatives10_5Optim/nsc4_6_16.png
A fence 4 feet tall runs parallel to a tall building at a distance of 4 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

________

Here are some hints for finding a solution:
Use the angle that the ladder makes with the ground to define the position of the ladder and draw a picture of the ladder leaning against the wall of the building and just touching the top of the fence.

If the ladder makes an angle 0.5 radians with the ground, touches the top of the fence and just reaches the wall, calculate the distance along the ladder from the ground to the top of the fence.

The distance along the ladder from the top of the fence to the wall is ______

Using these hints write a function $L(x)$ which gives the total length of a ladder which touches the ground at an angle $x$, touches the top of the fence and just reaches the wall.

$L(x)$ = __________

Use this function to find the length of the shortest ladder which will clear the fence.

The length of the shortest ladder is ______ feet.

7. (1 pt) setDerivatives10_5Optim/s3_8_6.png
A rancher wants to fence in an area of 500000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?

________

8. (1 pt) setDerivatives10_5Optim/dereco1.png
For the given cost function $C(x) = 54\sqrt{x} + \frac{x^2}{42900}$ find:

a) The cost at the production level 1900 ______

b) The average cost at the production level 1900 ______

c) The marginal cost at the production level 1900 ______

d) The production level that will minimize the average cost ______

e) The minimal average cost ______

9. (1 pt) setDerivatives10_5Optim/dereco2.png
For the given cost function $C(x) = 72900 + 300x + x^2$ find:

a) The cost at the production level 1050 ______

b) The average cost at the production level 1050 ______

c) The marginal cost at the production level 1050 ______

d) The production level that will minimize the average cost ______

e) The minimal average cost ______

10. (1 pt) setDerivatives10_5Optim/dereco3.png
For the given cost function $C(x) = 8450 + 260x + 0.1x^2$ and the demand function $p(x) = 780$. Find the production level that will maximize profit.

11. (1 pt) setDerivatives10_5Optim/dereco4.png
A manufacture has been selling 1300 television sets a week at 450 each. A market survey indicates that for each 29 rebate offered to a buyer, the number of sets sold will increase by 290 per week.

a) Find the demand function $p(x)$, where $x$ is the number of the television sets sold per week.

$p(x)$ = ______

b) How large rebate should the company offer to a buyer, in order to maximize its revenue? ______

c) If the weekly cost function is $97500 + 150x$, how should it set the size of the rebate to maximize its profit? ______

12. (1 pt) setDerivatives10_5Optim/dereco5.png
A baseball team plays in he stadium that holds 58000 spectators. With the ticket price at 11 the average attendence has been
26000. When the price dropped to 10, the average attendance rose to 29000.

a) Find the demand function \( p(x) \), where \( x \) is the number of spectators. (assume \( p(x) \) is linear) \( p(x) = \) ____________

b) How should be set a ticket price to maximize revenue?

---

13.(1 pt) setDerivatives10_5Optim/nsc4_7_16.png

The manager of a large apartment complex knows from experience that 110 units will be occupied if the rent is 324 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 6 dollar increase in rent. Similarly, one additional unit will be occupied for each 6 dollar decrease in rent. What rent should the manager charge to maximize revenue?

---

14.(1 pt) setDerivatives10_5Optim/s3_8_26.png

A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 41 feet?

---

15.(1 pt) setDerivatives10_5Optim/osu_dr_10_5_1.png

Let \( Q = (0, 7) \) and \( R = (11, 8) \) be given points in the plane. We want to find the point \( P = (x, 0) \) on the x-axis such that the sum of distances \( PQ + PR \) is as small as possible. (Before proceeding with this problem, draw a picture!)

To solve this problem, we need to minimize the following function of \( x \):

---

16.(1 pt) setDerivatives10_5Optim/osu_dr_10_5_2.png

Centerville is the headquarters of Greedy Cablevision Inc. The cable company is about to expand service to two nearby towns, Springfield and Shelbyville. There needs to be cable connecting Centerville to both towns. The idea is to save on the cost of cable by arranging the cable in a Y-shaped configuration. Centerville is located at \((12, 0)\) in the \(xy\)-plane, Springfield is at \((0, 6)\), and Shelbyville is at \((0, -6)\). The cable runs from Centerville to some point \((x, 0)\) on the x-axis where it splits into two branches going to Springfield and Shelbyville. Find the location \((x, 0)\) that will minimize the amount of cable between the 3 towns and compute the amount of cable needed. Justify your answer.

To solve this problem we need to minimize the following function of \( x \):

---
1. (1 pt) setDerivatives11Newton/s2_10_3.pg
Use Newton’s method to approximate a root of the equation
\[ x^3 + x + 4 = 0 \]
as follows.
Let \( x_1 = -1 \) be the initial approximation.
The second approximation \( x_2 \) is ________
and the third approximation \( x_3 \) is ________.

2. (1 pt) setDerivatives11Newton/s2_10_4.pg
Use Newton’s method to approximate a root of the equation
\[ 4x^3 + 8x + 2 = 0 \]
as follows.
Let \( x_1 = -1 \) be the initial approximation.
The second approximation \( x_2 \) is ______
and the third approximation \( x_3 \) is ______.

3. (1 pt) setDerivatives11Newton/s2_10_20.pg
Use Newton’s method to approximate a root of the equation
\[ 4x^7 + 2x^4 + 2 = 0 \]
as follows.
Let \( x_1 = 3 \) be the initial approximation.
The second approximation \( x_2 \) is ______
and the third approximation \( x_3 \) is ______.

4. (1 pt) setDerivatives11Newton/s2_10_11.pg
Use Newton’s method to approximate a root of the equation
\[ 2 \sin(x) = x \]
as follows.
Let \( x_1 = 2 \) be the initial approximation.
The second approximation \( x_2 \) is ______
and the third approximation \( x_3 \) is ______.

5. (1 pt) setDerivatives11Newton/s2_10_22.pg
Use Newton’s method to approximate a root of the equation
\[ \cos(x^2 + 2) = x^3 \]
as follows.
Let \( x_1 = 1 \) be the initial approximation.
The second approximation \( x_2 \) is ______
and the third approximation \( x_3 \) is ______.

6. (1 pt) setDerivatives11Newton/s2_10_22a.pg
Use Newton’s method to approximate a root of the equation
\[ \cos(x^2 - 7) = x^3 \]
as follows.
Let \( x_1 = 1 \) be the initial approximation.
The second approximation \( x_2 \) is ______
and the third approximation \( x_3 \) is ______.

7. (1 pt) setDerivatives11Newton/s2_10p1.pg
Find the positive value of \( x \) which satisfies \( x = 4.300 \sin(x) \).
Give the answer to four places of accuracy ________
Remember to calculate the trig functions in radian mode.

8. (1 pt) setDerivatives11Newton/s2_10p2.pg
Find the positive value of \( x \) which satisfies \( x = 0.1 \cos(x) \).
Give the answer to six places of accuracy ________
Remember to calculate the trig functions in radian mode.

9. (1 pt) setDerivatives11Newton/s2_10p3.pg
Find the smallest positive value of \( x \) which satisfies
\[ x = 2.700 \cos(2.300x) \]
Give the answer to four places of accuracy ________
Remember to calculate the trig functions in radian mode.
1. (1 pt) setDerivatives12MVT/s3_2_11.pg
Consider the function \( f(x) = 2 - 2x^2 \) on the interval \([-2, 8]\).
Find the average or mean slope of the function on this interval, i.e.
\[
\frac{f(8) - f(-2)}{8 - (-2)} = \]
By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-2, 8)\) such that \( f'(c) \) is equal to this mean slope.
For this problem, there is only one \( c \) that works. Find it.

2. (1 pt) setDerivatives12MVT/c3s2p1.pg
Consider the function
\[
f(x) = 3x^3 + 2x^2 - 1x + 3
\]
Find the average slope of this function on the interval \((-3, -1)\).
By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-3, -1)\) such that \( f'(c) \) is equal to this mean slope.
Find the value of \( c \) in the interval which works ________

3. (1 pt) setDerivatives12MVT/s3_2_14.pg
Consider the function \( f(x) = 4\sqrt{x} + 8 \) on the interval \([2, 8]\).
Find the average or mean slope of the function on this interval.
By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-4, 4)\) such that \( f'(c) \) is equal to this mean slope.
For this problem, there are two values of \( c \) that work. The smaller one is ________, and the larger one is ________

4. (1 pt) setDerivatives12MVT/s3_2_12.pg
Consider the function \( f(x) = 2x^3 - 12x^2 - 30x + 2 \) on the interval \([-6, 7]\). Find the average or mean slope of the function on this interval.
By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-6, 7)\) such that \( f'(c) \) is equal to this mean slope.
For this problem, there are two values of \( c \) that work. The smaller one is ________, and the larger one is ________

5. (1 pt) setDerivatives12MVT/s3_2_13.pg
Consider the function \( f(x) = \frac{1}{x} \) on the interval \([-6, 7]\). Find the average or mean slope of the function on this interval.
By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-6, 7)\) such that \( f'(c) \) is equal to this mean slope.
For this problem, there is only one \( c \) that works. Find it.

6. (1 pt) setDerivatives12MVT/s3_2_14.pg
Consider the function \( f(x) = 4\sqrt{x} + 8 \) on the interval \([-6, 7]\). Find the average or mean slope of the function on this interval.
By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-6, 7)\) such that \( f'(c) \) is equal to this mean slope.
For this problem, there is only one \( c \) that works. Find it.
1. (1 pt) setDerivatives13Higher/ur_dr_13_2f.pg
If \( f(x) = 7x^3 - 7e^x \), find \( f'(x) \).

\[ f''(x) = \]

2. (1 pt) setDerivatives13Higher/ur_dr_13_2g.pg
If \( f(x) = 6x^2 - 3e^x \), find \( f'(x) \).

\[ f'(1) = \]

Find \( f''(x) \).

3. (1 pt) setDerivatives13Higher/ur_dr_13_4a.pg
Let \( f(x) = x^8 - 7e^x \).

(a) \( f'(-1) = \)

(b) \( f''(-1) = \)

4. (1 pt) setDerivatives13Higher/s2_7_3a.pg
Let \( f(x) = x^4 + 4x^3 + 7x^2 + 2x \).
Then \( f'(x) = \)
and \( f''(3) = \)
If \( f''(3) = \)
and \( f''(x) = \)

5. (1 pt) setDerivatives13Higher/s2_7_4a.pg
Let \( f(x) = x^7 - 4x^5 + 4x^3 - 3x - 4 \).
Then \( f'(x) = \)
\( f'(2) = \)
\( f''(x) = \)
and \( f''(2) = \)

6. (1 pt) setDerivatives13Higher/ur_dr_13_1a.pg
Let \( f(x) = 7x^8 - 3x^5 - 7x^3 + 6x \), find \( f'(x) \).

Find \( f'(5) \).

Find \( f''(x) \).

Find \( f''(5) \).

7. (1 pt) setDerivatives13Higher/ur_dr_13_3a.pg
If \( f(x) = 7 + \frac{5}{x} + \frac{2}{x^2} \), find \( f'(x) \).

Find \( f'(3) \).

Find \( f''(x) \).

Find \( f''(3) \).

8. (1 pt) setDerivatives13Higher/s2_7_10a.pg
Let \( h(t) = 4t^3.2 - 7t^{-3.2} \).
Then \( h'(t) = \)
\( h''(t) = \)
and \( h'''(5) = \)

9. (1 pt) setDerivatives13Higher/s2_7_18a.pg
Let \( f(x) = \frac{1-x}{1+2x} \).
Then \( f'(5) = \)
and \( f''(5) = \)
and \( f'''(5) = \)

10. (1 pt) setDerivatives13Higher/ur_dr_13_5a.pg
Let \( f(x) = \frac{x^2+5x+6}{6x+18} \).
(a) \( f'(4) = \)
(b) \( f''(4) = \)

[NOTE: There are two ways to do this problem. The first is the quotient rule. The second is much easier and does not use the quotient rule.]

11. (1 pt) setDerivatives13Higher/ur_dr_13_6a.pg
If \( g(t) = -3t^4 + 3t^2 + 2 \) find
\( g(0) = \)
\( g'(0) = \)
\( g''(0) = \)
\( g'''(0) = \)
\( g''''(0) = \)
\( g(4) = \)
\( g(5) = \)

12. (1 pt) setDerivatives13Higher/ur_dr_13_7a.pg
Let \( f(x) = x \sin(x) \).
Find \( f''(5.1) \).
(Re sean riad mode!)

13. (1 pt) setDerivatives13Higher/ur_dr_13_8a.pg
Let \( h(t) = \tan(4x + 3) \).
Then \( h'(1) = \)
and \( h''(1) = \)

14. (1 pt) setDerivatives13Higher/s2_7_7a.pg
Let \( g(x) = (5x - 1)^6 \).
Then \( g'(x) = \)
\( g'(3) = \)
\( g''(x) = \)
and \( g''(3) = \)

15. (1 pt) setDerivatives13Higher/s2_7p1a.pg
If \( g(t) = (6 - t^2)^2 \) find
\( g(0) = \)
\( g'(0) = \)
\( g''(0) = \)

16. (1 pt) setDerivatives13Higher/s2_7_5a.pg
Let \( f(x) = \sqrt{x^2 + 3} \).
Then \( f'(x) = \)
17. (1 pt) setDerivatives13Higher/ur_dr_13_14.png
Let
\[ f(x) = -5e^{-x/3} \]
\[ f^{(7)}(1) = \quad \] 

18. (1 pt) setDerivatives13Higher/ur_dr_13_9.png
\[ \frac{d^4}{dx^4} \left( \frac{7x^5}{1-x} \right) = \quad \] Note: There is a way of doing this problem without using the quotient rule 4 times.

19. (1 pt) setDerivatives13Higher/ur_dr_13_10.png
Let
\[ f(x) = -8x^4 \frac{1}{1-x} \]
\[ f^{(4)}(x) = \quad \] Note: There is a way of doing this problem without using the quotient rule 4 times.

20. (1 pt) setDerivatives13Higher/ur_dr_13_13.png
Let
\[ f(x) = \frac{9x}{1-x} \]
\[ f^{(5)}(x) = \quad \]

Let
\[ f(x) = -5 \ln[\cos(x)] \]
\[ f''(x) = \quad \]

22. (1 pt) setDerivatives13Higher/ur_dr_13_12.png
Let
\[ f(x) = 6 \ln[\sec(x) + \tan(x)] \]
\[ f''(x) = \quad \] HINT: Simplify the first derivative before you find the second derivative.

23. (1 pt) setDerivatives13Higher/hdr1.png
Find the 88th derivative of the function \( f(x) = \cos(x) \).
The answer is function \( \quad \)

24. (1 pt) setDerivatives13Higher/ur_dr_13_15.png
If \( f(x) = 6x^3 \ln(4x) \), then
\[ f'(x) = \quad \]
\[ f''(x) = \quad \]
\[ f'''(x) = \quad \]
\[ f^{(4)}(x) = \quad \]
\[ f^{(5)}(x) = \quad \]
Consider the function \( f(x) = 28x^3 - 27x^2 + 16x - 7 \). Enter an antiderivative of \( f(x) \)

Consider the function \( f(x) = 5x^3 - 9x^2 + 2x - 7 \). An antiderivative of \( f(x) \) is \( F(x) = Ax^4 + Bx^3 + Cx^2 + Dx \) where \( A \) is _____ and \( B \) is _____ and \( C \) is _____ and \( D \) is _____

Consider the function \( f(x) = 3x^5 + 5x^3 - 10x^2 - 7 \). Enter an antiderivative of \( f(x) \)

Consider the function \( f(x) = 2x^9 + 10x^5 - 7x^4 - 3 \). An antiderivative of \( f(x) \) is \( F(x) = Ax^n + Bx^m + Cx^p + Dx^q \) where \( A \) is _____ and \( n \) is _____ and \( B \) is _____ and \( m \) is _____ and \( C \) is _____ and \( p \) is _____ and \( D \) is _____ and \( q \) is _____

Consider the function \( f(x) = \frac{6}{x^2} + \frac{3}{x^5} \). Let \( F(x) \) be the antiderivative of \( f(x) \) with \( F(1) = 0 \). Then \( F(x) = \) _____

Consider the function \( f(x) = \frac{4}{x^3} - \frac{8}{x^5} \). Let \( F(x) \) be the antiderivative of \( f(x) \) with \( F(1) = 0 \). Then \( F(3) = \) _____

Consider the function \( f(t) = 4 \sec^2(t) - 5t^3 \). Let \( F(t) \) be the antiderivative of \( f(t) \) with \( F(0) = 0 \). Then \( F(t) = \) _____

Consider the function \( f(t) = 10 \sec^2(t) - 6t^2 \). Let \( F(t) \) be the antiderivative of \( f(t) \) with \( F(0) = 0 \). Then \( F(4) = \) _____

Consider the function \( f(x) \) whose second derivative is \( f''(x) = 3x + 2 \sin(x) \). If \( f(0) = 3 \) and \( f'(0) = 2 \), what is \( f(x) \)?

Consider the function \( f(x) \) whose second derivative is \( f''(x) = 4x + 2 \sin(x) \). If \( f(0) = 2 \) and \( f'(0) = 3 \), what is \( f(3) \)?

Given \( f''(x) = -36 \sin(6x) \) and \( f'(0) = -6 \) and \( f(0) = 3 \).

Find \( f(\pi/4) = \) _____

Given \( f''(x) = 7x + 1 \) and \( f'(-3) = -2 \) and \( f(-3) = -6 \). Find \( f''(x) = \) _____ and find \( f(4) = \) _____

Given that the graph of \( f(x) \) passes through the point \((6, 5)\) and that the slope of its tangent line at \((x, f(x))\) is \( 5x + 6 \), what is \( f(2) \)?

A particle is moving with acceleration \( a(t) = 12t + 10 \). Its position at time \( t = 0 \) is \( s(0) = 13 \) and its velocity at time \( t = 0 \) is \( v(0) = 1 \). What is its position at time \( t = 6 \)?

A car traveling at 48 ft/sec decelerates at a constant 6 feet per second squared. How many feet does the car travel before coming to a complete stop?

A ball is shot straight up into the air with initial velocity of 45 ft/sec. Assuming that the air resistance can be ignored, how high does it go?

How far away does it land?

A ball is shot at an angle of 45 degrees into the air with initial velocity of 41 ft/sec. Assuming no air resistance, how high does it go?

How far away does it land?

A stone is thrown straight up from the edge of a roof, 825 feet above the ground, at a speed of 10 feet per second.

A stone is thrown straight up from the edge of a roof, 900 feet above the ground, at a speed of 11 feet per second.
A. Remembering that the acceleration due to gravity is -32 feet per second squared, how high is the stone 2 seconds later?

B. At what time does the stone hit the ground?

C. What is the velocity of the stone when it hits the ground?

20. (1 pt) setDerivatives20Antideriv/s3_10_67.pg
A stone is dropped from the edge of a roof, and hits the ground with a velocity of -200 feet per second. How high (in feet) is the roof?

21. (1 pt) setDerivatives20Antideriv/ur_dr_20_1.pg
Let $f(x) = \frac{3}{x} - 5e^x$.

22. (1 pt) setDerivatives20Antideriv/ur_dr_20_2.pg
Let $f(x) = \frac{15}{\sqrt{1-x^2}}$.

23. (1 pt) setDerivatives20Antideriv/ur_dr_20_3.pg
Let $f(x) = \frac{-13}{x^2 + 1}$.

Enter an antiderivative of $f(x)$

Enter an antiderivative of $f(x)$

Enter an antiderivative of $f(x)$
1. Evaluate the limit
\[ \lim_{x \to \infty} \sqrt{x^2 + 7x + 4 - x} \]

2. The function \[ \sqrt{x^2 + 3x + 7 - x} \]
has one horizontal asymptote at \( y = \) ___________.

3. Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to \infty} \frac{8x^3}{e^{6x}} \]

4. Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to -\infty} 8\cos(-5x)\sec(3x) \]

5. Compute the following limit using L'Hopital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} (\sqrt{x^3 - 9x^2 - x}) = \]

6. Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to 0} \frac{e^x - 1}{\sin(12x)} \]

7. Evaluate the limit using L'Hopital's rule if necessary
\[ \lim_{x \to -\infty} \frac{\sin(2x)}{\sin(8x)} \]

8. Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to 0} \frac{\sin(10x)}{\tan(3x)} \]

9. Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to 0} \frac{13^x - 8^x}{x} \]

10. Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{x^2 - 1}{x^2} \]
\[ \lim_{x \to -\infty} \frac{\tan^{-1}(x)}{(1/x) - 8} = \]

11. Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} 1 - \cos(3x) = \]
\[ \lim_{x \to -\infty} x^2 - 1 = \]

12. Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{\ln(x^2 - 3)}{e^{3x}} = \]
\[ \lim_{x \to -\infty} e^{3x} - e^{-4x} = \]

13. Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{\sin(7x)\ln(x)}{x^2} = \]

14. Find the following limits, using l'Hôpital's rule if appropriate
\[ \lim_{x \to -\infty} \frac{\arctan(x^2)}{x^7} = \]
\[ \lim_{x \to -\infty} \sqrt{x\ln(x)} = \]

15. Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to -\infty} 5xe^{1/x} - 5x \]

16. Evaluate the limit using L'Hopital's rule if necessary
\[ \lim_{x \to -\infty} \frac{(14x)^{4x}}{(14x + 10)^{4x}} \]

17. Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{1 - \frac{4}{x}}{} = \]

Evaluate the limit using L'Hospital's rule
\[ \lim_{x \to -\infty} x \]

Evaluate the limit using L'Hospital's rule if necessary
\[ \lim_{x \to -\infty} \frac{13^x - 8^x}{x} \]

Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{x^2 - 1}{x^2} \]
\[ \lim_{x \to -\infty} \frac{\tan^{-1}(x)}{(1/x) - 8} = \]

Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} 1 - \cos(3x) = \]
\[ \lim_{x \to -\infty} x^2 - 1 = \]

Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{\ln(x^2 - 3)}{e^{3x}} = \]
\[ \lim_{x \to -\infty} e^{3x} - e^{-4x} = \]

Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{\sin(7x)\ln(x)}{x^2} = \]

Compute the following limits, using l'Hôpital's rule if appropriate
\[ \lim_{x \to -\infty} \frac{\arctan(x^2)}{x^7} = \]
\[ \lim_{x \to -\infty} \sqrt{x\ln(x)} = \]

Evaluate the limit using L'Hopital's rule
\[ \lim_{x \to -\infty} 5xe^{1/x} - 5x \]

Evaluate the limit using L'Hopital's rule if necessary
\[ \lim_{x \to -\infty} \frac{(14x)^{4x}}{(14x + 10)^{4x}} \]

Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
\[ \lim_{x \to -\infty} \frac{1 - \frac{4}{x}}{} = \]
Evaluate the limit using L'Hospital’s rule if necessary

\[ \lim_{x \to \infty} \left( 1 + \frac{5}{x} \right)^{\frac{1}{x}} \]

Evaluate the limit using L'Hospital’s rule if necessary

\[ \lim_{x \to \infty} \ln(6x+1) \ln(x)+1 \]

For each of the following forms determine whether the following limit type is indeterminate, always has a fixed finite value, or never has a fixed finite value. In the first case answer IND, in the second case enter the numerical value, and in the third case answer DNE. For example

| IND | 0 | 0 |
| DNE | 1 |

To discourage blind guessing, this problem is graded on the following scale

0-9 correct = 0
10-13 correct = .3
14-16 correct = .5
17-19 correct = .7

Find the following limits, using L'Hôpital’s rule, if appropriate. Use INF to denote \( \infty \) and MINF to denote \(-\infty\)

(a) \( \lim_{x \to \infty} \frac{\tan^{-1}(x/4)}{\sin^{-1}(1/x)} = \) 
(b) \( \lim_{x \to 0} \frac{x \cos^5(\pi e^x)}{\ln(1 + 4x)} = \) 

Evaluate the limit using L'Hospital’s rule

\[ \lim_{x \to 0} \frac{e^x + 3x - 1}{4x} \]
1. (1 pt) setIntegrals0Theory/osu_in_0_14.pg
You are given the four points in the plane \( A = (-1, -1), B = (3, 1), C = (8, -7), \) and \( D = (12, 4). \) The graph of the function \( f(x) \) consists of the three line segments \( AB, BC \) and \( CD. \) Find the integral \( \int_{-1}^{12} f(x) \, dx \) by interpreting the integral in terms of sums and/or differences of areas of elementary figures.
\[ \int_{-1}^{12} f(x) \, dx = \ldots \]

2. (1 pt) setIntegrals0Theory/sc5_2_24.pg
Evaluate the integral below by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.
\[ \int_{-4}^{7} \sqrt{49 - x^2} \, dx \]

3. (1 pt) setIntegrals0Theory/sc5_2_2a.pg
Use the Midpoint Rule to approximate
\[ \int_{-2.5}^{5.5} x^3 \, dx \]
with \( n = 8. \)

4. (1 pt) setIntegrals0Theory/sc5_2_28.pg
Evaluate the integral by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.
\[ \int_{0}^{4} |3x - 8| \, dx \]

5. (1 pt) setIntegrals0Theory/sc5_2_5.pg
Use the Midpoint Rule to approximate the integral
\[ \int_{7}^{17} (2x - 6x) \, dx \]
with \( n = 3. \)

6. (1 pt) setIntegrals0Theory/sc5_2_3.pg
Consider the integral
\[ \int_{0}^{6} (3x^2 + 3x + 4) \, dx \]
(a) Find the Riemann sum for this integral using right endpoints and \( n = 3. \)
\[ R_3 = \ldots \]
(b) Find the Riemann sum for this same integral, using left endpoints and \( n = 3. \)
\[ L_3 = \ldots \]

7. (1 pt) setIntegrals0Theory/ur_in_0_l11.pg
Consider the integral
\[ \int_{4}^{8} \left( \frac{3}{x} + 2 \right) \, dx \]
(a) Find the Riemann sum for this integral using right endpoints and \( n = 4. \)
(b) Find the Riemann sum for this same integral, using left endpoints and \( n = 4 \)

8. (1 pt) setIntegrals0Theory/ur_in_0_l13.pg
Let \( \int_{6}^{9} f(x) \, dx = 7, \int_{6}^{9} f(x) \, dx = 6, \int_{8}^{9} f(x) \, dx = 5. \)
Find \( \int_{7}^{8} f(x) \, dx = \ldots \)
and \( \int_{8}^{9} (7f(x) - 6) \, dx = \ldots \)

9. (1 pt) setIntegrals0Theory/sc5_2_3b0.pg
\[ \int_{10}^{25} f(x) - \int_{15}^{10} f(x) = \int_{a}^{b} f(x) \]
where \( a = \ldots \) and \( b = \ldots \)

10. (1 pt) setIntegrals0Theory/ur_in_0_l12.pg
Consider the function \( f(x) = \frac{x^2}{3} - 6. \)
In this problem you will calculate \( \int_{0}^{2} \left( \frac{x^2}{3} - 6 \right) \, dx \) by using the definition
\[ \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \left[ \sum_{i=1}^{n} f(x_i) \Delta x \right] \]
The summation inside the brackets is \( R_n \) which is the Riemann sum where the sample points are chosen to be the right-hand endpoints of each sub-interval.
Calculate \( R_n \) for \( f(x) = \frac{x^2}{3} - 6 \) on the interval \([0, 2]\) and write your answer as a function of \( n \) without any summation signs. You will need the summation formulas on page 381 of your textbook (page 364 in older texts).
\[ R_n = \ldots \]
\[ \lim_{n \to \infty} R_n = \ldots \]

11. (1 pt) setIntegrals0Theory/osu_in_0_15.pg
The following sum
\[ \frac{1}{1 + \frac{4}{n}} + \frac{1}{1 + \frac{2}{n}} + \frac{1}{1 + \frac{4}{n}} + \ldots + \frac{1}{1 + \frac{4}{n}} \]
is a right Riemann sum for a certain definite integral
\[ \int_{1}^{b} f(x) \, dx \]
using a partition of the interval \([1, b]\) into \(n\) subintervals of equal length.

Then the upper limit of integration must be: \(b = \underline{}\) and the integrand must be the function \(f(x) = \underline{}\).

12. (1 pt) setIntegrals0Theory/osu_in_0_16.png
The following sum
\[
\sqrt{11 + \frac{2}{n}} \cdot \left(\frac{2}{n}\right) + \sqrt{11 + \frac{4}{n}} \cdot \left(\frac{2}{n}\right) + \ldots + \sqrt{11 + \frac{2n}{n}} \cdot \left(\frac{2}{n}\right)
\]
is a right Riemann sum for the definite integral
\[
\int_0^b f(x) \, dx
\]
where \(b = \underline{}\) and \(f(x) = \underline{}\).

It is also a Riemann sum for the definite integral
\[
\int_1^c g(x) \, dx
\]
where \(c = \underline{}\) and \(g(x) = \underline{}\).

The limit of these Riemann sums as \(n \to \infty\) is \(\underline{}\).

13. (1 pt) setIntegrals0Theory/osu_in_0_19.png
The following sum
\[
\sqrt{25 - \left(\frac{2}{n}\right)^2} \cdot \frac{5}{n} + \sqrt{25 - \left(\frac{4}{n}\right)^2} \cdot \frac{5}{n} + \ldots + \sqrt{25 - \left(\frac{5n}{n}\right)^2} \cdot \frac{5}{n}
\]
is a right Riemann sum for the definite integral
\[
\int_0^b f(x) \, dx
\]
where \(b = \underline{}\) and \(f(x) = \underline{}\).

The limit of these Riemann sums as \(n \to \infty\) is \(\underline{}\).

14. (1 pt) setIntegrals0Theory/osu_in_0_17.png
Suppose \(f(x)\) is continuous and decreasing on the closed interval \(3 \leq x \leq 11\), that \(f(3) = 10, f(11) = 6\) and that \(\int_3^{11} f(x) \, dx = 61.374798\).

Then \(\int_6^{10} f^{-1}(x) \, dx = \underline{}\).

15. (1 pt) setIntegrals0Theory/osu_in_0_18.png
Consider the function
\[
f(x) = x^3 - 12x^2 + 75x + 10
\]
By drawing a suitable picture, find a relation between the definite integrals \(\int_1^2 f(x) \, dx\) and \(\int_{74}^{120} f^{-1}(x) \, dx\). Use this relation to find the second of these two integrals
\[
\int_{74}^{120} f^{-1}(x) \, dx = \underline{}.
\]

16. (1 pt) setIntegrals0Theory/ur_in_0_1.png
Estimate the area under the graph of \(f(x) = x^2 + 4x\) from \(x = 3\) to \(x = 7\) using 4 approximating rectangles and left endpoints.

17. (1 pt) setIntegrals0Theory/ur_in_0_2.png
Evaluate the definite integral by interpreting it in terms of areas.
\[
\int_3^6 (4x - 16) \, dx
\]

18. (1 pt) setIntegrals0Theory/ur_in_0_2.png
Given that \(4 \leq f(x) \leq 5\) for \(-5 \leq x \leq 4\), use property 8 on page 387 to estimate the value of \(\int_{-5}^{4} f(x) \, dx\)
\[
\leq \int_{-5}^{4} f(x) \, dx \leq \underline{}.
\]
1. (1 pt) setIntegrals3Definite/osu_in_3_4.png
\[ \int_{b}^{2b} x^4 \, dx = \]

2. (1 pt) setIntegrals3Definite/c4s4p5.pg
The value of \( \int_{-2}^{0} (x - 5)^2 \, dx \) is

3. (1 pt) setIntegrals3Definite/c4s4p6.pg
The value of \( \int_{9}^{5} \frac{1}{x^2} \, dx \) is

4. (1 pt) setIntegrals3Definite/s4_4_17.png
Evaluate the definite integral
\[ \int_{-4}^{7} (6x + 2) \, dx \]

5. (1 pt) setIntegrals3Definite/s4_4_20.png
Evaluate the definite integral
\[ \int_{3}^{6} (3x^2 - 4x + 6) \, dx \]

6. (1 pt) setIntegrals3Definite/s4_4_21.png
Evaluate the definite integral
\[ \int_{8}^{-8} (64 - x^2) \, dx \]

7. (1 pt) setIntegrals3Definite/s4_4_27.png
Evaluate the definite integral
\[ \int_{4}^{6} \frac{4x^2 + 8}{\sqrt{x}} \, dx \]

8. (1 pt) setIntegrals3Definite/osu_in_3_2.png
Evaluate the definite integral
\[ \int_{2}^{5} \frac{6}{\sqrt{x}} \, dx \]

9. (1 pt) setIntegrals3Definite/osu_in_3_3.png
\[ \int_{2}^{5} \frac{4x^2 + 5}{x^2} \, dx = \]

10. (1 pt) setIntegrals3Definite/s4_4_41.png
Evaluate the definite integral
\[ \int_{0}^{\pi} 4 \sin(x) \, dx \]

11. (1 pt) setIntegrals3Definite/se5_3_17.png
Evaluate the integral
\[ \int_{6}^{5} \sin(t) \, dt \]

12. (1 pt) setIntegrals3Definite/se5_3_25.png
Evaluate the integral
\[ \int_{1}^{\sqrt{3}} \frac{6}{1 + x^2} \, dx \]

13. (1 pt) setIntegrals3Definite/se5_3_26.png
Evaluate the integral
\[ \int_{0}^{0.9} \frac{dx}{\sqrt{1 - x^2}} \]

14. (1 pt) setIntegrals3Definite/ur_in_3_1.png
The velocity function is \( v(t) = -t^2 + 4t - 3 \) for a particle moving along a line. Find the displacement and the distance traveled by the particle during the time interval \([-3, 6]\).

Displacement =

Distance traveled =

If needed, see page 405 of your textbook (378 in older books) for definitions of these terms.

15. (1 pt) setIntegrals3Definite/osu_in_3_1a.png
The velocity function is \( v(t) = -t^2 + 5t - 6 \) for a particle moving along a line. Find the displacement (net distance covered) of the particle during the time interval \([0, 5]\).

Displacement =

16. (1 pt) setIntegrals3Definite/osu_in_3_5.png
Note: You can get full credit for this problem by just answering the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

The integral \( \int_{-1}^{6} |14x^2 - x^3 - 45x| \, dx \) MUST be evaluated by breaking it up into a sum of three integrals:

\[ \int_{-1}^{a} |14x^2 - x^3 - 45x| \, dx + \int_{a}^{c} |14x^2 - x^3 - 45x| \, dx + \int_{c}^{6} |14x^2 - x^3 - 45x| \, dx \]

Where

\[ a = \]
\[ c = \]

\[ \int_{-1}^{a} |14x^2 - x^3 - 45x| \, dx = \]
\[ \int_{a}^{c} |14x^2 - x^3 - 45x| \, dx = \]
\[ \int_{c}^{6} |14x^2 - x^3 - 45x| \, dx = \]

Thus \( \int_{-1}^{6} |14x^2 - x^3 - 45x| \, dx = \)
Consider the function
\[ f(x) = \begin{cases} 
  x & \text{if } x < 1 \\
  \frac{1}{x} & \text{if } x \geq 1 
\end{cases} \]

Evaluate the definite integral.
\[ \int_{-3}^{5} f(x) \, dx \]
1. (1 pt) setIntegrals4FTC/c4s4p1.png
If \( f(x) = \int_4^x t^8 dt \)
then
\( f'(x) = \) _________
\( f'(-2) = \) _________

2. (1 pt) setIntegrals4FTC/c4s4p2.png
If \( f(x) = \int_x^{17} t^3 dt \)
then
\( f'(x) = \) _________

3. (1 pt) setIntegrals4FTC/c4s4p2B.png
If \( f(x) = \int_x^{12} t^4 dt \)
then
\( f'(x) = \) _________

4. (1 pt) setIntegrals4FTC/c4s4p3.png
If \( f(x) = \int_3^x t^2 dt \)
then
\( f'(x) = \) _________

5. (1 pt) setIntegrals4FTC/c4s4p4.png
If \( f(x) = \int_x^{17} 2t dt \)
then
\( f'(x) = \) _________
\( f'(6) = \) _________

6. (1 pt) setIntegrals4FTC/c4s4p4B.png
If \( f(x) = \int_x^{12} t^4 dt \)
then
\( f'(x) = \) _________

7. (1 pt) setIntegrals4FTC/c4s4p1a.png
If \( f(x) = \int_0^x (t^3 + 2t^2 + 7) dt \)
then
\( f''(x) = \) _________

8. (1 pt) setIntegrals4FTC/c4s4p7.png
Given
\( f(x) = \int_0^x \frac{t^2 - 49}{1 + \cos^2(t)} dt \)
At what value of \( x \) does the local max of \( f(x) \) occur?
\( x = \) _________

9. (1 pt) setIntegrals4FTC/ur_in_a4_10.png
Given
\( f(t) = \int_0^t \frac{t^2 + 11t + 24}{1 + \cos^2(t)} dt \)
At what value of \( t \) does the local max of \( f(t) \) occur?
\( t = \) _________

10. (1 pt) setIntegrals4FTC/ur_in_a4_12.png
NOTE: It will be easier to see the function \( f(x) \) if you use the
display mode "typeset". Keep in mind, though, that loading the
problem into your computer using this display mode will take
longer.

Let
\[
f(x) = \begin{cases} 
0 & \text{if } x < -3 \\
2 & \text{if } -3 \leq x < -1 \\
-5 & \text{if } -1 \leq x < 3 \\
0 & \text{if } x \geq 3 
\end{cases}
\]
and
\[
g(x) = \int_{-3}^x f(t) dt
\]
Determine the value of each of the following:
(a) \( g(-6) = \) _________
(b) \( g(-2) = \) _________
(c) \( g(0) = \) _________
(d) \( g(4) = \) _________
(e) The absolute maximum of \( g(x) \) occurs when \( x = \) _________ and is the value _________

It may be helpful to make a graph of \( f(x) \) when answering these questions.

11. (1 pt) setIntegrals4FTC/sc5_4_11.png
Use part I of the Fundamental Theorem of Calculus to find the derivative of
\[
f(x) = \int_{-4}^x \left( \frac{1}{4} t^2 - 1 \right)^5 dt
\]
\( f'(x) = \) _________

[NOTE: Enter a function as your answer. Make sure that your syntax is correct, i.e. remember to put all the necessary * , (, ), etc.]

12. (1 pt) setIntegrals4FTC/sc5_4_12.png
Use part I of the Fundamental Theorem of Calculus to find the derivative of
\[
f(x) = \int_{-2}^x \sqrt{1 + t^3 + 8t} dt
\]
\( f'(x) = \) _________

[NOTE: Enter a function as your answer. Make sure that your syntax is correct, i.e. remember to put all the necessary * , (, ), etc.]

13. (1 pt) setIntegrals4FTC/sc5_4_13.png
Use part I of the Fundamental Theorem of Calculus to find the derivative of
\[
f(x) = \int_{-2}^x \frac{1}{1 + t^2} dt
\]
\( f'(x) = \) _________

14. (1 pt) setIntegrals4FTC/sc5_4_14.png
Use part I of the Fundamental Theorem of Calculus to find the derivative of
\[
F(x) = \int_x^6 \sin(t^3) dt
\]
\( F'(x) = \) _________
15. Use part I of the Fundamental Theorem of Calculus to find the derivative of
   \[ h(x) = \int_{-5}^{\sin(x)} (\cos(t^4) + t) \, dt \]
   \[ h'(x) = \int_{-5}^{\sin(x)} (\cos(t^4) + 1) \, dt \]
   [NOTE: Enter a function as your answer. Make sure that your syntax is correct, i.e. remember to put all the necessary *, (, ), etc.]

16. Use part I of the Fundamental Theorem of Calculus to find the derivative of
   \[ h(x) = \int_{-3}^{\sin(x)} (\cos(t^3) + t) \, dt \]
   \[ h'(x) = \int_{-3}^{\sin(x)} (\cos(t^3) + 1) \, dt \]
   [NOTE: Enter a function as your answer. Make sure that your syntax is correct, i.e. remember to put all the necessary *, (, ), etc.] For more help see: WeBWorK functions

17. Find the derivative of
   \[ g(x) = \int_{3}^{9x} u \, du \]

18. Use part I of the Fundamental Theorem of Calculus to find the derivative of
   \[ g(x) = \int_{8}^{9x} u \, du \]
   [NOTE: Enter a function as your answer. Make sure that your syntax is correct, i.e. remember to put all the necessary *, (, ), etc.] For more help see: WeBWorK functions

19. Find the derivative of the following function
   \[ F(x) = \int_{x^4}^{3} (2t - 1)^3 \, dt \]
   using the Fundamental Theorem of Calculus.

20. Find the derivative of the following function
   \[ F(x) = \int_{\sqrt{\frac{1}{4} + 5x^2}}^{1} \frac{s^2}{4} \, ds \]
   using the appropriate form of the Fundamental Theorem of Calculus.
   \[ F'(x) = \int_{\sqrt{\frac{1}{4} + 5x^2}}^{1} \frac{s}{2} \, ds \]

21. Find a function \( f \) and a number \( a \) such that
   \[ 2 + \int_{a}^{f(x)} \frac{t}{t^2} \, dt = 4x \]
   \[ f(x) = \]
   \[ a = \]

22. Evaluate the definite integral
   \[ \int_{5}^{8} \left( \frac{d}{dt} \sqrt{3 + 2t^2} \right) \, dt \]
   using the Fundamental Theorem of Calculus.
   You will need accuracy to at least 4 decimal places for your numerical answer to be accepted. You can also leave your answer as an algebraic expression involving square roots.
   \[ \int_{5}^{8} \left( \frac{d}{dt} \sqrt{3 + 2t^2} \right) \, dt = \]

23. Evaluate the following definite integrals using the Fundamental Theorem of Calculus.
   \[ \int_{-10}^{3} |s - 5^2| \, ds = \]
   \[ \int_{0}^{2\pi} \sin(\sqrt{x}) \, dx = \]
   \[ \int_{5}^{10} \frac{t - 5}{t^2 + 10t + 26} \, dt = \]

24. Compute the following limit. Use INF to denote \( \infty \) and MINF to denote \( -\infty \).
   \[ \lim_{x \to 0} \int_{x^2}^{729} \sqrt{729 - 7t^3} \, dt = \]
1. (1 pt) setIntegrals5Trig/c3s10p6.pg
Find the value of \( \int_{0}^{\pi/4} \cos(2x) \, dx \).

Note: The notation \( \int_{0}^{a} f(x) \, dx \) is read "the integral from 0 to a of f(x)".
Remember: The angles for sin and cosine are always (well... almost always) in radians!

2. (1 pt) setIntegrals5Trig/c4s5p1.pg
Find the value of \( \int_{0}^{\pi/2} \cos(2x) \, dx \).

Note: The notation \( \int_{0}^{a} f(x) \, dx \) is read "the integral from 0 to a of f(x)".
Remember: The angles for sin and cosine are always (well... almost always) in radians!

3. (1 pt) setIntegrals5Trig/c4s5p3.pg
Find the value of \( \int_{0}^{\pi/3} \sin(2x) \sin(x) \, dx \).

4. (1 pt) setIntegrals5Trig/c4s5p4.pg
Find the value of \( \int_{0}^{\pi/3} \cos(x) \sin(sin(x)) \, dx \).

5. (1 pt) setIntegrals5Trig/sc5_5_97.pg
Evaluate the definite integral.
\[
\int_{0}^{\pi/2} \sin^2(6x) \cos^2(6x) \, dx
\]

6. (1 pt) setIntegrals5Trig/sc5_5_98.pg
Evaluate the definite integral.
\[
\int_{0}^{\pi/2} \sin^5 x \cos^{16} x \, dx
\]

7. (1 pt) setIntegrals5Trig/ur_in_5_7.pg
Evaluate the definite integral.
\[
\int_{0}^{\pi} \tan^2(3x) \, dx
\]

8. (1 pt) setIntegrals5Trig/sc5_5_99.pg
Evaluate the indefinite integral.
\[
\int 77 \cos^3(88x) \, dx
\]

9. (1 pt) setIntegrals5Trig/sc5_5_100.pg
Evaluate the indefinite integral.
\[
\int 68 \cos^2(2x) \, dx
\]

10. (1 pt) setIntegrals5Trig/ur_in_5_1.pg
Evaluate the indefinite integral.
\[
\int \sin^2(13x) \cos^4(13x) \, dx + C
\]

11. (1 pt) setIntegrals5Trig/ur_in_5_7.pg
Evaluate the indefinite integral.
\[
\int \sin^4(5x) \, dx =
\]

12. (1 pt) setIntegrals5Trig/ur_in_5_2.pg
Evaluate the indefinite integral.
\[
\int 72 \cos^4(4x) \, dx + C
\]

13. (1 pt) setIntegrals5Trig/ur_in_5_3.pg
Evaluate the indefinite integral.
\[
\int \sin(2x) \sin(4x) \, dx
\]

[Note: Remember to enter all necessary *, (, and )!! Enter \text{arctan}(x) for \tan^{-1} x, \sin(x) for \sin x ... ]

14. (1 pt) setIntegrals5Trig/ur_in_5_4.pg
Evaluate the definite integral.
\[
\int_{0}^{\pi/3} \sec^3(15x) \cot(15x) \, dx
\]

[Note: Remember to enter all necessary *, (, and )!! Enter \text{arctan}(x) for \tan^{-1} x, \sin(x) for \sin x. ]

15. (1 pt) setIntegrals5Trig/ur_in_5_5.pg
Evaluate the indefinite integral.
\[
\int \sin(9x) \cos(16x) \, dx
\]

[Note: Remember to enter all necessary *, (, and )!! Enter \text{arctan}(x) for \tan^{-1} x, \sin(x) for \sin x. ]

16. (1 pt) setIntegrals5Trig/ur_in_5_6.pg
Evaluate the indefinite integral.
\[
\int 28x^3 \sec^4(x^4) \, dx + C
\]
1. (1 pt) setIntegrals10InvTrig/invtrigs1.pg
Evaluate the definite integral.
\[ \int_{0}^{\frac{11\sin(\frac{\pi}{17})}{\sqrt{121 - x^2}}} x^3 \, dx \]

2. (1 pt) setIntegrals10InvTrig/invtrigs2.pg
Evaluate the definite integral.
\[ \int_{0}^{12} \frac{1}{\sqrt{4 + x^2}} \, dx \]

3. (1 pt) setIntegrals10InvTrig/invtrigs3.pg
Evaluate the indefinite integral.
\[ \int \frac{1}{x^2\sqrt{36 - x^2}} \, dx \]

4. (1 pt) setIntegrals10InvTrig/invtrigs4.pg
Evaluate the indefinite integral.
\[ \int \frac{\sqrt{81x^2 - 289}}{x} \, dx \]

5. (1 pt) setIntegrals10InvTrig/invtrigs5.pg
Evaluate the indefinite integral.
\[ \int \sqrt{18x - x^2} \, dx \]

6. (1 pt) setIntegrals10InvTrig/invtrigs6.pg
Evaluate the indefinite integral.
\[ \int \frac{66}{x^2\sqrt{400x^2 - 256}} \, dx \]

7. (1 pt) setIntegrals10InvTrig/invtrigs7.pg
Evaluate the indefinite integral.
\[ \int \frac{99}{\sqrt{28 - 36x - 9x^2}} \, dx \]

8. (1 pt) setIntegrals10InvTrig/osu_in_10_6.pg
Evaluate the indefinite integral.
\[ \int \frac{1}{\sqrt{16 - 64x^2}} \, dx = \frac{x}{4\sqrt{1 - x^2}} + C \]
WeBWorK notation for \( \sin^{-1}(x) \) is \( \text{arcsin}(x) \) or \( \text{asin}(x) \), and for \( \tan^{-1}(x) \) it’s \( \text{arctan}(x) \) or \( \text{atan}(x) \).

9. (1 pt) setIntegrals10InvTrig/osu_in_10_7.pg
Evaluate the indefinite integral.
\[ \int \frac{1}{x^2 + 8x + 97} \, dx = \frac{1}{\sqrt{97}} \ln \left| x + 4 + \frac{1}{\sqrt{97}} \right| + C \]
WeBWorK notation for \( \sin^{-1}(x) \) is \( \text{arcsin}(x) \) or \( \text{asin}(x) \), and for \( \tan^{-1}(x) \) it’s \( \text{arctan}(x) \) or \( \text{atan}(x) \).

10. (1 pt) setIntegrals10InvTrig/ur_in_10_2.pg
For each of the indefinite integrals below, choose which of the following substitutions would be most helpful in evaluating the integral. Enter the appropriate letter (A, B, or C) in each blank. DO NOT EVALUATE THE INTEGRALS.

A. \( x = 9\tan q \)
B. \( x = 9\sin q \)
C. \( x = 9\sec q \)

11. (1 pt) setIntegrals10InvTrig/ur_in_10_2.pg
Match each of the trigonometric expressions below with the equivalent non-trigonometric function from the following list. Enter the appropriate letter (A,B,C,D, or E) in each blank.

A. \( \tan(\arcsin(x/5)) \)
B. \( \cos(\arcsin(x/5)) \)
C. \( (1/2) \sin(2\arcsin(x/5)) \)
D. \( \sin(\arctan(x/5)) \)
E. \( \cos(\arctan(x/5)) \)

12. (1 pt) setIntegrals10InvTrig/ur_in_10_3.pg
Evaluate the indefinite integral.
\[ \int \frac{x^{10}}{(25 - x^2)^{13/2}} \, dx \]

13. (1 pt) setIntegrals10InvTrig/ur_in_10_4.pg
Evaluate the definite integral.
\[ \int_{0}^{3\sqrt{2}/2} \sqrt{9 - x^2} \, dx \]

14. (1 pt) setIntegrals10InvTrig/ur_in_10_5.pg
Evaluate the indefinite integral.
\[ \int \frac{dx}{(36 + x^2)^2} \]
1. (1 pt) setIntegrals12Methods/ur_int1_1.pg
Evaluate the indefinite integral.
\[ \int x^2 \arctan(2x) \, dx \]
[NOTE: Remember to enter all necessary ( and ) !! Enter \arctan(x) for \tan^{-1}x, \arcsin(x) for \sin^{-1}x. ]

2. (1 pt) setIntegrals12Methods/ur_int1_2.pg
Evaluate the indefinite integral.
\[ \int \ln(x^2 + 16x + 60) \, dx \]

3. (1 pt) setIntegrals12Methods/mec_int1.pg
Evaluate the indefinite integral.
\[ \int x \cos^2(6x) \, dx \]

4. (1 pt) setIntegrals12Methods/mec_int2.pg
Evaluate the indefinite integral.
\[ \int \frac{(\arcsin x)^2}{\sqrt{1-x^2}} \, dx \]

5. (1 pt) setIntegrals12Methods/mec_int3.pg
Evaluate the indefinite integral.
\[ \int e^{5x} \cdot \frac{e^{10x} + 49}{e^{10x}} \, dx \]

6. (1 pt) setIntegrals12Methods/osu_int1_3.pg
Find the indicated integrals.
(a) \[ \int \ln(x^3) \, dx = \quad + C \]
(b) \[ \int \frac{e^x \cos(x)}{3 + 4 \sin(x)} \, dt = \quad + C \]
(c) \[ \int_{3/4}^{x} \frac{\sin^{-1} \left( \frac{4}{x} \right)}{\sqrt{9 - 16x^2}} \, dx = \quad \]

7. (1 pt) setIntegrals12Methods/osu_int1_4.pg
Find the indicated integrals (if they exist)
\[ \int x^2 \sqrt{2x + 6} \, dx = \]
\[ \int \frac{e^{6x}}{3x + 7} \, dx = \]
\[ \int \frac{2x^3 + 13x + 6}{x^2} \, dx = \]
\[ \int \frac{\ln(x)}{x^9} \, dx = \]
1. (1 pt) setIntegrals14Substitution/sc5_5_1.pg
Evaluate the integral
\[ \int x^3 (x^4 - 2)^5 \, dx, \]
by making the substitution \( u = x^4 - 2 \).

NOTE: Your answer should be in terms of \( x \) and not \( u \).

2. (1 pt) setIntegrals14Substitution/ou_in_14_2.pg
Note: You can get full credit for this problem by just answering
the last question correctly. The initial questions are meant as
hints towards the final answer and also allow you the opportu-
nity to get partial credit.

Consider the indefinite integral \( \int x^7 (7 + 12x^8) \, dx \)
Then the most appropriate substitution to simplify this integral is
\( u = \quad \)
Then \( dx = f(x) \, du \) where
\( f(x) = \quad \)
After making the substitution we obtain the integral
\( \int g(u) \, du \) where
\( g(u) = \quad \)
This last integral is: \( = \quad + C \)
(Leave out constant of integration from your answer.)
After substituting back for \( u \) we obtain the following final form of the answer:
\( = \quad + C \)
(Leave out constant of integration from your answer.)
18. (1 pt) setIntegrals14Substitution/sec5_5_32.png
Evaluate the indefinite integral.
\[
\int \frac{9x + 6}{x^2 + 1} \, dx
\]

19. (1 pt) setIntegrals14Substitution/sec5_5_33.png
Evaluate the indefinite integral.
\[
\int \frac{3x - 2}{(2x^2 - 4x + 5)^3} \, dx
\]

20. (1 pt) setIntegrals14Substitution/sec5_5_33a.png
Evaluate the indefinite integral.
\[
\int \frac{4x - 1}{(4x^2 - 7x + 1)^5} \, dx
\]

[NOTE: Remember to enter all necessary *, (, and )!!]

21. (1 pt) setIntegrals14Substitution/sec5_5_35.png
Evaluate the indefinite integral.
\[
\int 4\sin^2 x \cos x \, dx
\]

22. (1 pt) setIntegrals14Substitution/ossi_in_14_1.png
Note: You can get full credit for this problem by just entering the answer to the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the indefinite integral \[ \int \cos^3(3t) \sin(3t) \, dt \]
Then the most appropriate substitution to simplify this integral is
\[ u = \text{_____} \quad \text{Then } dt = f(t) \, du \text{ where} \]
\[ f(t) = \text{_____} \]
After making the substitution we obtain the integral
\[ \int g(u) \, du \]
\[ g(u) = \text{_____} \]
This last integral is:
\[ = \text{_____} + C \]
(Leave out constant of integration from your answer.)
After substituting back for \( u \) we obtain the following final form of the answer:
\[ = \text{_____} + C \]
(Leave out constant of integration from your answer.)

23. (1 pt) setIntegrals14Substitution/ossi_in_14_5.png
Note: You can get full credit for this problem by just answering the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral \[ \int_{\pi/6}^{\pi/2} \cos^3(z) \, dz \]
Then the most appropriate substitution to simplify this integral is
\[ u = \text{_____} \quad \text{Then } dz = f(z) \, du \text{ where} \]
\[ f(z) = \text{_____} \]
After making the substitution and simplifying we obtain the integral \[ \int_a^b g(u) \, du \text{ where} \]
\[ g(u) = \text{_____} \]
\[ a = \text{_____} \]
\[ b = \text{_____} \]
This definite integral has value = \( \text{_____} \)

24. (1 pt) setIntegrals14Substitution/ur_in_14_13.png
Evaluate the indefinite integral.
\[ \int \sin^3(13x) \cos^2(13x) \, dx \]

25. (1 pt) setIntegrals14Substitution/sec5_5_38.png
Evaluate the definite integral.
\[ \int_0^{\pi/3} e^{\sin(x)} \cos(x) \, dx \]

26. (1 pt) setIntegrals14Substitution/sec5_5_39.png
Evaluate the definite integral.
\[ \int_0^1 \frac{2}{1 + x^2} \, dx \]

27. (1 pt) setIntegrals14Substitution/sec5_5_44.png
Evaluate the definite integral.
\[ \int_0^{\pi/3} \sin(3t) \, dt \]

28. (1 pt) setIntegrals14Substitution/sec5_5_49.png
Evaluate the definite integral.
\[ \int_0^2 \frac{dx}{1x + 3} \]

29. (1 pt) setIntegrals14Substitution/sec5_5_51.png
Evaluate the definite integral.
\[ \int_1^e \frac{dx}{x \sqrt{\ln x}} \]

30. (1 pt) setIntegrals14Substitution/ossi_in_14_7.png
Evaluate the definite integral.
\[ \int_1^e \frac{dx}{x(1 + \ln x)} \]

31. (1 pt) setIntegrals14Substitution/sec5_5_57.png
Verify that
\[ \frac{1}{x^2 - 1} = \frac{1}{2} \left( \frac{1}{x - 1} - \frac{1}{x + 1} \right) \]
and use this equation to evaluate
\[ \int_2^6 \frac{6}{x^2 - 1} \, dx \]

32. (1 pt) setIntegrals14Substitution/mec_int3.png
Evaluate the indefinite integral.
\[
\int \frac{e^{2x}}{e^{6x} + 9} \, dx
\]

33. (1 pt) setIntegrals14Substitution/osu_in_14_12.pg
\[
\int_0^1 6^x \, dx = \text{________}
\]

34. (1 pt) setIntegrals14Substitution/sec5_5_101.png
Use the substitution \( x = 6 \tan(\theta) \) to evaluate the indefinite integral
\[
\int \frac{40 \, dx}{x^2 \sqrt{x^2 + 36}}
\]

35. (1 pt) setIntegrals14Substitution/osu_in_14_14.pg

\textbf{Note:} You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral
\[
\int \frac{1}{\sqrt{1 + (6x - 7)^2}} \, dx
\]

Then the most appropriate substitution to simplify this integral is \( x = g(t) \) where
\( g(t) = \text{________} \)

Note: We are using \( t \) as variable for angles instead of \( \theta \), since there is no standard way to type \( \theta \) on a computer keyboard.

After making this substitution and simplifying (using trig identities), we obtain the integral \( \int f(t) \, dt \) where
\( f(t) = \text{________} \)

This integrates to the following function of \( t \)
\[
\int f(t) \, dt = \text{________} + C
\]

After substituting back for \( t \) in terms of \( x \) we obtain the following final form of the answer:
\( + C \)

36. (1 pt) setIntegrals14Substitution/osu_in_14_15.pg

\textbf{Note:} You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral
\[
\int_{\sqrt{2}/3}^{\sqrt{5}} \frac{x^3}{\sqrt{5x^2 - 1}} \, dx
\]

Then the most appropriate substitution to simplify this integral is \( x = g(t) \) where
\( g(t) = \text{________} \)

Note: We are using \( t \) as variable for angles instead of \( \theta \), since there is no standard way to type \( \theta \) on a computer keyboard.

After making this substitution and simplifying (using trig identities), we obtain the integral \( \int_a^b f(t) \, dt \) where
\( f(t) = \text{________} \)

\( a = \text{________} \)
\( b = \text{________} \)

After evaluating this integral we obtain:
\[
\int_{\sqrt{2}/3}^{\sqrt{5}} \frac{x^3}{\sqrt{5x^2 - 1}} \, dx = \text{________}
\]

37. (1 pt) setIntegrals14Substitution/osu_in_14_16.pg

\textbf{Note:} You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral
\[
\int_4^8 x \sqrt{8x - x^2} \, dx
\]

Then the most appropriate substitution to simplify this integral is \( x = g(t) \) where
\( g(t) = \text{________} \)

Note: We are using \( t \) as variable for angles instead of \( \theta \), since there is no standard way to type \( \theta \) on a computer keyboard.

After making this substitution and simplifying (using trig identities), we obtain the integral \( \int_a^b f(t) \, dt \) where
\( f(t) = \text{________} \)
\( a = \text{________} \)
\( b = \text{________} \)

After evaluating this integral we obtain:
\[
\int_4^8 x \sqrt{8x - x^2} \, dx = \text{________}
\]

38. (1 pt) setIntegrals14Substitution/mec_int2.png

Evaluate the indefinite integral.
\[
\int \frac{(\arctan(x))^3}{1 + x^2} \, dx
\]

39. (1 pt) setIntegrals14Substitution/osu_in_14_3.pg

\textbf{Note:} You can get full credit for this problem by just answering the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral
\[
\int \frac{1}{5x + 7} \, dx
\]

Then the most appropriate substitution to simplify this integral is
\( u = \text{________} \)

Then \( dx = f(x) \, du \) where
\( f(x) = \text{________} \)

After making the substitution and simplifying we obtain the integral \( \int g(u) \, du \) where
\( g(u) = \text{________} \)

This last integral is: \( = \text{________} + C \)

(Leave out constant of integration from your answer.)

After substituting back for \( u \) we obtain the following final form of the answer:
\( = \text{________} + C \)

(Leave out constant of integration from your answer.)

40. (1 pt) setIntegrals14Substitution/osu_in_14_4.png

\textbf{Note:} You can get full credit for this problem by just answering the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit. Also the appropriate way to enter roots (except \( \sqrt{ } \)) into WeBWorK is to use fractional exponents.
Consider the definite integral $\int_0^1 \frac{dx}{\sqrt{x + 4} \sqrt[4]{x}}$

Then the most appropriate substitution to simplify this integral is $u = \sqrt[4]{x}$

Then $dx = f(x) du$ where $f(x) = \frac{1}{4} u^3$.

After making the substitution and simplifying we obtain the integral $\int_a^b g(u) du$ where 

$g(u) = \frac{u^3}{4}$

$a = \sqrt[4]{a}$

$b = \sqrt[4]{b}$

This definite integral has value $= \frac{b^3}{4} - \frac{a^3}{4}$.

**41. (1 pt) setIntegrals14Substitution/osu_in_14_6a.pg**

Calculate the following definite integral.

$$\int_0^1 x^2 \sqrt{x + 8} \, dx = \frac{50}{3}$$

**42. (1 pt) setIntegrals14Substitution/osu_in_14_6b.pg**

**Note:** You can get full credit for this problem by just entering the answer to the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral $\int_0^1 x^2 \sqrt{8x + 6} \, dx$.

Then the most appropriate substitution to simplify this integral is $u = \sqrt[4]{x}$

Then $dx = f(x) du$ where $f(x) = \frac{1}{4} u^3$.

After making the substitution and simplifying we obtain the integral $\int_a^b g(u) du$ where 

$g(u) = \frac{u^3}{4}$

$a = \sqrt[4]{a}$

$b = \sqrt[4]{b}$

This definite integral has value $= \frac{b^3}{4} - \frac{a^3}{4}$.

**43. (1 pt) setIntegrals14Substitution/osu_in_14_8a.pg**

Find the following indefinite integrals.

$$\int \frac{x}{\sqrt{x + 8}} \, dx = \{\text{ans}, \text{ule}(50)\}$$

$g + C$.

**WARNINGS:**

* ERROR in oldSafeAv, PGbasicmacros.pl: * There was an error occurring inside evaluation brackets.

**44. (1 pt) setIntegrals14Substitution/osu_in_14_8b.pg**

Find the following indefinite integrals.

$$\int \frac{x}{\sqrt{x + 2}} \, dx = \frac{44}{9} + C$$

**Hint:** This is similar to Problem 6 of WeBWorK Hw wk #2.

$$\int \frac{\cos(t)}{(2\sin(t) + 12)^2} \, dt = \frac{9}{4} + C$$

**45. (1 pt) setIntegrals14Substitution/osu_in_14_10a.png**

Note: You can get full credit for this problem by just entering the answer to the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral

$$\int \frac{3}{3 + e^x} \, dx$$

The most appropriate substitution to simplify this integral is $u = f(x)$ where $f(x) = \frac{1}{4} u^3$.

We then have $dx = g(u) du$ where $g(u) = \frac{1}{4} u^3$.

**Hint:** you need to back substitute for $x$ in terms of $u$ for this part.

After substituting into the original integral we obtain

$$\int h(u) \, du$$

where $h(u) = \frac{1}{4} u^3$.

To evaluate this integral rewrite the numerator as

$$3 = u - (u - 3)$$

simplify, then integrate, thus obtaining

$$\int h(u) \, du = H(u)$$

where $H(u) = \frac{1}{4} u^3 + C$.

After substituting back for $u$ we obtain our final answer

$$\int \frac{3}{3 + e^x} \, dx = \frac{9}{4} + C$$

**46. (1 pt) setIntegrals14Substitution/osu_in_14_17a.png**

Consider the integral

$$\int \frac{x}{\sqrt{x^2 + 10x}} \, dx$$

Then an appropriate trigonometric substitution to simplify this integral is $x = f(t)$ where

$$f(t) = \ldots$$

After making this substitution and simplifying, we obtain the integral $\int g(t) \, dt$ where

$$\int g(t) \, dt = \ldots$$
Note that this problem doesn’t ask you to evaluate this integral.

47. (1 pt) set Integrals 14 Substitution/ osu_in_14_18.pg

For each of the following integrals find an appropriate trigonometric substitution of the form $x = f(t)$ to simplify the integral.

$$\int (6x^2 - 7)^{3/2} \, dx$$

$$\int \frac{x^2}{\sqrt{5x^2 + 4}} \, dx$$

$$\int x\sqrt{6x^2 + 24x + 19} \, dx$$

$$\int \frac{x}{\sqrt{-23 - 3x^2 - 18x}} \, dx$$
1. Use integration by parts to evaluate the integral.
\[ \int xe^{4x} \, dx + C \]

2. \[ \int_0^1 x^2 \sqrt{x} \, dx = \]

3. Use integration by parts to evaluate the integral.
\[ \int 3x \sin(x) \, dx + C \]

4. Use integration by parts to evaluate the integral.
\[ \int 5x \cos(2x) \, dx + C \]

5. Use integration by parts to evaluate the integral.
\[ \int 5x \ln(6x) \, dx + C \]

6. Use integration by parts to evaluate the integral.
\[ \int 27x^2 \cos(3x) \, dx + C \]

7. Use integration by parts to evaluate the integral.
\[ \int (\ln(5x))^2 \, dx + C \]

8. Evaluate the indefinite integral.
\[ \int e^{5x} \sin(6x) \, dx \]

9. Use integration by parts to evaluate the definite integral.
\[ \int_1^e 4r^2 \ln r \, dt \]

10. Evaluate the definite integral.
\[ \int_0^6 t^4 \ln(2t) \, dt \]

11. Evaluate the definite integral.
\[ \int_0^1 te^{-t} \, dt \]

12. Use integration by parts to evaluate the integral.
\[ \int_0^1 \sqrt{7} \ln t \, dt \]

13. Use arctan() to denote \( \tan^{-1}(\cdot) \) in your answer.
\[ \int 2y \tan^{-1}(8y) \, dy + C \]

14. Evaluate the indefinite integral.
\[ \int x \arctan(6x) \, dx \]

15. Evaluate the indefinite integral.
\[ \int x \sin^2(3x) \, dx \]

16. First make a substitution and then use integration by parts to evaluate the integral.
\[ \int x^5 \cos(x^3) \, dx + C \]

17. Evaluate the indefinite integral.
\[ \int \ln(x^2 + 16x + 60) \, dx \]

18. A particle that moves along a straight line has velocity
\[ v(t) = t^2 e^{-2t} \]
meters per second after \( t \) seconds. How many meters will it travel during the first \( t \) seconds?

19. \[ \int \sin^{-1}(4x) \, dx \]
Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the definite integral \[ \int_0^{1/4} \sin^{-1}(4x) \, dx \]
The first step in evaluating this integral is to apply integration by parts:
\[ \int u \, dv = uv - \int v \, du \]
where
\[ u = \]
and \( dv = h(x) \, dx \) where \( h(x) = \)

Note: Use \( \text{arcsin}(x) \) for \( \sin^{-1}(x) \).

After integrating by parts, we obtain the integral
\[
\int_{0}^{1/4} v \, du = \int_{0}^{1/4} f(x) \, dx
\]
on the right hand side where
\[
f(x) = \text{__________________________}
\]
The most appropriate substitution to simplify this integral is
\[
x = g(t) \quad \text{where}
\]
\[
g(t) = \text{__________________________}
\]
Note: We are using \( t \) as variable for angles instead of \( \theta \), since there is no standard way to type \( \theta \) on a computer keyboard.

After making this substitution and simplifying (using trig identities), we obtain the integral
\[
\int_{a}^{b} k(t) \, dt
\]
where
\[
k(t) = \text{__________________________}
\]
\[
a = \text{__________________________}
\]
\[
b = \text{__________________________}
\]
After evaluating this integral and plugging back into the integration by parts formula we obtain:
\[
\int_{0}^{1/4} x \sin^{-1}(4x) \, dx = \text{__________________________}
\]
| (1 pt) setIntegrals16Tables/sc5_7_3.pg | \[ \int e^{2x} \sin 4xdx \]  
Use the Table of Integrals in the back of your textbook to evaluate the integral.

| (1 pt) setIntegrals16Tables/tab_int_102.pg | \[ \int \frac{2xdx}{(x^2 + 1) \ln(x^2 + 1)} \]  
Use the Table of Integrals in the back of your textbook to evaluate the integral.

| (1 pt) setIntegrals16Tables/tab_int_25.pg | \[ \int \frac{2xdx}{\sqrt{x^4 + 25}} \]  
Use the Table of Integrals in the back of your textbook to evaluate the integral.
Approximate \( \int_0^{\pi/2} \sin(x) \, dx \) by computing \( L_f(P) \) and \( U_f(P) \), using the partition \( \{0, \pi/6, \pi/4, \pi/3, \pi/2\} \).

Your answers should be accurate to at least 4 decimal places.

\[
L_f(P) = \quad U_f(P) =
\]
You may include a formula as an answer.

Approximate \( \int_0^{\pi/2} x \sin(x) \, dx \) by computing \( L_f(P) \) and \( U_f(P) \), using the partition \( \{0, \pi/6, \pi/4, \pi/3, \pi/2\} \).

Your answers should be accurate to at least 4 decimal places.

\[
L_f(P) = \quad U_f(P) =
\]
You may include a formula as an answer.

Approximate the definite integral \( \int_7^{10} |8 - t| \, dt \) using midpoint Riemann sums with the following partitions:
(a) \( P = \{7, 8, 10\} \). Then midpoint Riemann sum =
(b) Using 3 subintervals of equal length. Then midpoint Riemann sum =

Use the Midpoint Rule to approximate the integral
\[
\int_{-5}^{0} (10x + 7x^2) \, dx
\]
with \( n=3 \).

Given the following integral and value of \( n \), approximate the following integral using the methods indicated (round your answers to six decimal places):
\[
\int_0^1 e^{-2x^2} \, dx, \quad n = 4
\]
(a) Trapezoidal Rule
(b) Midpoint Rule
(c) Simpson’s Rule

Use Simpson’s Rule and all the data in the following table to estimate the value of the integral \( \int_{-1}^{5} y \, dx \).

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-8</td>
<td>1</td>
<td>9</td>
<td>8</td>
<td>-6</td>
<td>-1</td>
<td>-8</td>
</tr>
</tbody>
</table>
Find the area under the curve \( y = \frac{1}{3x^3} \) from \( x = 1 \) to \( x = t \) and evaluate it for \( t = 10 \), \( t = 100 \). Then find the total area under this curve for \( x \geq 1 \).

(a) \( t = 10 \)

(b) \( t = 100 \)

(c) Total area

Find the area under the curve 

\[ y = 1.5x^{-2.5} \]

from \( x = 7 \)

\( \) to \( x = t \) and evaluate it for \( t = 10 \)

\( \), \( t = 100 \).

Then find the total area under this curve for \( x \geq 7 \).

(a) \( t = 10 \)

(b) \( t = 100 \)

(c) Total area

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."

\[ \int_0^\infty 8e^{-3x}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, give the answer -1.

\[ \int_8^\infty xe^{-5x}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."

\[ \int_2^\infty \frac{2}{(x+3)^{3/2}}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".

\[ \int_4^\infty \frac{1}{x^{3/4}}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."

\[ \int_{-\infty}^1 \frac{8}{(2x-3)^2}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."

\[ \int_{-\infty}^\infty (8x^2 + 3x - 7)dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".

\[ \int_{-\infty}^{\infty} x^6e^{-x^7}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".

\[ \int_{-\infty}^{\infty} \frac{1}{x^2 + 1}dx \]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "divergent."

\[ \int_6^\infty \ln(x) \frac{1}{x}dx \]
13. Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, give the answer -1.
\[ \int_5^\infty \frac{\ln(5x)}{x} \, dx \]

14. Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, give the answer -1.
\[ \int_{\ln(x)}^{1} \frac{1}{x^{1/3}} \, dx \]

15. Define the functions \( F(x) \) and \( G(x) \) by
\[ F(x) = \int_x^{x+6} t^7 \, dt, \quad G(x) = \int_{-x}^{-x+6} t^7 \, dt \]
Determine whether each of the following improper integrals and limits is divergent or convergent. If it is convergent, evaluate it. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
(a) \( \int_{-\infty}^{\infty} x^7 \, dx \)
(b) \( \lim_{x \to \infty} F(x) \)
(c) \( \lim_{x \to \infty} G(x) \)

16. Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \int_0^1 \frac{1}{x^{1/3}} \, dx \]

17. Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \int_1^{12} \frac{12}{\sqrt{x-1}} \, dx \]

18. Consider the following integrals. Label each as "P", "C", "D", according as the integral is proper, improper but convergent, or improper and divergent.
1. \( \int_{-\infty}^{\infty} \sin(x) \tan^{-1}(x) \, dx \)
2. \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \)
3. \( \int_{-\infty}^{\infty} \sin(4x) \, dx \)
4. \( \int_0^{1/2} x^2 \, dx \)
5. \( \int_{-\pi/10}^{\pi/10} \tan^2(4x) \, dx \)
6. \( \int_{-\infty}^{\infty} \frac{x}{x^2 + 9} \, dx \)
7. \( \int_{-\infty}^{\infty} \frac{1}{\sqrt{x^2 - 16}} \, dx \)
8. \( \lim_{x \to \infty} \frac{1}{\sqrt{x^2 - 16}} \, dx \)

19. Let \( f(x) \) be a continuous function defined on the interval \([2, \infty)\) such that
\[ f(3) = 6 \]
\[ |f(x)| < x^3 + 2 \]
and
\[ \int_3^\infty f(x) e^{-x/8} \, dx = -4 \]
Determine the value of
\[ \int_3^\infty f'(x) e^{-x/8} \, dx \]
1. (1 pt) setIntegrals19Area/sc6_1_5.pg
Sketch the region enclosed by \( y = 6x \) and \( y = 4x^2 \). Decide whether to integrate with respect to \( x \) or \( y \). Then find the area of the region.

2. (1 pt) setIntegrals19Area/sc6_1_7.pg
Sketch the region enclosed by \( y = e^{3x} \), \( y = e^{4x} \), and \( x = 1 \). Decide whether to integrate with respect to \( x \) or \( y \). Then find the area of the region.

3. (1 pt) setIntegrals19Area/sc6_1_9.png
Sketch the region enclosed by the given curves. Decide whether to integrate with respect to \( x \) or \( y \). Then find the area of the region.

4. (1 pt) setIntegrals19Area/sc6_1_12.png
Sketch the region enclosed by \( x + y^2 = 42 \) and \( x + y = 0 \). Decide whether to integrate with respect to \( x \) or \( y \). Then find the area of the region.

5. (1 pt) setIntegrals19Area/sc6_1_14.png
Sketch the region enclosed by the given curves. Decide whether to integrate with respect to \( x \) or \( y \). Then find the area of the region.

6. (1 pt) setIntegrals19Area/ur_in_19_11.png
Sketch the region enclosed by the given curves. Decide whether to integrate with respect to \( x \) or \( y \). Then find the area of the region.

7. (1 pt) setIntegrals19Area/ns6_1_99.png
Find the area between the curves:
\( y = x^3 - 10x^2 + 21x \)
and \( y = -x^3 + 10x^2 - 21x \)

8. (1 pt) setIntegrals19Area/osu_in_19_15.png
The total area enclosed by the graphs of
\( y = 6x^2 - x^3 + x \)
\( y = x^2 + 5x \)
is

9. (1 pt) setIntegrals19Area/ns6_1_1.png
Find the area enclosed between \( f(x) = 0.8x^2 + 9 \) and \( g(x) = x \) from \( x = -8 \) to \( x = 8 \).

10. (1 pt) setIntegrals19Area/ns6_1_25.png
Find the area of the region enclosed by \( y = 3 \sin(x) \) and \( y = 2 \cos(x) \) from \( x = 0 \) to \( x = 0.3\pi \).
Hint: Notice that this region consists of two parts.

11. (1 pt) setIntegrals19Area/sc6_1_27.png
Use the parametric equations of an ellipse, \( x = 2 \cos(\theta) \), \( y = 4 \sin(\theta) \), \( 0 \leq \theta \leq 2\pi \), to find the area that it encloses.

12. (1 pt) setIntegrals19Area/ns6_1_27.png
Use the parametric equations of an ellipse
\( x = 9 \cos \theta \)
\( y = 10 \sin \theta \)
\( 0 \leq \theta \leq 2\pi \)
to find the area that it encloses.

13. (1 pt) setIntegrals19Area/ur_in_19_12.png
Find the area of the region enclosed by the parametric equation
\( x = t^3 - 8t \)
\( y = 3t^2 \)

14. (1 pt) setIntegrals19Area/ur_in_19_10.png
There is a line through the origin that divides the region bounded by the parabola \( y = 5x - 4x^2 \) and the x-axis into two regions with equal area. What is the slope of that line?

15. (1 pt) setIntegrals19Area/ns6_1_10.png
Farmer Jones, and his wife, Dr. Jones, decide to build a fence in
their field, to keep the sheep safe. Since Dr. Jones is a mathematician, she suggests building fences described by $y = 2x^2$ and $y = x^2 + 3$. Farmer Jones thinks this would be much harder than just building an enclosure with straight sides, but he wants to please his wife. What is the area of the enclosed region?

16. (1 pt) setIntegrals19Area/osu_in_19_13.pg

Note: You can get full credit for this problem by just answering the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Find the area bounded by the two curves:

$$x = 100000 \left(8 \sqrt{y} - 1\right)$$

$$x = 100000 \left(\frac{8 \sqrt{y} - 1}{\sqrt{7}}\right)$$

The appropriate definite integral for computing this area has integrand

lower limit of integration = __________;
and upper limit of integration = __________
This definite integral has value = __________

This is the area of the region enclosed by the two curves.

17. (1 pt) setIntegrals19Area/osu_in_19_14.pg

Consider the area between the graphs $x + 4y = 7$ and $x + 5 = y^2$. This area can be computed in two different ways using integrals

First of all it can be computed as a sum of two integrals

$$\int_a^b f(x) \, dx + \int_b^c g(x) \, dx$$

where $a = __________$, $b = __________$, $c = __________$ and
$f(x) = __________$;
$g(x) = __________$
Alternatively this area can be computed as a single integral

$$\int_\alpha^\beta h(y) \, dy$$

where $\alpha = __________$, $\beta = __________$ and
$h(y) = __________$
Either way we find that the area is __________

18. (1 pt) setIntegrals19Area/ur_in_19_1.pg

Find $c > 0$ such that the area of the region enclosed by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 360.

c = __________
Consider the blue vertical line shown above (click on graph for better view) connecting the graphs $y = g(x) = \sin(1x)$ and $y = f(x) = \cos(2x)$.

Referring to this blue line, match the statements below about rotating this line with the corresponding statements about the result obtained.

1. The result of rotating the line about the $x$-axis is
   A. a cylinder of radius $\pi - x$ and height $\cos(2x) - \sin(1x)$
   B. an annulus with inner radius $2 + \sin(1x)$ and outer radius $2 + \cos(2x)$
   C. an annulus with inner radius $\pi + \sin(1x)$ and outer radius $\pi + \cos(2x)$
   D. an annulus with inner radius $\pi - \cos(2x)$ and outer radius $\pi - \sin(1x)$
   E. an annulus with inner radius $1 - \cos(2x)$ and outer radius $1 - \sin(1x)$ is
   F. an annulus with inner radius $\sin(1x)$ and outer radius $\cos(2x)$
   G. a cylinder of radius $x$ and height $\cos(2x) - \sin(1x)$
   H. a cylinder of radius $x + 2$ and height $\cos(2x) - \sin(1x)$

2. The result of rotating the line about the $y$-axis is
   \[ y = f(x) = \sin(2y) \]
   \[ x = g(y) = \cos(1y) \]

3. The result of rotating the line about the line $y = 1$ is
4. The result of rotating the line about the line $x = -2$ is
5. The result of rotating the line about the line $x = \pi$ is
6. The result of rotating the line about the line $y = -2$ is
7. The result of rotating the line about the line $y = \pi$
8. The result of rotating the line about the line $y = -\pi$
   A. a cylinder of radius $y$ and height $\cos(1y) - \sin(2y)$
   B. an annulus with inner radius $2 + \sin(2y)$ and outer radius $2 + \cos(1y)$
   C. a cylinder of radius $\pi + y$ and height $\cos(1y) - \sin(2y)$
   D. a cylinder of radius $1 - y$ and height $\cos(1y) - \sin(2y)$
   E. a cylinder of radius $2 + y$ and height $\cos(1y) - \sin(2y)$
   F. a cylinder of radius $\pi - y$ and height $\cos(1y) - \sin(2y)$
   G. an annulus with inner radius $\pi - \cos(1y)$ and outer radius $\pi - \sin(2y)$ is
   H. an annulus with inner radius $\sin(2y)$ and outer radius $\cos(1y)$

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

3. The result of rotating the line about the $x$-axis is
   \[ y = 4x^2, x = 1, y = 0, \text{ about the } x\text{-axis} \]

4. The result of rotating the line about the $y$-axis is
   \[ y = x^2, \ y = 0, \ x = 3, \text{ about the } y\text{-axis} \]
5. (1 pt) setIntegrals20Volume/osu_in_20_11.png
Find the volume formed by rotating the region enclosed by:
\[ x = 3y \text{ and } y^2 = x \text{ with } y \geq 0 \]
about the y-axis.

6. (1 pt) setIntegrals20Volume/ur_in_20_2.png
Find the volume of the solid formed by rotating the region enclosed by
\[ y = e^{3x} + 4, \quad y = 0, \quad x = 0, \quad x = 0.2 \]
about the x-axis.

7. (1 pt) setIntegrals20Volume/ur_in_20_3.png
Find the volume of the solid formed by rotating the region enclosed by
\[ y = e^{2x} + 2, \quad y = 0, \quad x = 0, \quad x = 0.1 \]
about the y-axis.

8. (1 pt) setIntegrals20Volume/ur_in_20_4.png
Find the volume of the solid formed by rotating the region inside the first quadrant enclosed by
\[ y = x^2 \]
\[ y = 9x \]
about the x-axis.

Find the volume of the solid formed by rotating the region enclosed by
\[ x = 0, \quad x = 1, \quad y = 0, \quad y = 4 + x^2 \]
about the x-axis.

Find the volume of the solid formed by rotating the region enclosed by
\[ x = 0, \quad x = 1, \quad y = 0, \quad y = 5 + x^4 \]
about the y-axis.

11. (1 pt) setIntegrals20Volume/sc6_2_14a.png
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = \frac{1}{x}, \quad y = 0, \quad x = 1, \quad x = 9; \]
about the y-axis.

12. (1 pt) setIntegrals20Volume/sc6_3_99.png
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = 28x - 7x^2, \quad y = 0; \]
about the y-axis.

13. (1 pt) setIntegrals20Volume/sc6_2_9.png
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = x^4, \quad y = 1; \quad \text{about } y = 3 \]

14. (1 pt) setIntegrals20Volume/sc6_2_9a.png
Find the volume of the solid obtained by rotating the region in the first quadrant bounded by the curves
\[ x = 0, \quad y = 1, \quad x = y^2, \quad \text{about the line } y = 1. \]

15. (1 pt) setIntegrals20Volume/ur_in_20_12.png
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = x^4, \quad y = 1; \quad \text{about } y = 4 \]

16. (1 pt) setIntegrals20Volume/sc6_2_14.png
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = \frac{1}{x}, \quad y = 0, \quad x = 4, \quad x = 6; \]
about the y-axis.

17. (1 pt) setIntegrals20Volume/ur_in_20_5.png
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = 0, \quad y = \cos(1x), \quad x = \frac{3}{2}, \quad x = 0 \quad \text{about the axis } y = -3 \]

18. (1 pt) setIntegrals20Volume/osu_in_20_9.png
The region between the graphs of \( y = x^2 \) and \( y = 2x \) is rotated around the line \( y = 4 \).
The volume of the resulting solid is ___________.

19. (1 pt) setIntegrals20Volume/osu_in_20_10.png
The region between the graphs of \( y = x^2 \) and \( y = 2x \) is rotated around the line \( x = 2 \).
The volume of the resulting solid is ___________.

The base of a certain solid is the area bounded above by the graph of \( y = f(x) = 25 \) and below by the graph of \( y = g(x) = 16x^2 \). Cross-sections perpendicular to the x-axis are squares. (See picture above, click for a better view.)
Use the formula
\[ V = \int_a^b A(x) \, dx \]
to find the volume of the solid.
Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

The lower limit of integration is \( a = \) ________
The upper limit of integration is \( b = \) ________
The side \( s \) of the square cross-section is the following function of \( x \):
\[
A(x) = \quad \text{Thus the volume of the solid is } V = \quad \text{(1 pt) setIntegrals20Volume/osu_in_20_4/osu_in_20_4.pg}
\]

\[
\text{The base of a certain solid is the area bounded above by the graph of } y = f(x) = 16 \text{ and below by the graph of } y = g(x) = 9x^2. \text{ Cross-sections perpendicular to the } y\text{-axis are squares. (See picture above, click for a better view.)}
\]

Use the formula
\[
V = \int_{a}^{b} A(y) \, dy
\]

to find the volume of the formula.

Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

The lower limit of integration is \( a = \) ________
The upper limit of integration is \( b = \) ________
The diameter \( 2r \) of the semicircular cross-section is the following function of \( x \):
\[
A(x) = \quad \text{Thus the volume of the solid is } V = \quad \text{(1 pt) setIntegrals20Volume/ns6_2_9.pg}
\]

\[
\text{As a hardworking student, plagued by too much homework, you spend all night doing math homework. By 6am, you imagine yourself to be a region bounded by}
\]
\[
y = 7x^2 \quad x = 0 \quad x = 2 \quad y = 0
\]

As you grow more and more tired, the world begins to spin around you. However, according to Newton, there is no difference between the world spinning around you, and you spinning around the world. Unfortunately, you are so tired that you think the world is the \( x \)-axis. What is the volume of the solid you (the region) create by spinning about the \( x \)-axis?

\[
\text{(1 pt) setIntegrals20Volume/ns6_2_21.pg}
\]

You wake up one morning, and find yourself wearing a toga and scarab ring. Always a logical person, you conclude that you must have become an Egyptian pharaoh. You decide to honor yourself with a pyramid of your own design. You decide it should have height \( h = 3840 \) and a square base with side \( s = 1690 \)

To impress your Egyptian subjects, find the volume of the pyramid.

\[
\text{(1 pt) setIntegrals20Volume/ur_in_20_1.pg}
\]

A ball of radius 13 has a round hole of radius 6 drilled through its center. Find the volume of the resulting solid.

\[
\text{(1 pt) setIntegrals20Volume/ur_in_20_11.pg}
\]

Find the volume of a pyramid with height 25 and rectangular base with dimensions 4 and 10.

\[
\text{(1 pt) setIntegrals20Volume/mecl1.pg}
\]

Find the volume of a pyramid with height 22 and rectangular base with dimensions 4 and 10.
29. (1 pt) setIntegrals20Volume/osu_in_20_8.png
The base of a certain solid is the triangle with vertices at \((-4, 2), (2, 2), \) and the origin. Cross-sections perpendicular to the \(y\)-axis are squares. Then the volume of the solid is 

30. (1 pt) setIntegrals20Volume/osu_in_20_6.png
A soda glass has the shape of the surface generated by revolving the graph of \(y = 6x^2\) for \(0 \leq x \leq 1\) about the \(y\)-axis. Soda is extracted from the glass through a straw at the rate of \(1/2\) cubic inch per second. How fast is the soda level in the glass dropping when the level is 3 inches? (Answer should be implicitly in units of inches per second. Do not put units in your answer. Also your answer should be positive, since we are asking for the rate at which the level DROPS rather than rises.)

answer: 

Coffee is poured into one of mugs above at a constant rate (constant volume per unit time). The graph below shows the depth of coffee in the mug as a function of time. (Click on images for better view.)

Which mug was filled with coffee? 

Be prepared to explain your choice (offline).

32. (1 pt) setIntegrals20Volume/osu_in_20_11.png
As viewed from above, a swimming pool has the shape of the ellipse 

\[
\frac{x^2}{3600} + \frac{y^2}{2500} = 1
\]

The cross sections perpendicular to the ground and parallel to the \(y\)-axis are squares. Find the total volume of the pool. (Assume the units of length and area are feet and square feet respectively. Do not put units in your answer.)

\[V = \text{__________} \]

33. (1 pt) setIntegrals20Volume/ur_in_20_6.png
Find the volume of the solid obtained by rotating the region bounded by the curve \(y = \sin(10x^2)\) and the \(x\)-axis, \(0 \leq x \leq \sqrt{\frac{\pi}{10}}\), about the \(y\)-axis.

\[V = \text{__________} \]

34. (1 pt) setIntegrals20Volume/ur_in_20_7.png
Find the volume of a right circular cone with height 16 and base radius 4.

\[V = \text{__________} \]
Consider the parametric curve given by the equations
\[ x(t) = t^2 + 29t + 43 \]
\[ y(t) = t^2 + 29t - 48 \]
from point A to point B, where A = (0, 0) and B = (36, 6).

Find the length of the curve for \( \theta = 0 \) to \( \theta = \frac{1}{8} \pi \).

Consider the parametric equation
\[ x = 5(\cos \theta + \theta \sin \theta) \]
\[ y = 5(\sin \theta - \theta \cos \theta) \]
What is the length of the curve for \( \theta = 0 \) to \( \theta = \frac{1}{8} \pi \)?

If \( f(\theta) \) is given by: \( f(\theta) = 8 \cos^3 \theta \) and \( g(\theta) \) is given by: \( g(\theta) = 8 \sin^3 \theta \)
Find the total length of the astroid described by \( f(\theta) \) and \( g(\theta) \).
(The astroid is the curve swept out by \((f(\theta), g(\theta))\) as \( \theta \) ranges from 0 to \( 2\pi \).)

Given the equation: \( xy = 12 \), set up an integral to find the length of the path from \( x = a \) to \( x = b \) and enter the integrand below.
(i.e. if your integral is \( L = \int_a^b \frac{x^2 + 2x^2}{n} \), enter \( \frac{x^2 + 2x^2}{n} \) as your answer.)
\[ L = \int_a^b \frac{x^2 + 2x^2}{n} \, dx \]

You and your best friend Janine decide to play a game. You are in a land of make-believe, where you are a function \( f(t) \), and she will be a function \( g(t) \). In this make-believe land, the two
of you are posing as parametric equations, (to keep other equations from interfering). As parametric equations, your joint path is dependent on decisions that each of you make. Janine decides how you will move in the North and South (y-axis) directions, and you control East and West (x-axis). If your identity, \( f(t) \) is given by:

\[
f(t) = \frac{(t^2 + 50)^{\frac{3}{2}}}{3}
\]

and Janine’s identity, \( g(t) \) is given by:

\[g(t) = 25t\]

How many units of distance do the two of you cover between the Most Holy Point o’ Beginnings \((t=0)\), and The Buck Stops Here \((t=29)\)?

14. (1 pt) setIntegrals21Length/ur_in_21_11.pg
A cable hangs between two poles of equal height and 38 feet apart. At a point on the ground directly under the cable and \( x \) feet from the point on the ground halfway between the poles the height of the cable in feet is

\[h(x) = 10 + (0.3)(x^{1.5}).\]

The cable weighs 19 pounds per linear foot. Find the weight of the cable.
1. (1 pt) setIntegrals22Average/ur_in_22_1.pg
A car drives down a road in such a way that its velocity (in m/s) at time t (seconds) is
\[ v(t) = 2t^{1/2} + 4 \]
Find the car’s average velocity (in m/s) between \( t = 5 \) and \( t = 7 \).

2. (1 pt) setIntegrals22Average/ur_in_22_2.pg
A solid lies between two parallel planes 4 feet apart and has a volume of 37 cubic feet. What is the average area of cross-sections of the solid by planes that lie between the given ones?

3. (1 pt) setIntegrals22Average/ur_in_22_3.pg
Find the average value of: \( f(x) = 4 \sin x + 4 \cos x \) on the interval \([0, 19\pi/6]\).
Average value = ________________

4. (1 pt) setIntegrals22Average/osu_in_22_4.pg
Find the mean value of the function \( f(x) = |9 - x| \) on the closed interval \([7, 12]\).
mean value = __________

5. (1 pt) setIntegrals22Average/ur_in_22_11.pg
In a certain city the temperature (in degrees Fahrenheit) \( t \) hours after 9am was approximated by the function
\[ T(t) = 60 + 8 \sin \left( \frac{\pi t}{12} \right) \]
Determine the temperature at 9 am. __________
Determine the temperature at 3 pm. __________
Find the average temperature during the period from 9 am to 9 pm. __________

6. (1 pt) setIntegrals22Average/ur_in_22_12.pg
One fine day in Rochester the low temperature occurs at 5 a.m. and the high temperature at 5 p.m. The temperature varies sinusoidally all day.
The temperature \( t \) hours after midnight is
\[ T(t) = A + B \sin \left( \frac{\pi(t - C)}{12} \right) \]
where \( A, B, \) and \( C \) are certain constants.
The low temperature is 25 and the high temperature is 37.
Find the average temperature during the first 3 hours after noon. Hint: The high and low temperatures can be used together to find \( A \) and \( B \).
Determine \( C \) from the fact that it is hottest at 5 p.m.
1. (1 pt) setIntegrals23Work/eva5_1.pg
The force on a particle is described by $3x^3 - 5$ at a point $x$ along the $x$-axis. Find the work done in moving the particle from the origin to $x = 5$.

2. (1 pt) setIntegrals23Work/eva5_2.pg
A force of 3 pounds is required to hold a spring stretched 0.5 feet beyond its natural length. How much work (in foot-pounds) is done in stretching the spring from its natural length to 0.7 feet beyond its natural length?

3. (1 pt) setIntegrals23Work/eva5_3.pg
Work of 4 Joules is done in stretching a spring from its natural length to 18 cm beyond its natural length. What is the force (in Newtons) that holds the spring stretched at the same distance (18 cm)?

4. (1 pt) setIntegrals23Work/eva6_1.pg
A tank in the shape of an inverted right circular cone has height 11 meters and radius 19 meters. It is filled with 1 meters of hot chocolate.

Find the work required to empty the tank by pumping the hot chocolate over the top of the tank. Note: the density of hot chocolate is $\delta = 1450 \text{kg/m}^3$.

5. (1 pt) setIntegrals23Work/lanpri_7_6a.pg
A trough is 2 feet long and 1 foot high. The vertical cross-section of the trough parallel to an end is shaped like the graph of $y = x^{10}$ from $x = -1$ to $x = 1$. The trough is full of water.

Find the amount of work in foot-pounds required to empty the trough by pumping the water over the top. Note: The weight of water is 62 pounds per cubic foot.

6. (1 pt) setIntegrals23Work/lanpri_7_8a.pg
A trough is 7 meters long, 2.5 meters wide, and 4 meters deep. The vertical cross-section of the trough parallel to an end is shaped like an isosceles triangle (with height 4 meters, and base, on top, of length 2.5 meters). The trough is full of water (density 1000 $\text{kg/m}^3$). Find the amount of work in joules required to empty the trough by pumping the water over the top. (Note: Use $g = 9.8 \text{m/s}^2$ as the acceleration due to gravity.)

7. (1 pt) setIntegrals23Work/ns6_5_12.pg
You are visiting your friend Fabio’s house. You find that, as a joke, he filled his swimming pool with Kool-Aid, which dissolved perfectly into the water. However, now that you want to swim, you must remove all of the Kool-Aid contaminated water.

The swimming pool is round, with a 8 foot radius. It is 6.5 feet tall and has 6.5 feet of water in it. How much work is required to remove all of the water by pumping it over the side? Use the physical definition of work, and the fact that the weight of the Kool-Aid contaminated water is $\sigma = 63.7 \text{lbs/ft}^3$.

8. (1 pt) setIntegrals23Work/ur_in_23_11.pg
A circular swimming pool has a diameter of 12 m, the sides are 4 m high, and the depth of the water is 2.5 m. How much work (in Joules) is required to pump all of the water over the side? (The acceleration due to gravity is $9.8 \text{ m/s}^2$ and the density of water is 1000 $\text{kg/m}^3$.)
1. (1 pt) setIntegrals23_5Pressure/ur_in_23.5_1.pg
An aquarium 9 m long, 5 m wide, and 4 m deep is full of water. Find the following:
- the hydrostatic pressure on the bottom of the aquarium
- the hydrostatic force on the bottom of the aquarium
- the hydrostatic force on one end of the aquarium

2. (1 pt) setIntegrals23_5Pressure/benny14.pg
The Deligne Dam on the Cayley River is built so that the wall facing the water is shaped like the region above the curve
\( y = 0.4x^2 \) and below the line \( y = 240 \). (Here, distances are measured in meters.) The water level is 40 meters below the top of the dam. Find the force (in Newtons) exerted on the dam by water pressure. (Water has a density of 1000 kg/m³, and the acceleration of gravity is 9.8 m/sec².)
1. (1 pt) setIntegrals24Centroid/ur_in_24_1.pg
The masses $m_i$ are located at the points $P_i$. Find the center of mass of the system.

$m_1 = 4, m_2 = 5, m_3 = 9.$
$P_1 = (-8, 6), P_2 = (0, 7), P_3 = (-4, 2).$

$x = \bar{x} = \frac{\sum m_i x_i}{\sum m_i}$
$y = \bar{y} = \frac{\sum m_i y_i}{\sum m_i}$

2. (1 pt) setIntegrals24Centroid/ur_in_24_2.pg
Find the centroid $(\bar{x}, \bar{y})$ of the triangle with vertices at $(0,0)$, $(10,0)$, and $(0,4)$.

$\bar{x} = \frac{\sum x_i}{3}$
$\bar{y} = \frac{\sum y_i}{3}$

3. (1 pt) setIntegrals24Centroid/centroid6_5.1.pg
Find the centroid $(\bar{x}, \bar{y})$ of the region bounded by:

$y = 4x^2 + 3x, \quad y = 0, \quad x = 0, \quad$ and $\quad x = 3$

$\bar{x} = \frac{\int_0^3 (4x^2 + 3x) \, dx}{\int_0^3 (4x^2 + 3x) \, dx}$
$\bar{y} = \frac{\int_0^3 \frac{1}{2} (4x^2 + 3x)^2 \, dx}{\int_0^3 (4x^2 + 3x) \, dx}$
1. Evaluate the integral.
\[ \int \frac{1}{(x-1)(x+4)} \, dx \]

2. Evaluate the indefinite integral.
\[ \int \frac{1}{(x-3)(x+1)} \, dx \]

3. Evaluate the indefinite integral.
\[ \int \frac{-2}{x^2 + 12x + 36} \, dx \]

4. Evaluate the integral.
\[ \int_{-2}^{1} \frac{1}{(x+6)(x^2+4)} \, dx \]

5. Evaluate the integral.
\[ \int_{-1}^{1} \frac{1}{(x^2+0x+4)} \, dx \]

6. Evaluate the integral.
\[ \int_{5}^{6} \frac{7x-9}{x^2 - 2x - 3} \, dx \]

7. Evaluate the integral.
\[ \int_{5}^{6} \frac{6x-8}{x^2 - 1x - 6} \, dx \]

8. Evaluate the integral.
\[ \int_{2}^{3} \frac{8x+3}{x^2 + 1x + 0} \, dx \]

9. Evaluate the integral.
\[ \int \frac{2x+3}{x^2+2x+10} \, dx \]

10. Evaluate the indefinite integral.
\[ \int \frac{x+1}{x^2 + 2x + 2} \, dx + C \]

11. Write out the form of the partial fraction decomposition of the function appearing in the integral:
\[ \int \frac{-4x - 196}{x^2 + 2x - 63} \, dx \]

Determine the numerical values of the coefficients, A and B, where A ≤ B.
\[ A = \frac{\text{denominator}}{A} \quad B = \frac{\text{denominator}}{B} \]

12. Write out the form of the partial fraction decomposition of the function:
\[ Q = \int \frac{1x}{x^2 + 5x + 6} \, dx \]

Determine the numerical values of the coefficients, A and B, where B ≤ A.
\[ A = \frac{\text{denominator}}{A} \quad B = \frac{\text{denominator}}{B} \]

13. Write out the form of the partial fraction decomposition of the function:
\[ Q = \int \frac{3x + 11}{x^2 + 4x + 4} \, dx \]

Determine the numerical values of the coefficients, A and B, where A ≤ B.
\[ A = \frac{\text{denominator}}{A} \quad B = \frac{\text{denominator}}{B} \]

14. Evaluate the integral.
\[ \int \frac{6x^2 - 25x - 38}{x^3 - 6x^2 + 0x + 32} \, dx \]

15. The answer to this question contains absolute values. The absolute value of a quantity w should be written \( |w| \). Evaluate the integral.
\[ \int \frac{3x^2 - 13x - 40}{x^3 - 7x^2 + 41x - 87} \, dx \]

16. Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.
Consider the indefinite integral \( \int \frac{3x^3 + 4x^2 - 7x - 14}{x^2 - 4} \, dx \). 
Then the integrand decomposes into the form 
\[
a x + b + \frac{c}{x - 2} + \frac{d}{x + 2}
\]
where 
\[
a = \\
b = \\
c = \\
d =
\]
Integrating term by term, we obtain that 
\[
\int \frac{3x^3 + 4x^2 - 7x - 14}{x^2 - 4} \, dx = +C
\]

17. (1 pt) setIntegrals25RationalFunctions/ur_in_25_12.pg

The form of the partial fraction decomposition of a rational function is given below. 
\[
\frac{5x^2 + 4x + 54}{(x - 4)(x^2 + 9)} = \frac{A}{x - 4} + \frac{Bx + C}{x^2 + 9}
\]
\[
A = \quad B = \quad C =
\]
Now evaluate the indefinite integral. 
\[
\int \frac{5x^2 + 4x + 54}{(x - 4)(x^2 + 9)} \, dx
\]

18. (1 pt) setIntegrals25RationalFunctions/ur_in_25_12B.pg

The form of the partial fraction decomposition of a rational function is given below. 
\[
\frac{-3x^2 + 10x - 16}{(x - 2)(x^2 + 4)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 4}
\]
\[
A = \quad B = \quad C =
\]
Now evaluate the indefinite integral. 
\[
\int \frac{-3x^2 + 10x - 16}{(x - 2)(x^2 + 4)} \, dx
\]


Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the indefinite integral \( \int \frac{10x^3 + 9x^2 + 80x + 80}{x^4 + 16x^2} \, dx \). 
Then the integrand has partial fractions decomposition 
\[
a \frac{b}{x^2} + \frac{cx + d}{x^2 + 16}
\]
where 
\[
a = \\
b = \\
c = \\
d =
\]
Integrating term by term, we obtain that 
\[
\int \frac{10x^3 + 9x^2 + 80x + 80}{x^4 + 16x^2} \, dx = +C
\]


Evaluate the indefinite integral. 
\[
\int \frac{-1x^3 + 4x^2 - 16}{(x^4 + 2x^3)} \, dx
\]


Evaluate the indefinite integral. 
\[
\int \frac{x^3 + 69}{x^2 + 7x + 12} \, dx
\]

22. (1 pt) setIntegrals25RationalFunctions/ur_in_25_14B.pg

Evaluate the indefinite integral. 
\[
\int \frac{3x^3 - 16x^2 + 18x + 8}{(x^4 + 4x^3)} \, dx
\]

23. (1 pt) setIntegrals25RationalFunctions/ur_in_25_1.pg

Evaluate the integral. 
\[
\int_{-1}^{4} \frac{x^3 + 3}{(x + 5)(x + 3)} \, dx
\]

24. (1 pt) setIntegrals25RationalFunctions/ur_in_25_25.pg

Evaluate the integral. 
\[
\int \frac{-27e^x - 36}{e^{2x} + 5e^x + 4} \, dx
\]

25. (1 pt) setIntegrals25RationalFunctions/ur_in_25_5.pg

Let \( f(x) \) be a quadratic function such that \( f(0) = -8 \) and 
\[
\int \frac{f(x)}{x^2(x - 3)^2} \, dx
\]
is a rational function. 
Determine the value of \( f'(0) \). 
\[
f'(0) =
\]

26. (1 pt) setIntegrals25RationalFunctions/osu_in_25_9a.pg

Consider the integral 
\[
\int \frac{x^{21} - 8x^{14} + 5x^7 - 14}{(x^3 - 3x^2 + 2x)^7(x^4 - 16)^2} \, dx
\]
Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the integrand. 
\( A_1, A_2, A_3 \ldots \) and \( B_1, B_2, B_3, \ldots \) denote constants. 
You must get all of the answers correct to receive credit.

1. \( \frac{B_1}{x+2} \) 
2. \( \frac{B_1}{x+1} \) 
3. \( \frac{B_1}{x+2} \) 
4. \( \frac{A_3+B_2}{(x-2)^2} \) 
5. \( \frac{B_3}{x^2+4} \) 
6. \( \frac{B_1}{x-2} \) 
7. \( \frac{A_3+B_3}{x^2-4} \)
27. (1 pt) setIntegrals25RationalFunctions/osu_in_25_9b.pg
(Continuation of Problem 7) Consider the integral
\[ \int \frac{x^2 + 8x^{14} + 5x^7 - 14}{(x^3 - 3x^2 + 2x)^3 (x^4 - 16)^2} \, dx \]
Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the integrand. 
A_1, A_2, A_3, … and B_1, B_2, B_3, … denote constants.
You must get all of the answers correct to receive credit.

1. \( B_3 \)
2. \( B_1 \)
3. \( A_2x^4 \)
4. \( A_3x^5 \)
5. \( B_4 \)
6. \( B_5 \)
7. \( A_1x \)

28. (1 pt) setIntegrals25RationalFunctions/osu_in_25_9c.pg
(Continuation of Problem 7) Consider the integral
\[ \int \frac{x^2 + 8x^{14} + 5x^7 - 14}{(x^3 - 3x^2 + 2x)^3 (x^4 - 16)^2} \, dx \]
Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the integrand. 
A_1, A_2, A_3, … and B_1, B_2, B_3, … denote constants.
You must get all of the answers correct to receive credit.

1. \( B_3 \)
2. \( B_1 \)
3. \( A_2x^4 \)
4. \( A_3x^5 \)
5. \( B_4 \)
6. \( B_5 \)
7. \( A_2x \)

29. (1 pt) setIntegrals25RationalFunctions/osu_in_25_9d.pg
(Continuation of Problem 7) Consider the integral
\[ \int \frac{x^2 + 8x^{14} + 5x^7 - 14}{(x^3 - 3x^2 + 2x)^3 (x^4 - 16)^2} \, dx \]
Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the integrand. 
A_1, A_2, A_3, … and B_1, B_2, B_3, … denote constants.

1. \( A_{3x+B_1} \)
2. \( B_2 \)
3. \( B_4 \)
4. \( A_{3x+B_0} \)
5. \( A_{1x} \)
6. \( B_3 \)
7. \( A_{x^2+B_2} \)
8. \( \frac{B_4}{x} \)

30. (1 pt) setIntegrals25RationalFunctions/osu_in_25_10.pg
Consider the integral
\[ \int \frac{(1 + 12x)^{10}}{(x^3 - x)^2 (x^2 - 9x + 8)} \, dx \]
Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the integrand. 
A_1, A_2, A_3, … and B_1, B_2, B_3, … denote constants.

1. \( B_1 \)
2. \( A_2 \)
3. \( B_1 \)
4. \( A_1 \)
5. \( B_3 \)
6. \( B_5 \)
7. \( A_{10x+B_1} \)
8. \( A_4x^2 \)
9. \( B_3 \)
10. \( A_{3x^3} \)
11. \( A_{4x^2+B_1} \)
12. \( A_6x \)
13. \( B_3 \)
14. \( B_3 \)
15. \( B_8 \)
16. \( A_{3x^3+B_0} \)
17. \( A_4x^2 \)
18. \( A_4x^2 \)
19. \( \frac{A_{4x+B_0}}{x^2} \)
20. \( \frac{B_4}{x^2} \)
1. Find the area of the region inside \( r = 10 \sin \theta \) but outside: \( r = 4 \)

2. Find the area of the region bounded by the given curve: \( r = 7e^{\theta} \) on the interval \( \frac{-\pi}{4} \leq \theta \leq 2\pi \).

3. Find the exact length of the polar curve described by: \( r = 4e^{-\theta} \) on the interval \( \frac{\pi}{4} \leq \theta \leq 7\pi \).

4. Find the area of the region bounded by: \( r = 7 - 1 \sin \theta \)

5. A curve with polar equation
   \[
   r = \frac{4}{9 \sin \theta + 37 \cos \theta}
   \]
   represents a line. This line has a Cartesian equation of the form \( y = mx + b \), where \( m \) and \( b \) are constants. Give the formula for \( y \) in terms of \( x \). For example, if the line had equation \( y = 2x + 3 \) then the answer would be \( 2x + 3 \).

6. Match each polar equation below to the best description. Possible answers are C,E,F,H,L,P, and S.

   **DESCRIPTIONS**
   C. Circle, E. Ellipse, F. Figure eight, H. Hyperbola, L. Line, P. Parabola, S. Spiral

   **POLAR EQUATIONS**
   
   \[
   r = \frac{1}{3 + 3 \cos \theta}
   \]
   
   \[
   r = \frac{1}{13 + 3 \cos \theta}
   \]
   
   \[
   r = 3 \sin \theta + 17 \cos \theta
   \]
   
   \[
   r = \frac{1}{3 + 17 \cos \theta}
   \]
   
   \[
   r = \frac{1}{3 \sin \theta + 17 \cos \theta}
   \]

7. Match each polar equation below to the best description. Each answer should be C,E,F,H,L,O,P,R,S,T, or W.

   **DESCRIPTIONS**
   C. Cardioid, E. Ellipse, F. Figure eight, H. Hyperbola, L. Line, O. Oval, P. Parabola, R. Rose with four petals, S. Spiral, T. Three-petaled rose, W. A pair of wings

   **POLAR EQUATIONS**
   
   \[
   r^2 = \csc 2\theta
   \]
   
   \[
   r = \tan \theta
   \]
   
   \[
   r^2 = 14 \sin \theta
   \]

8. A circle \( C \) has center at the origin and radius 3. Another circle \( K \) has a diameter with one end at the origin and the other end at the point \((0,17)\). The circles \( C \) and \( K \) intersect in two points. Let \( P \) be the point of intersection of \( C \) and \( K \) which lies in the first quadrant. Let \((r, \theta)\) be the polar coordinates of \( P \), chosen so that \( r \) is positive and \( 0 \leq \theta \leq 2 \). Find \( r \) and \( \theta \).

   \[
   r = \_
   \]

   \[
   \theta = \_
   \]

9. Find the area of the region bounded by: \( r = 8 - 5 \sin \theta \)

10. Find the exact length of the polar curve described by:
    \[
    r = 8e^{0.4\theta}
    \]
    on the interval \( 0 \leq \theta \leq \frac{1}{4} \).

11. Find the area enclosed by the polar curve
    \[
    r = 10e^{0.8\theta}
    \]
    on the interval \( 0 \leq \theta \leq \frac{1}{2} \) and the straight line segment between its ends.

12. Find the area of the region inside
    \[
    r = 20 \sin \theta
    \]
    but outside \( r = 2 \).

13. Find the length of the entire perimeter of the region inside
    \[
    r = 10 \sin \theta
    \]
    but outside \( r = 1 \).

14. Find the area enclosed by the closed curve obtained by joining the ends of the spiral
    \[
    r = 70, 0 \leq \theta \leq 3.9
    \]
    by a straight line segment.

15. Find the area of the region bounded by: \( r = 5 \cos 6\theta \)

16. Find the area of the region bounded by: \( r^2 = 72 \cos 2\theta \)

17. Find the area inside the loop of the following limacon: \( r = 5 - 10 \sin \theta \)
18. Find the arc length of the polar curve described by: \( r = 5 + 5 \cos \theta \)

19. Find the area of the region outside \( r = 7 + 7 \sin \theta \), but inside \( r = 21 \sin \theta \).
1. (1 pt) setIntegrals27SurfaceArea/ur_in_27_1.pg
Find the area of the surface obtained by rotating the curve 
\[ y = 2x^3 \]
from \( x = 0 \) to \( x = 3 \) about the \( x \)-axis.

2. (1 pt) setIntegrals27SurfaceArea/ur_in_27_2.pg
Find the area of the surface obtained by rotating the curve 
\[ y = \sqrt{6x} \]
from \( x = 0 \) to \( x = 5 \) about the \( x \)-axis.

3. (1 pt) setIntegrals27SurfaceArea/ur_in_27_3.pg
Find the area of the surface obtained by rotating the curve 
\[ y = 1 + 4x^2 \]
from \( x = 0 \) to \( x = 6 \) about the \( y \)-axis.

4. (1 pt) setIntegrals27SurfaceArea/ur_in_27_4.pg
Find the area of the surface obtained by rotating the curve 
\[ x = 2e^{2y} \]
from \( y = 0 \) to \( y = 4 \) about the \( y \)-axis.
1. (1 pt) setIntegrationProjects/proj0/proj0.pg

Consider the function \( y = f(x) \) specified by the following table:

\begin{align*}
-1 & \quad -0.707106781186547 \\
-0.8 & \quad 0.827080574274562 \\
-0.6 & \quad 0.612907053652977 \\
-0.4 & \quad -0.562083377852131 \\
-0.2 & \quad -0.999506560365732 \\
0 & \quad -0.707106781186547 \\
0.2 & \quad -0.278991106039229 \\
0.4 & \quad 0.181991106039229 \\
0.6 & \quad 0.562083377852131 \\
0.8 & \quad 0.827080574274562 \\
1 & \quad -0.707106781186547
\end{align*}

(The first column contains \( x \) values, while the second column contains the corresponding \( y \) values.) Find a numerical approximation to the function

\[ F(x) = \int_{0.2}^{x} f(t)\,dt \]

using the following variant of midpoint Riemann sums: instead of computing

\[ f\left(\frac{x_1 + x_2}{2}\right) \]

(whose value is not given) compute

\[ \frac{f(x_1) + f(x_2)}{2} \]

\((x_1 \text{ and } x_2 \text{ denote adjacent } x \text{ values in the table.}) \) Use a spreadsheet to do the calculation.

Then answer the following questions.

When \( x = 0.8 \) then \( F(x) \approx \quad \) 
When \( x = 0 \) then \( F(x) \approx \quad \) 
When \( x = -0.6 \) then \( F(x) \approx \quad \)

Look at the eight graphs below and choose the one which most closely resembles the graph of

\[ F(x) = \int_{0.2}^{x} f(t)\,dt \]

ANSWER = \quad (Enter the label of the graph you think is right: 1, 2, 3, 4, 5, 6, 7 or 8.)
Consider the function
\[ F(x) = \int_0^x \sin(-1.0903t^2) \, dt \]

Find a numerical approximation to this function using midpoint Riemann sums and plot it using a spreadsheet. Use equally spaced partitions of size \( \Delta x = 0.1 \). Then numerically compute an approximation to the derivative \( F'(x) \) and compare the resulting graph to that of \( \sin(-1.0903x^2) \).

To get credit for this problem you need to submit separate printouts (during lecture or office hours) of each of the following:

1. The spreadsheet Tables 1, 2 and 3 with numerical values as indicated below. To conserve paper, just print the first two pages of each table.
2. The spreadsheet Tables 1, 2 and 3 with spreadsheet formulas displayed instead of numbers. To conserve paper, just print the first two pages of each table.
3. The graph of \( y = F(x) \) using the spreadsheet chart facility.
4. The graphs of \( y = F'(x) \) and \( y = \sin(-1.0903x^2) \) plotted together, also using the spreadsheet chart facility.
5. A printout of this page, after you have clicked the Submit button, indicating that you have answered the questions below correctly. (Don’t bother submitting anything else without a WeBWorK printout showing you have at least computed the \( F(x) \) values correctly.)

The spreadsheet Tables 1 and 2 should have the following format:
- The A column should contain \( x \) values starting with \( x = 0 \) going up to \( x = 10 \) in increments of the step size \( \pm 0.1 \).
- The B column should contain the step size (interval width) \( \Delta x \).
- The C column should contain the midpoint of the interval whose left endpoint is in column A and whose width is in column B.
- The D column should contain the value of \( f(x) = \sin(-1.0903x^2) \) at the midpoint in column C.
- The E column should contain the (signed) area of the approximating rectangle over the interval whose left endpoint is in column A.
- The F column should contain an approximation to the value of \( y = F(x) \) corresponding to the \( x \) value in column A. This approximation is obtained by summing up the areas of the approximating rectangles in column E in all the rows above the current one (EXCLUDING the current row).

You spreadsheet should consist of two separate tables, Table 1 using a step size of \( \Delta x = 0.1 \) (stepping forward), Table 2 using a step size of \( \Delta x = -0.1 \) (stepping backward).

For the plot chart of \( y = F(x) \) follow these basic instructions, modifying as needed to suit your spreadsheet software.

1. Select all the cells in your spreadsheet and "Copy"
2. Open a new blank worksheet
3. Click on the first cell and "Paste Special — Values" into the new worksheet.
Consider the function
\[ F(x) = \int_0^x \sin(-1.0903t^2)dt \]

of Part 1.

Now we are ready for the second part of the problem: the comparison between the graphs of \( y = F'(x) \) and \( y = \sin(-1.0903x^2) \). Recall that

\[ F'(x) = \lim_{h \rightarrow 0} \frac{F(x+h)-F(x)}{h} \]

Since we have only computed values of \( F(x) \) in steps of size 0.1, the smallest value of \( h \) we can use is \( h = 0.1 \) giving us an approximation:

\[ F'(x) \approx \frac{F(x+0.1)-F(x)}{0.1} \]

Start with the spreadsheet you saved from the previous part, whose first column contains \( x \) values from -10 to 10 going up in increments of 0.1. The second column contains the corresponding values \( F(x) \). Now start a third column computing approximations to \( F'(x) \) using the above approximation.

The third column should end one row early, corresponding to \( x = 9.9 \), since computing cell C201 = \( F'(10) \) by this method, would require knowing \( F(10+0.1) \) = \( F(10.1) \), which we haven’t computed.

Finally fill in the fourth column by computing the values of \( \sin(-1.0903x^2) \) corresponding to the \( x \) values in the first column. This completes the Table 3 spreadsheet, which you are supposed to hand in (in two forms, one displaying numbers, the second displaying formulas).

You will notice that the values in the third column \( F'(x) \) roughly match those in the fourth column \( \sin(-1.0903x^2) \) for \( x \) values in the range \(-4 \leq x \leq 4 \), but tend to diverge from each other outside this interval. Select this region \(-4 \leq x \leq 4 \) in your table and paste it into a new spreadsheet using the "Paste Special→ Values" command. Then delete the second column (containing \( F(x) \) values), leaving a spreadsheet containing \( x \) values from \(-4 \) to \( 4 \) in the first column, corresponding \( F'(x) \) values in the second column, and corresponding \( \sin(-1.0903x^2) \) values in the third column. Select all the cells in this spreadsheet and, using the appropriate menu or task bar icon, create a chart plotting the selected cells fas in the first part of problem.

The following questions are designed to tell you whether your methods are correct, and to debug your spreadsheets. Answer them by referring to Tables 1, 2 and 3 prepared according to the above instructions. You will need to submit a printout of the response from Webwork indicating that your answers are correct.

When \( x = 4.3 \) then \( F'(x) \approx \)
When \( x = 8.6 \) then \( F'(x) \approx \)
When \( x = -4 \) then \( F'(x) \approx \)
When \( x = -8.1 \) then \( F'(x) \approx \)

3. (1 pt) setIntegrationProjects/proj1/prob2.pg

EXTRA BONUS PROBLEM: You can improve the accuracy of the above calculations (especially the second graph, also extending the range of \( x \) values to a bigger interval than \(-4 \leq x \leq 4 \)) by using a smaller step size than 0.1. Redo the entire problem using a smaller step size and explaining what you think is going on. (Be sure to explain what you think the value of the shift will be for this smaller step size.) Hand in the results separately writing your name and BONUS PROJECT 1 on the cover sheet. (You still have to hand in the regular Project 1.) To conserve paper, print out just the first two pages of Tables 1, 2 and 3. If you do this correctly, you will get 10 more bonus points added to your course total, after the grading curve for the class has been constructed.
Consider the exponential equation
\[ 1.1^x = 2 \]

Let \( N \) and \( N + 1 \) be the two INTEGERS which bracket the solution (ie. \( 1.1^N < 2 \) and \( 1.1^{N+1} > 2 \)). Then
\[ N = \ldots \]

Now consider the exponential equation
\[ 1.1^y = 3 \]

Let \( N \) and \( N + 1 \) be the two INTEGERS which bracket the solution (ie. \( 1.1^N < 3 \) and \( 1.1^{N+1} > 3 \)). Then
\[ N = \ldots \]

Using the integers \( N \) as approximations to the actual solutions \( x \) and \( y \) of the exponential equations above we find that the exponential equation
\[ 2^x = 3 \]

has approximate solution \( z \approx \frac{n}{m} \) where
\[ m = \ldots \]
and \( n = \ldots \]

To see how good an approximation \( \frac{n}{m} \) is to the actual solution we compute
\[ 2^n = \ldots \]
and check how close it is to 3.

This problem illustrates John Napier’s (1550-1617) approach to solving exponential equations and how he came to discover natural logarithms. Note that computing INTEGER powers of 1.001 can be done easily by hand (given enough free time), as multiplication by 1.001 requires only shifting and adding.

HINT FOR PARTS 3 AND 4: \((1.001)^a = 1.1^{ab}\)

This problem is a reprise of Problem 1 with 1.1 replaced everywhere by 1.001

Consider the exponential equation
\[ 1.001^x = 2 \]

Let \( N \) and \( N + 1 \) be the two INTEGERS which bracket the solution (ie. \( 1.001^N < 2 \) and \( 1.001^{N+1} > 2 \)). Then
\[ N = \ldots \]

Now consider the exponential equation
\[ 1.001^y = 3 \]

Let \( N \) and \( N + 1 \) be the two INTEGERS which bracket the solution (ie. \( 1.001^N < 3 \) and \( 1.001^{N+1} > 3 \)). Then
\[ N = \ldots \]

Using the integers \( N \) as approximations to the actual solutions \( x \) and \( y \) of the exponential equations above we find that the exponential equation
\[ 2^x = 3 \]

has approximate solution \( z \approx \frac{n}{m} \) where
\[ m = \ldots \]
and \( n = \ldots \]

To see how good an approximation \( \frac{n}{m} \) is to the actual solution we compute
\[ 2^n = \ldots \]
and check how close it is to 3.

This problem illustrates John Napier’s (1550-1617) approach to solving exponential equations and how he came to discover natural logarithms. Note that computing INTEGER powers of 1.001 can be done easily by hand (given enough free time), as multiplication by 1.001 requires only shifting and adding.

HINT FOR PARTS 3 AND 4: \((1.001)^a = 1.1^{ab}\)

The solution \( x \) to the exponential equation
\[ a^x = b \]

with \( a \) and \( b \) given numbers is denoted \( x = \log_a b \). Thus in problems 1 and 2 you were computing \( \text{Int}(\log_{1.1} 2) \), \( \text{Int}(\log_{1.001} 2) \), \( \text{Int}(\log_{1.1} 3) \) and \( \text{Int}(\log_{1.001} 3) \) (where \( \text{Int}(\ ) \) denotes the integer part), as well as approximations to \( \log_{1.1} 3 \).

Continue your calculations from the first part of problem 2 and compare your results with those you get computing natural logarithms (using a calculator or spreadsheet).

\[ \text{Int}(\log_{1.001} 2) = \ldots \]
while \( \ln 2 = \ldots \)
\[ \text{Int}(\log_{1.001} 3) = \ldots \]
while \( \ln 3 = \ldots \)
\[ \text{Int}(\log_{1.001} 4) = \ldots \]
while \( \ln 4 = \ldots \)
\[ \text{Int}(\log_{1.001} 5) = \ldots \]
while \( \ln 5 = \ldots \)
\[ \text{Int}(\log_{1.001} 6) = \ldots \]
while \( \ln 6 = \ldots \)
\[ \text{Int}(\log_{1.001} 7) = \ldots \]
while \( \ln 7 = \ldots \)
\[ \text{Int}(\log_{1.001} 8) = \ldots \]
while \( \ln 8 = \ldots \)
\[ \text{Int}(\log_{1.001} 9) = \ldots \]
while \( \ln 9 = \ldots \)
\[ \text{Int}(\log_{1.001} 10) = \ldots \]
while \( \ln 10 = \ldots \)

Referring back to the results of the previous problem we find that there is a nice relation between natural logarithms and \( \text{Int}(\log_{1.001} (-)) \) namely
\[ \ln(x) \approx \frac{\text{Int}(\log_{1.001} x)}{n} \]

where \( n = \ldots \)

Let’s try to explain this relation. Use the same procedure as in the preceding two problems to find \( \text{Int}(\log_{1.001} (e)) \) where \( e = 2.718281828459 \ldots \) is the base of natural logarithms. We find
\[ \text{Int}(\log_{1.001} (e)) = \ldots \]

Now use the same procedure as you used in problem 2 (to solve \( 2^x = 3 \)) to find an approximate solution of the exponential equation
\[ e^x = 3 \]
You find that $z \approx \frac{m}{n}$ where
\[ m = \ldots \]
and
\[ n = \ldots \]
Now recall that by definition $z = \ln(3)$.

Let’s now explore this a little further. Suppose that we replace 1.001 in our calculations by 1.0005. Then we find
\[ \text{Int}(\log_{1.0005}(e)) = \ldots \]
and
\[ \text{Int}(\log_{1.0005}(3)) = \ldots \]
which gives the approximate solution $z \approx \frac{m}{n}$ to $e^z = 3$ where
\[ m = \ldots \]
and
\[ n = \ldots \]

Generalizing this we get the following relation:
\[ \ln(x) \approx \frac{\text{Int}(\log_{1.0005} x)}{n} \]
where $n = \ldots$

Similarly we find that
\[ \text{Int}(\log_{1+1/2013}(e)) = \ldots \]
and that
\[ \ln(x) \approx \frac{\text{Int}(\log_{1+1/2013} x)}{n} \]
where $n = \ldots$

By the same method we find that
\[ \ln(x) \approx \frac{\text{Int}(\log_{1+1/38784} x)}{n} \]
where $n = \ldots$

The reason we get this pattern is because of the following limit formula
\[ e = \lim_{n \to \infty} \ldots \]
which is usually taken to be the definition of the number $e$.

9. (1 pt) setIntegrationProjects/proj2/prob5.pg
Compute an approximation to
\[ \int_{1}^{2} \frac{1}{x} \, dx, \]
which gives the area under $y = \frac{1}{x}$ for $1 \leq x \leq 2$, using a modified Riemann sum with the (NOT equally spaced) partition
\[ 1, 1.001, 1.001^2, 1.001^3, 1.001^4, 1.001^5, 1.001^6, 1.001^7, 2 \]
and left hand endpoints EXCEPT neglecting the area of the last rectangle. This amounts to computing the sum of the areas of the rectangles as shown in the following figure:

As you can see in the figure, the area of the last rectangle is relatively small compared to the others, and the other rectangles already give an overestimate of the area.

Please note that the problem is NOT asking for the value of $\int_{1}^{2} \frac{1}{x} \, dx$. Rather it is asking for the EXACT values of the areas of the 7 approximating rectangles and for the EXACT value of the sum of the areas of the rectangles. Calculator approximations (no matter how accurate) will NOT be accepted. Do the calculations by hand using fractions (until you notice the pattern in the areas).

The area of the first rectangle =
The area of the second rectangle =
The area of the third rectangle =
The area of the fourth rectangle =
The area of the fifth rectangle =
The area of the sixth rectangle =
The area of the seventh rectangle =
The sum of the areas of the 7 rectangles =

10. (1 pt) setIntegrationProjects/proj2/prob6.pg
This problem is a reprise of problem 5 with 1.001 replaced by 1.0001

Compute an approximation to
\[ \int_{1}^{10} \frac{1}{x} \, dx, \]
which gives the area under $y = \frac{1}{x}$ for $1 \leq x \leq 10$, using a modified Riemann sum with the (NOT equally spaced) partition
\[ 1, 1.001, 1.001^2, 1.001^3, \ldots, 1.001^N, 10 \]
and left hand endpoints EXCEPT neglecting the area of the last rectangle. Here $N$ denotes the largest possible power which fits in the interval $1 \leq x \leq 10$.

Please note that the problem is NOT asking for the value of $\int_{1}^{10} \frac{1}{x} \, dx$. Rather it is asking for the EXACT values of the areas of approximating rectangles and for the EXACT value of the sum of the areas of the rectangles. Calculator approximations (no matter how accurate) will NOT be accepted. Do the calculations by hand using fractions (until you notice the pattern in the areas).

The number of approximating rectangles is:
\[ N = \ldots \]

The area of the first rectangle = \ldots
The area of the second rectangle = \ldots
The area of the third rectangle = \ldots
The sum of the areas of the \( N \) rectangles = \ldots

11. (1 pt) setIntegrationProjects/proj2/prob7.pg

This problem is a reprise of problem 6 with 1.001 replaced by 1.000001

Compute an approximation to

\[ \int_{1}^{5} \frac{1}{x} \, dx, \]

which gives the area under \( y = \frac{1}{x} \) for \( 1 \leq x \leq 5 \), using a modified Riemann sum with the (NOT equally spaced) partition

\[ 1, 1.0000001, 1.000001^2, 1.000001^3, \ldots, 1.000001^N, 5 \]

and left hand endpoints EXCEPT neglecting the area of the last rectangle. Here \( N \) denotes the largest possible power which fits in the interval \( 1 \leq x \leq 5 \).

Please note that the problem is NOT asking for the value of \( \int_{1}^{5} \frac{1}{x} \, dx \). Rather it is asking for the EXACT values of the areas of approximating rectangles and for the EXACT value of the sum of the areas of the rectangles. Calculator approximations (no matter how accurate) will NOT be accepted. Do the calculations by hand using fractions (until you notice the pattern in the areas).

The number of approximating rectangles is:
\[ N = \ldots \]
The area of the first rectangle = \ldots
The area of the second rectangle = \ldots
The area of the third rectangle = \ldots
The sum of the areas of the \( N \) rectangles = \ldots

The ingenious idea of using these unusual nonequally spaced partitions to compute the area under \( y = \frac{1}{x} \) thus relating it to Napier’s logarithms is due to a Belgian monk, Gregory St. Vincent around 1647.

12. (1 pt) setIntegrationProjects/proj3/prob1.pg

Suppose that a hawk, whose initial position is \((a, 0) = (8000, 0)\) on the \( x \)-axis, spots a pigeon at \((0, -5000)\) on the \( y \)-axis. Suppose that the pigeon flies at a constant speed of 60 ft/sec in the direction of the \( y \)-axis (oblivious to the hawk), while the hawk flies at a constant speed of 80 ft/sec, always in the direction of the pigeon.

The problem is to find an equation for the flight path of the hawk (the curve of pursuit) and to find the time and place where the hawk will catch the pigeon. Assume that in this problem all distances are measured in feet and all times measured in seconds. Leave out all dimensions from your answers.

Consider the diagram above (click on it for a better view) which represents the situation at an arbitrary time \( t \) during the pursuit. The points \( P \) and \( Q \) represent the positions of the hawk and pigeon respectively at that time instant \( t \), with \( y = f(x) \) representing the flight path of the hawk.

The pigeon’s position \( Q = (0, g(t)) \) is given by the following function of time

\[ g(t) = \ldots \]

The fact that the hawk is always headed in the direction of the pigeon means that the line \( PQ \) is tangent to the pursuit curve \( y = f(x) \). This tells us that \( \frac{dy}{dx} = h(x, y, t) \) where

\[ h(x, y, t) = \ldots \]

(Your answer must involve the three variables \( x \), \( y \), and \( t \)).

If we solve the equation

\[ p = h(x, y, t), \]

where \( p = \frac{dy}{dx} \), for time we obtain that \( t = k(x, y, p) \) where

\[ k(x, y, p) = \ldots \]

(Your answer must involve the three variables \( x \), \( y \), and \( p \), where \( p \) stands for \( \frac{dy}{dx} \)).
Again referring to the diagram above (click on it for a better view) we see that the distance that the hawk has flown in time $t$ is given by the integral $\int_c^d F\,dx$ where $c =$_________ $d =$_________

(Hints: Note that this integral computes the length of a curve. Also recall that the hawk’s initial position is at $(a,0) = (8000,0)$.)

and $F =$_________

(Use $p$ to denote $\frac{dy}{dx}$ in your last answer above.)

On the other hand the hawk is ying at a constant speed of 80 for time $t$. Hence the total distance it has flown is_________.

If we equate this to the distance we just computed and solve for $t$ we obtain

$$t = \int_c^d G\,dx$$

where $G =$_________

(Remember to use $p$ to represent $\frac{dy}{dx}$.)

Equating the two expressions for $t$ from Problems 1 and 2 we obtain the integral equation

$$k(x,y,p) = \int_c^d G\,dx$$

To get rid of the integral, we differentiate both sides of the equation with respect to $x$. On the left hand side of the resulting equation we obtain the following expression (which might involve $x$, $y$, $p = \frac{dy}{dx}$ and $q = \frac{dp}{dx} = \frac{d^2y}{dx^2}$)

(remember to separate different variables in a product with spaces or multiplication signs)

while on the right hand side (after applying the Fundamental Theorem of Calculus) we obtain

The resulting differential equation we obtained above is a separable differential equation in the variables $p$ and $x$. We can rewrite it in the form

$$K(p)\,dp = L(x)\,dx$$

(with all numerical factors moved to the right hand side of the equation so that $L(1)=60/80$.)

where $K(p) =$_________ $L(x) =$_________

Integrating the left hand side of the equation $\int K(p)\,dp$, using the methods of sections 7.3 and 7.3, we obtain

$\int K(p)\,dp =$_________

while on the right hand side we obtain $\int L(x)\,dx =$_________ + $C$

Plugging in the initial positions of the hawk and pigeon, and recalling that $p = \frac{dy}{dx}$ is the slope of the tangent line, we find that $C =$_________

Solving the equation we obtained in Problem 4 for $p$ in terms of $x$, we obtain

$p =$_________

(Hints for solving for $p$: Exponentiate to get rid of the logarithm. Then isolate the square root on one side of the equation and square both sides.)

Recalling that $p = \frac{dy}{dx}$ and integrating, we obtain that $y =$_________ + $C$

Plugging in the initial position of the hawk we obtain that the constant of integration is given by $C =$_________

Hence the hawk catches the pigeon at the point $(0,c)$ where $c =$_________

at time $t =$_________
1. (1 pt) setSequences1Definitions/ns8_1_5.pg
For each sequence, find a formula for the general term, \( a_n \). For example, answer \( n^2 \) if given the sequence:
\[ \{1, 4, 9, 16, 25, 36, \ldots \} \]

1. \[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \]

2. \[ \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots \]

2. (1 pt) setSequences1Definitions/ns8_1_5a.pg
For each sequence, find a formula for the general term, \( A_n \). For example, answer \( n^2 \) if given the sequence:
\[ \{1, 4, 9, 16, 25, 36, \ldots \} \]

1. \[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \]

2. \[ \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots \]
1. Find the limit of the sequence 
\[ a_n = \frac{5n - 4}{4n + 5} \]

2. Find the limit of the sequence:
\[ a_n = \frac{2n^2 + 5n + 6}{4n^2 + 11n + 1} \]

3. Find the limit of the sequence \( a_n = \frac{(\cos n)}{2^n} \).

4. Find the limit of the sequence whose terms are given by \( a_n = (n^2)(1 - \cos(\frac{\pi}{n})) \).

5. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} -16n + \frac{9}{7n} \]

6. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} \frac{6}{2^n} + 7 \arctan(n^3) \]

7. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} \frac{15(2^n) + 14}{6(5^n)} \]

8. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges without being infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".

9. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} \frac{6n^3 + \sin^2(3n)}{n^4 + 10} \]

10. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} \frac{16 + 30 \arctan(n!)}{5^n} \]

11. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} \frac{n^4}{e^{-5n}} \]

12. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[ \lim_{n \to \infty} \frac{-10(n!)}{(-6)^n} \]

13. Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as "INF" (without the quotation marks). If it diverges to negative infinity, state your answer as "MINF". If it diverges without being infinity or negative infinity, state your answer as "DIV".
\[
\lim_{n \to \infty} -\frac{4(n!)}{(n)^n}
\]

14. (1 pt) setSequencesLimits/ur_sq_2_22.pg
Find the limit of the sequence whose terms are given by
\[ a_n = \left( e^{2n} + 6n \right)^{1/n}. \]

15. (1 pt) setSequencesLimits/ur_sq_2_23.pg
Find the limit of the sequence whose terms are given by
\[ a_n = \left( \frac{1}{e^{2n} + n^2} \right)^{1/n}. \]

16. (1 pt) setSequencesLimits/ur_sq_2_18.pg
If a sequence \( c_1, c_2, c_3, \ldots \) has limit \( K \) then the sequence \( e^{c_1}, e^{c_2}, e^{c_3}, \ldots \) has limit \( e^K \). Use this fact together with l’Hôpital’s rule to compute the limit of the sequence given by \( b_n = \left( 1 + \frac{1/n}{n} \right)^n \).

17. (1 pt) setSequencesLimits/ur_sq_2_19.pg
If a sequence \( c_1, c_2, c_3, \ldots \) has limit \( K \) then the sequence \( e^{c_1}, e^{c_2}, e^{c_3}, \ldots \) has limit \( e^K \). Use this fact together with l’Hôpital’s rule to compute the limit of the sequence given by \( b_n = \left( n + \frac{1}{n} \right)^n \).

18. (1 pt) setSequencesLimits/ur_sq_2_21.pg
Match each sequence below to statement that BEST fits it.

STATEMENTS
Z. The sequence converges to zero;
I. The sequence diverges to infinity;
F. The sequence has a finite non-zero limit;
D. The sequence diverges.

SEQUENCES
___1. \( \sin(n) \)
___2. \( \frac{n^3 - 3n}{5n - n^2} \)
___3. \( \frac{(\ln(n))}{n^{100}} \)
___4. \( \ln(\ln(\ln(n))) \)
___5. \( n \sin(\frac{1}{n}) \)
___6. \( \frac{n!}{n^{100}} \)

19. (1 pt) setSequencesLimits/ur_sq_2_24.pg
Match each sequence below to statement that BEST fits it.

STATEMENTS
Z. The sequence converges to zero; I. The sequence diverges to positive infinity;
F. The sequence has a finite non-zero limit; D. The sequence diverges, but not to infinity.

SEQUENCES
___1. \( \frac{e^n}{n^2} \)
___2. \( \left( -\frac{1}{5} \right)^n \)
___3. \( \frac{100n^2 + 1}{3n^2} \)
___4. \( (-1)^n \frac{2n}{\ln(n)} \)
___5. \( \left( \frac{3}{10} \right)^n \)
___6. \( \cos^2(n) + \sin^2(n) \)
___7. \( (\sqrt{n})^n \)
___8. \( \sqrt{n^3 + 4n - \sqrt{n^2}} \)
1. \( a_n = \cos \frac{n\pi}{4} \)

2. \( a_n = \frac{n-4}{n+4} \)

3. \( a_n = \frac{1}{4n+6} \)

4. \( a_n = \frac{n+4}{6n+4} \)

---

2. \( f(x) = \frac{x}{x^2 + 9x + 41} \)

A. Find the smallest real number \( r \) such that \( f(x) \) is decreasing for all \( x \) greater than \( r \).

\( r = \) 

B. Find the smallest integer \( s \) such that \( f(n) \) is decreasing for all integers \( n \) greater than or equal to \( s \).

\( s = \)
1. (1 pt) setSequences4Arithmetic/ur_sq4_1.pg
Write down the first five terms of the sequence \( \left\{ \frac{7n}{n+13} \right\} \)

\( \_\_\_ \)

\( \_\_\_ \)

\( \_\_\_ \)

\( \_\_\_ \)

2. (1 pt) setSequences4Arithmetic/ur_sq4_2.pg
Find the 5th term of the arithmetic sequence 3, 2, 1, ...
Answer: _____

3. (1 pt) setSequences4Arithmetic/ur_sq4_3.pg
Find the sum 
\(-5 - 2 + 1 + ... + (-8 + 3n)\)
Answer: ____________

4. (1 pt) setSequences4Arithmetic/ur_sq4_4.pg
Find the sum 
\(-6 + 1 + 8 + ... + 15\)
Answer: ________

5. (1 pt) setSequences4Arithmetic/ur_sq4_5.pg
Find the common difference and write out the first four terms of the arithmetic sequence \( \left\{ \frac{1}{n} - \frac{4}{5} \right\} \)
Common difference is ______
\( a_1 = \_\_\_ \)
\( a_2 = \_\_\_ \)
\( a_3 = \_\_\_ \)
\( a_4 = \_\_\_ \)

6. (1 pt) setSequences4Arithmetic/ur_sq4_6.pg
Find the nth term of the arithmetic sequence whose initial term is 3 and common difference is 5.
(Your answer must be a function of \( n \).)

7. (1 pt) setSequences4Arithmetic/ur_sq4_7.pg
Find the first term and the common difference of the arithmetic sequence whose 4th term is \(-18\) and 18th term is \(-46\).
First term is _____
Common difference is _____

8. (1 pt) setSequences4Arithmetic/ur_sq4_8.pg
Find \( x \) such that 2\( x + 1 \), 3\( x - 1 \), and 8\( x - 75 \) are consecutive terms of an arithmetic sequence.
\( x = \_\_\_ \)

9. (1 pt) setSequences4Arithmetic/ur_sq4_9.pg
Write down the first five terms of the following recursively defined sequence.
\( a_1 = 4; \ a_{n+1} = -2a_n + 8 \)
\( \_\_\_ \)
\( \_\_\_ \)
\( \_\_\_ \)
\( \_\_\_ \)
1. (1 pt) setSequences5Geometric/ur_sq_5_1.pg
Find the common ratio and write out the first four terms of the
geometric sequence \( \left\{ \frac{6^n - 3}{8} \right\} \)
Common ratio is ___.
\( a_1 = \) ___.
\( a_2 = \) ___.
\( a_3 = \) ___.
\( a_4 = \) ___.

2. (1 pt) setSequences5Geometric/ur_sq_5_2.pg
Find the 7th term of the geometric sequence
0, 0, 0, ...
Answer: ____

3. (1 pt) setSequences5Geometric/ur_sq_5_3.pg
Find the nth term of the geometric sequence whose initial term
is 1 and common ration is 8.
__________ (Your answer must be a function of \( n \)).

4. (1 pt) setSequences5Geometric/ur_sq_5_4.pg
Daniel and Carrie want to purchase a house. Suppose they in-vest 500 dollars per month into a mutual fund. How much will
they have for a downpayment after 4 years if the per annum rate
of return of the mutual fund is assumed to be 9 percent com-pounded monthly?
1. (1 pt) 
\[ s_k = \sum_{n=1}^{k} n(1)^n \]
Find \( s_4 \).
\[ s_4 = \]

2. (1 pt) 
Let \( a_n \) be the \( n \)th digit after the decimal point in \( 6\pi + 8e \). Evaluate 
\[ \sum_{n=1}^{\infty} a_n (1)^n. \]
Find \( s_4 \) and \( s_8 \).
\[ s_4 = \]
\[ s_8 = \]

3. (1 pt) 
Let \( r = \frac{45}{25} \).
For both of the following answer blanks, decide whether the given sequence or series is convergent or divergent. If convergent, enter the limit (for a sequence) or the sum (for a series). If divergent, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, or DIV otherwise.
A. Consider the sequence \( \{nr^n\} \).
\[ \lim_{n \to \infty} nr^n = \]
B. Take my word for it that it can be shown that 
\[ \sum_{i=1}^{n} ir^i = \frac{n r^{n+2} - (n+1) r^{n+1} + r}{(1-r)^2}. \]
Now consider the series \( \sum_{n=1}^{\infty} nr^n. \)

4. (1 pt) 
Consider the series \( \sum_{n=1}^{\infty} \frac{9}{n+8} \). Let \( s_n \) be the \( n \)-th partial sum; that is, 
\[ s_n = \sum_{i=1}^{n} \frac{9}{i+8}. \]
Find \( s_4 \) and \( s_8 \).
\[ s_4 = \]
\[ s_8 = \]

5. (1 pt) 
Two boys on bicycles, 69 miles apart, began racing directly toward each other. The instant they started, a fly on the handle bar of one bicycle started flying straight toward the other cyclist. As soon as it reached the other handle bar it turned and started back. The fly flew back and forth in this way, from handle bar to handle bar, until the two bicycles met. If each bicycle had a constant speed of 12 miles an hour, and the fly flew at a constant speed of 18 miles an hour, how far did the fly fly?

6. (1 pt) 
Evaluate the sum:
\[ \sum_{k=0}^{4} (-1)^k (k+3)^2 + 4 \]
1. (1 pt) setSeries2Telescope/ns8_2_21.pg
If the following series converges, compute its sum. Otherwise, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, and DIV otherwise.

\[ \sum_{n=1}^{\infty} \frac{2}{n(n+2)} \]

(Hint: try breaking the summands up partial fractions-style.)

2. (1 pt) setSeries2Telescope/ns8_2_24.pg
For the following series, if it converges, enter the limit of convergence. If not, enter "DIV" (unquoted).

\[ \sum_{n=1}^{\infty} \ln(2(n+1)) - \ln(2n) \]

3. (1 pt) setSeries2Telescope/ns8_2_25.pg
Determine the sum of the following series.

\[ \sum_{n=1}^{\infty} \left( \sin \frac{9}{n} - \sin \frac{9}{n+1} \right) \]

4. (1 pt) setSeries2Telescope/ns8_2_25a.pg
Decide whether each of the following series converges. If a given series converges, compute its sum. Otherwise, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, and DIV otherwise.

1. \[ \sum_{n=1}^{\infty} \left( \sin(-2n) - \sin(-2(n+1)) \right) \]

2. \[ \sum_{n=1}^{\infty} \left( \sin \left( -\frac{2}{n} \right) - \sin \left( -\frac{2}{n+1} \right) \right) \]

3. \[ \sum_{n=1}^{\infty} \left( e^{9n} - e^{9(n+1)} \right) \]

5. (1 pt) setSeries2Telescope/ns8_2_26.pg
If the following series converges, compute its sum. Otherwise, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, and DIV otherwise.

\[ \sum_{n=1}^{\infty} \left( e^{-7n} - e^{-7(n+1)} \right) \]
1. (1 pt) setSeries3Convergent/ns8_2_9.pg
Given: \[ A_n = \frac{3n}{4n+5} \]
For both of the following answer blanks, decide whether the given sequence or series is convergent or divergent. If convergent, enter the limit (for a sequence) or the sum (for a series). If divergent, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, or DIV otherwise.
(a) The series \( \sum_{n=1}^{\infty} (A_n) \). __________
(b) The sequence \( \{A_n\} \). __________

2. (1 pt) setSeries3Convergent/ns8_2_9b.pg
Given: \[ A_n = \frac{256^\frac{n}{2}}{4^n} \]
Determine:
(a) whether \( \sum_{n=1}^{\infty} (A_n) \) is convergent. _____
(b) whether \( \{A_n\} \) is convergent. _____
If convergent, enter the limit of convergence. If not, enter "DIV" (unquoted).

3. (1 pt) setSeries3Convergent/ns8_2_9c.pg
Given: \[ A_n = \frac{70}{7^n} \]
Determine:
(a) whether \( \sum_{n=1}^{\infty} (A_n) \) is convergent. __________
(b) whether \( \{A_n\} \) is convergent. __________
If convergent, enter the limit of convergence. If not, enter "DIV" (unquoted).

4. (1 pt) setSeries3Convergent/ns8_2_9d.pg
Given: \[ A_n = \frac{8^n}{80} \]
Determine:
(a) whether \( \sum_{n=1}^{\infty} (A_n) \) is convergent. __________
(b) whether \( \{A_n\} \) is convergent. __________
If convergent, enter the limit of convergence. If not, enter 'DIV' (unquoted).

5. (1 pt) setSeries3Convergent/ns8_2_17.pg
Match each of the following with the correct statement. C stands for Convergent, D stands for Divergent.

\[
\begin{align*}
\text{1. } & \sum_{n=1}^{\infty} \frac{7}{n^{10} - 81} \\
\text{2. } & \sum_{n=1}^{\infty} \frac{1}{3 + \sqrt[3]{n^6}} \\
\text{3. } & \sum_{n=1}^{\infty} \frac{8+4^n}{10+4^n} \\
\text{4. } & \sum_{n=1}^{\infty} \frac{n(n+7)}{n} \\
\text{5. } & \sum_{n=1}^{\infty} \frac{\ln(n)}{10n}
\end{align*}
\]

6. (1 pt) setSeries3Convergent/ns8_3_17BB.pg
Match each of the following with the correct statement. C stands for Convergent, D stands for Divergent.

\[
\begin{align*}
\text{1. } & \sum_{n=1}^{\infty} \frac{\ln(n)}{4n} \\
\text{2. } & \sum_{n=1}^{\infty} \frac{4+8^n}{7+5^n} \\
\text{3. } & \sum_{n=1}^{\infty} \frac{1}{3 + \sqrt[3]{n^6}} \\
\text{4. } & \sum_{n=1}^{\infty} \frac{3}{n^4 - 81} \\
\text{5. } & \sum_{n=1}^{\infty} \frac{3}{n(n+6)}
\end{align*}
\]

7. (1 pt) setSeries3Convergent/ns8_3_6.pg
Determine whether the series is convergent or divergent.
\[
\sum_{n=1}^{\infty} \left( \frac{6}{\sqrt{n^1}} + \frac{-3}{n^6} \right)
\]
If convergent, enter the 4th partial sum to estimate the sum of the series; otherwise, enter DIV. (Note: if you have trouble reading this problem, try selecting typeset mode below and then hitting the submit answer button.)

8. (1 pt) setSeries3Convergent/ns8_3_7.pg
Match each of the following with the correct statement. C stands for Convergent, D stands for Divergent.

\[
\begin{align*}
\text{1. } & \sum_{n=1}^{\infty} \frac{6}{n^3 + n^2} \\
\text{2. } & \sum_{n=2}^{\infty} \frac{10}{2n \ln(n)} \\
\text{3. } & \sum_{n=1}^{\infty} \frac{n^2}{n^3 + 4} \\
\text{4. } & \sum_{n=1}^{\infty} ne^{-n^2} \\
\text{5. } & \sum_{n=1}^{\infty} \frac{9 + 4^n}{7^n}
\end{align*}
\]

9. (1 pt) setSeries3Convergent/ns8_3_7BB.pg
Match each of the following with the correct statement. C stands for Convergent, D stands for Divergent.
1. \( \sum_{n=1}^{\infty} \frac{3}{6n \ln(n)} \)
2. \( \sum_{n=1}^{\infty} \frac{4 + 8n}{3^n} \)
3. \( \sum_{n=1}^{\infty} ne^{-n^2} \)
4. \( \sum_{n=1}^{\infty} \frac{8}{n^2 + n^6} \)
5. \( \sum_{n=1}^{\infty} \frac{n^4}{n^2 + 4} \)

10. (1 pt) setSeries3Convergent/ur_srr_3_1.pg
For each of the following series, tell whether or not you can apply the 3-condition test (i.e. the alternating series test). If you can apply this test, enter D if the series diverges, or C if the series converges. If you can’t apply this test (even if you know how the series behaves by some other test), enter N.

1. \( \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!} \)
2. \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \)
3. \( \sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n!} \)
4. \( \sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^5} \)
5. \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 5} \)
6. \( \sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n} \)

11. (1 pt) setSeries3Convergent/ur_srr_3_10.pg
For the following alternating series,
\[ \sum_{n=1}^{\infty} a_n = 1 - \frac{1}{10} + \frac{1}{100} - \frac{1}{1000} + \ldots \]
how many terms do you have to go for your approximation (your partial sum) to be within 1e-08 from the convergent value of that series?

12. (1 pt) setSeries3Convergent/ur_srr_3_2.pg
For each of the following series, tell whether or not you can apply the 3-condition test (i.e. the alternating series test). If you can apply this test, enter D if the series diverges, or C if the series converges. If you can’t apply this test (even if you know how the series behaves by some other test), enter N.

1. \( \sum_{n=1}^{\infty} \frac{3}{6n \ln(n)} \)
2. \( \sum_{n=1}^{\infty} \frac{4 + 8n}{3^n} \)
3. \( \sum_{n=1}^{\infty} ne^{-n^2} \)
4. \( \sum_{n=1}^{\infty} \frac{8}{n^2 + n^6} \)
5. \( \sum_{n=1}^{\infty} \frac{n^4}{n^2 + 4} \)

13. (1 pt) setSeries3Convergent/ur_srr_3_7.pg
If the following series converges, compute its sum. Otherwise, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, and DIV otherwise.
\[ \sum_{n=1}^{\infty} \frac{3 + 10^n}{10^n} \]

14. (1 pt) setSeries3Convergent/ur_srr_3_8.pg
For the following alternating series,
\[ \sum_{n=1}^{\infty} a_n = 0.6 - \frac{(0.6)^3}{3!} + \frac{(0.6)^5}{5!} - \frac{(0.6)^7}{7!} + \ldots \]
how many terms do you have to go for your approximation (your partial sum) to be within 0.0000001 from the convergent value of that series?

15. (1 pt) setSeries3Convergent/ur_srr_3_9.pg
For the following alternating series,
\[ \sum_{n=1}^{\infty} a_n = 1 - \frac{(0.4)^2}{2!} + \frac{(0.4)^4}{4!} - \frac{(0.4)^6}{6!} + \frac{(0.4)^8}{8!} - \ldots \]
how many terms do you have to go for your approximation (your partial sum) to be within 0.0000001 from the convergent value of that series?

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Let \( m \) be the six digit integer formed by the first six digits of the new list. Let \( r \) be the number given by the decimal obtained by putting a decimal point at the start of the new infinite list. The number \( r \) is rational and can be written as a fraction \( p/q \), where \( p \) and \( q \) are positive integers and have no common factor greater than one. Find \( p \) and \( q \).

A. Find the smallest number \( M \) such that \( s_k \leq M \) for every positive integer \( k \).

\( M = \)__________

B. Find \( s_3 \).

\( s_3 = \)__________

C. Note that \( 1 - r = \frac{20}{29} \). Then \( -\ln(1 - r) = \ln\left(\frac{20}{29}\right) \).

Suppose \( s_3 \) is used to approximate \( \ln\left(\frac{20}{29}\right) \).

The error is \( \sum_{n=4}^{\infty} \left(\frac{1}{n}\right) r^n \), which is less than \( \frac{1}{4} \sum_{n=4}^{\infty} r^n \).

Use the formula for the sum of a geometric series to calculate this last sum and thereby to estimate the error in the approximation.

ERROR \( \leq \)__________

Your answer to C. should be more than the actual error which is 0.0264805138932787.

11.(1 pt) setSeries4Geometric/ur_sr_4_14.pg

The geometric series can be used to approximate the reciprocal of a number by using a nearby number whose reciprocal is known. For example, \( \frac{1}{24} = \frac{1}{25} (1 + \frac{1}{25}) \).

\[ \frac{1}{25} \sum_{n=0}^{\infty} \left(\frac{1}{25}\right)^n \]

leads to the approximation \( \frac{1}{25} \left(1 + \frac{1}{25}\right) \) of \( \frac{1}{24} \) by truncating the series.

This approximation to \( \frac{1}{24} \) is easily expressed as a decimal: \( .04(1 + .04) = .0416 \).

Use the fact that 96 is near 100 to get a similar four place decimal approximation of \( \frac{1}{96} \).
The error in approximating a number A by a number a is $e = a - A$. The relative error is $\frac{e}{A}$. The relative percent error is $100\frac{e}{A}$.

Find the relative percent error in the approximation of $\frac{1}{5}$ described above.

12. (1 pt) setSeries4Geometric/ur_sr_d_15.pg

A ball drops from a height of 22 feet. Each time it hits the ground, it bounces up 25 percents of the height it fell. Assume it goes on forever, find the total distance it travels.


Find the sum

$$8 + \frac{8}{5} + \frac{8}{25} + \ldots + \frac{8}{5^n}$$

Answer: __________

14. (1 pt) setSeries4Geometric/ur_sr_d_1.pg

The following series are geometric series.

Determine whether each series converges or not. For the series which converge, enter the sum of the series. For the series which diverges enter “DIV” (without quotes).

(a) $\sum_{n=1}^{\infty} \frac{11^n}{10^n} = \underline{_________}$

(b) $\sum_{n=2}^{\infty} \frac{1}{3^n} = \underline{_________}$

(c) $\sum_{n=0}^{\infty} \frac{3^n}{6^{n+1}} = \underline{_________}$

(d) $\sum_{n=3}^{\infty} \frac{10^n}{11^n} = \underline{_________}$

(e) $\sum_{n=1}^{\infty} \frac{4^n}{4^{n+4}} = \underline{_________}$

(f) $\sum_{n=1}^{\infty} \frac{10^n + 3^n}{11^n} = \underline{_________}$

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1. (1 pt) setSeries5IntegralTest/eva8_3_1.pg

Compute the value of the following improper integral if it converges. If it diverges, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, or DIV otherwise (hint: integrate by parts).

\[ \int_1^\infty \frac{7 \ln(x)}{x^3} \, dx \]

Determine whether \( \sum_{n=1}^{\infty} \frac{7 \ln(n)}{n^3} \) is a convergent series. Enter C if the series is convergent, or D if it is divergent.

2. (1 pt) setSeries5IntegralTest/eva8_3_2.pg

Find the value of

\[ \int_2^\infty \frac{dx}{3x(\ln(9x))^2} \]

Determine whether \( \sum_{n=2}^{\infty} \frac{1}{3n(\ln(9n))^2} \) is convergent. Enter A if series is convergent, B if series is divergent.

3. (1 pt) setSeries5IntegralTest/eva8_3_3.pg

Find the value of

\[ \int_2^\infty \frac{dx}{(4x - 2)^4} \]

Determine whether \( \sum_{n=2}^{\infty} \frac{1}{(4n - 2)^4} \) is convergent. Enter C if series is convergent, D if series is divergent.

4. (1 pt) setSeries5IntegralTest/ns8_2_24eva.pg

(a) Compute \( s_3 \) (the 3rd partial sum) of \( s = \sum_{n=1}^{\infty} \frac{3}{4n^8} \)

(b) Estimate the error in using \( s_3 \) as an approximation of the sum of the series. (i.e. use \( \int_3^\infty f(x) \, dx \geq R_3 \))

5. (1 pt) setSeries5IntegralTest/ur_sr_5_11.pg

Test each of the following series for convergence by the Integral Test. If the Integral Test can be applied to the series, enter CONV if it converges or DIV if it diverges. If the integral test cannot be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the integral Test cannot be applied to it, then you must enter NA rather than CONV.)

1. \( \sum_{n=1}^{\infty} \frac{5}{n(\ln(5n))^3} \)

2. \( \sum_{n=1}^{\infty} \frac{\ln(6n)}{n} \)

3. \( \sum_{n=1}^{\infty} \frac{n^2}{n+7} \)

4. \( \sum_{n=1}^{\infty} \frac{5}{n \ln(5n)} \)

6. (1 pt) setSeries5IntegralTest/ur_sr_5_12.pg

Find the value of \( \int_1^\infty \frac{9 \, dx}{x^2 + 1} \)

Determine whether \( \sum_{n=1}^{\infty} \frac{9}{n^2 + 1} \) is convergent. Enter A if series is convergent, B if series is divergent.

7. (1 pt) setSeries5IntegralTest/ur_sr_5_13.pg

Find the value of \( \int_1^\infty 7x^2 e^{-x^3} \, dx \)

Determine whether \( \sum_{n=1}^{\infty} \frac{(7n^2 e^{-n^3})}{n} \) is convergent. Enter C if series is convergent, D if series is divergent.
Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If either test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

1. \( \sum_{n=1}^{\infty} \frac{\cos^2(n) \sqrt{n}}{n^2} \)
2. \( \sum_{n=1}^{\infty} \frac{(-1)^n}{7n} \)
3. \( \sum_{n=1}^{\infty} \frac{\cos(n) \sqrt{n}}{4n + 7} \)
4. \( \sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n + 6} \)
5. \( \sum_{n=1}^{\infty} \frac{4n^2}{n^3 + 7} \)

Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If either test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

1. \( \sum_{n=1}^{\infty} \frac{(\ln(n))^3}{n + 3} \)
2. \( \sum_{n=1}^{\infty} \frac{4n^6 - n^3 + 4 \sqrt{n}}{5n^8 - n^2 + 3} \)
3. \( \sum_{n=1}^{\infty} \frac{(-1)^n}{5n} \)
4. \( \sum_{n=1}^{\infty} \frac{4n^2}{n^6 + 7} \)
5. \( \sum_{n=1}^{\infty} \frac{4n^2}{n^6 + 7} \)

Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If at least one test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

1. \( \sum_{n=1}^{\infty} \frac{3n^2}{n^4 + 6} \)
2. \( \sum_{n=1}^{\infty} \frac{6n^4 - n^3 + 3 \sqrt{n}}{2n^4 + 1} \)
3. \( \sum_{n=1}^{\infty} \frac{2n^6 - n^2 + 2}{3n + 1} \)
4. \( \sum_{n=1}^{\infty} \frac{\cos(n) \sqrt{n}}{n^2} \)
5. \( \sum_{n=1}^{\infty} \frac{\cos^2(n) \sqrt{n}}{n^2} \)

Each of the following statements is an attempt to show that a given series is convergent or divergent not using the Comparison Test (NOT the Limit Comparison Test.) For each statement, enter C (for “correct”) if the argument is valid, or enter I (for “incorrect”) if any part of the argument is flawed. (Note: if the conclusion is true but the argument that led to it was wrong, you must enter I.)

1. For all \( n > 2 \), \( \frac{\pi}{n^2 - 7} < \frac{2}{n^2} \), and the series \( 2 \sum \frac{1}{n^2 - 7} \) converges, so by the Comparison Test, the series \( \sum \frac{1}{n^2 - 7} \) converges.
2. For all \( n > 1 \), \( \frac{\arctan(n)}{n^3} < \frac{\pi}{2n^3} \), and the series \( \frac{1}{2} \sum \frac{\arctan(n)}{n^3} \) converges, so by the Comparison Test, the series \( \sum \frac{\arctan(n)}{n^3} \) converges.
3. For all \( n > 2 \), \( \frac{1}{n^2 - 6} < \frac{1}{n^2} \), and the series \( \sum \frac{1}{n^2} \) converges, so by the Comparison Test, the series \( \sum \frac{1}{n^2 - 6} \) converges.
4. For all \( n > 2 \), \( \frac{\ln(n)}{n^7} > \frac{1}{n^7} \), and the series \( \sum \frac{1}{n^7} \) converges, so by the Comparison Test, the series \( \sum \frac{\ln(n)}{n^7} \) converges.
The three series

<table>
<thead>
<tr>
<th>Statement</th>
<th>Series</th>
<th>Comparison</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. For all ( n &gt; 1, \frac{n}{n^2 - 3} &lt; \frac{1}{n^2} ), and the series ( \sum_{n=1}^{\infty} \frac{1}{n^2 - 3} ) converges, so by the Comparison Test, the series ( \sum_{n=1}^{\infty} \frac{1}{n^2} ) diverges.</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>7. For all ( n &gt; 1, \frac{n}{n^2 - 3} &lt; \frac{1}{n^2} ), and the series ( \sum_{n=1}^{\infty} \frac{1}{n^2 - 3} ) converges, so by the Comparison Test, the series ( \sum_{n=1}^{\infty} \frac{1}{n^2} ) diverges.</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

The three series \( \sum A_n, \sum B_n, \) and \( \sum C_n \) have terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{n+1}{n} )</td>
<td>( A_n = \frac{1}{n^2} )</td>
</tr>
<tr>
<td>( \frac{n}{n^2} )</td>
<td>( B_n = \frac{1}{n^2} )</td>
</tr>
<tr>
<td>( \frac{1}{n \ln(n)} )</td>
<td>( C_n = \frac{1}{n} )</td>
</tr>
</tbody>
</table>

Use the Limit Comparison Test to compare the following series to any of the above series. For each of the series below, you must enter two letters. The first is the letter (A, B, or C) of the series above that it can be legally compared to with the Limit Comparison Test. The second is C if the given series converges, or D if it diverges. So for instance, if you believe the series converges and can be compared with series C above, you would enter CC; or if you believe it diverges and can be compared with series A, you would enter AD.

<table>
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<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \sum_{n=1}^{\infty} \frac{3n^2 + 2n^5}{n^2} )</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>2. ( \sum_{n=1}^{\infty} \frac{2n^6 + n^5}{n^2} )</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>3. ( \sum_{n=1}^{\infty} \frac{2n^5 + 2n^3}{n^3} )</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

The three series \( \sum A_n, \sum B_n, \) and \( \sum C_n \) have terms

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</tr>
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<tbody>
<tr>
<td>( \sqrt{n} )</td>
<td>( A_n = \frac{1}{n^2} )</td>
</tr>
<tr>
<td>( \ln(n) )</td>
<td>( B_n = \frac{1}{n^2} )</td>
</tr>
<tr>
<td>( \frac{1}{n} )</td>
<td>( C_n = \frac{1}{n} )</td>
</tr>
</tbody>
</table>

Use the Limit Comparison Test to compare the following series to any of the above series. For each of the series below, you must enter two letters. The first is the letter (A, B, or C) of the series above that it can be legally compared to with the Limit Comparison Test. The second is C if the given series converges, or D if it diverges. So for instance, if you believe the series converges and can be compared with series C above, you would enter CC; or if you believe it diverges and can be compared with series A, you would enter AD.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Series</th>
<th>Comparison</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+9} )</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>2. ( \sum_{n=1}^{\infty} \frac{(-1)^n \ln(e^n)}{n^3 \cos(n \pi)} )</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>3. ( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4 - 9} )</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

Select the FIRST correct reason why the given series converges.

A. Convergent geometric series
B. Convergent p series
C. Comparison (or Limit Comparison) with a geometric or p series
D. Converges by alternating series test

<table>
<thead>
<tr>
<th>Statement</th>
<th>Series</th>
<th>Comparison</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \sum_{n=1}^{\infty} \frac{7(6)^n}{112^n} )</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>2. ( \sum_{n=1}^{\infty} \frac{1}{n(2n+2)} )</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>3. ( \sum_{n=1}^{\infty} \frac{(n+1)(8)^n}{3^n} )</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>
Select the FIRST correct reason why the given series converges.

1. \[ \sum_{n=1}^{\infty} \frac{7(7)^n}{11^{2n}} \]
2. \[ \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n + 8} \]
3. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{7n + 6} \]
4. \[ \sum_{n=1}^{\infty} \frac{3^n}{2^{2n}} \]
5. \[ \sum_{n=1}^{\infty} \frac{(-1)^n e^n}{3^n} \]
6. \[ \sum_{n=1}^{\infty} \frac{n^2 + \sqrt{n}}{n^4 - 8} \]

11. (1 pt) setSeries6CompTests/ur_sr_6.10.pg
Select the FIRST correct reason why the given series converges.
A. Convergent geometric series
B. Convergent p series
C. Comparison (or Limit Comparison) with a geometric or p series
D. Cannot apply any test done so far in class

1. \[ \sum_{n=1}^{\infty} \frac{\sin^2(3n)}{n^2} \]
2. \[ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(7n)} \]
3. \[ \sum_{n=1}^{\infty} \frac{(-1)^n \ln(e^n)}{n^3 \cos(n\pi)} \]
4. \[ \sum_{n=1}^{\infty} \frac{(n + 1)(3)^n}{2^{2n}} \]
5. \[ \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n + 1} \]
6. \[ \sum_{n=1}^{\infty} \frac{5(7)^n}{T^2 2^{2n}} \]

12. (1 pt) setSeries6CompTests/ur_sr_6.12a.pg
Select the FIRST correct reason why the given series converges.
A. Convergent geometric series
B. Convergent p series
C. Comparison (or Limit Comparison) with a geometric or p series
D. Alternating Series Test
E. Cannot apply any test done so far in class

1. \[ \sum_{n=1}^{\infty} \frac{1}{n \ln(n)} \]
2. \[ \sum_{n=1}^{\infty} \frac{4n + 2}{(-1)^n} \]
3. \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]
4. \[ \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{(n!)^2} \]
5. \[ \sum_{n=1}^{\infty} \frac{(n + 1)(4^2 + 1)^n}{4^{2n}} \]
6. \[ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(5)} \]
C. \( M = \frac{1}{e} \) and it is decreasing for all \( x \). Its values may be found in tables. Make the change of variables \( y = x \ln(2) \) to express \( I = \int_{0}^{\infty} x^4 e^{-x} \, dx \) as a constant \( C \times h(4) \). Find \( C \).

C. Let \( g(x) = x^4 e^{-x} \). Find the smallest number \( M \) such that the function \( g \) is decreasing for all \( x > M \).

M =

D. Does \( \sum_{n=1}^{\infty} n^4 e^{-n} \) converge or diverge?

Answer with one letter, C or D.

16. (1 pt) setSeries6CompTests/ur_fs_14.png

For each sequence \( a_n \), find a number \( k \) such that \( n^k a_n \) has a finite non-zero limit. (This is of use, because by the limit comparison test the series \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} n^{-k} \) both converge or both diverge.)

A. \( a_n = (5 + 2n)^{-5} \)

\( k = \)

B. \( a_n = \frac{5}{n^{1/n}} \)

\( k = \)

C. \( a_n = \frac{5n^2 + 5n + 5}{4n^2 + 3n + 2} \)

\( k = \)

D. \( a_n = \left( \frac{5n^2 + 5n + 5}{4n^2 + 3n + 2} \right)^4 \)

\( k = \)

17. (1 pt) setSeries6CompTests/ur_fs_15.png

For each sequence \( a_n \), find a number \( r \) such that \( n^r a_n \) has a finite non-zero limit. (This is of use, because by the limit comparison test the series \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} n^{1/r} \) both converge or both diverge.)

A. \( a_n = (7 + 5n)^{-6} \)

\( r = \)

B. \( a_n = \frac{5^n}{2n^n} \)

\( r = \)

C. \( a_n = \frac{7^n + 4}{15n^2 + 2n + 5} \)

\( r = \)

18. (1 pt) setSeries6CompTests/ur_fs_16.png

The series \( \sum_{n=1}^{\infty} n^k r^n \) converges when \( 0 < r < 1 \) and diverges when \( r > 1 \). This is true regardless of the value of the constant \( k \). When \( r = 1 \) the series is a \( p \)-series. It converges if \( k < -1 \) and diverges otherwise. Each of the series below can be compared to a series of the form \( \sum_{n=1}^{\infty} n^k r^n \). For each series determine the best value of \( r \) and decide whether the series converges.

A. \( \sum_{n=1}^{\infty} (4 + n)(3^n)^{-1} \)

\( r = \)

B. \( \sum_{n=1}^{\infty} \frac{n^2}{3^n + n^3} \)

\( r = \)

C. \( \sum_{n=1}^{\infty} n^5 + 5 \)

\( r = \)

D. \( \sum_{n=1}^{\infty} \left( \frac{5n^2 + 4n + 4}{5n^2 + 3n + 3} \right)^8 \)

\( r = \)

19. (1 pt) setSeries6CompTests/ur_fs_17.png

For each of the series below select the letter from a to c that best applies and the letter from d to k that best applies. A possible answer is af, for example.

A. The series is absolutely convergent.

B. The series diverges, but not absolutely.

C. The series diverges.

D. The alternating series test shows the series converges.

E. The series is a \( p \)-series.

F. The series is a geometric series.

G. We can decide whether this series converges by comparison with a \( p \) series.

H. We can decide whether this series converges by comparison with a geometric series.

I. Partial sums of the series telescope.

J. The terms of the series do not have limit zero.

K. None of the above reasons applies to the convergence or divergence of the series.

\( r = \)

\( r = \)

\( r = \)

\( r = \)

\( r = \)
20. Select the FIRST correct reason why the given series diverges.

For each of the series below select the letter from a to c that best applies and the letter from d to k that best applies. A possible correct answer is af, for example.

A. The series is absolutely convergent.
B. The series converges, but not absolutely.
C. The series diverges.
D. The alternating series test shows the series converges.
E. The series is a p-series.
F. The series is a geometric series.
G. We can decide whether this series converges by comparison with a geometric series.
H. We can decide whether this series converges by comparison with a p-series.
I. Partial sums of the series telescope.
J. The terms of the series do not have limit zero.

1. \[ \sum_{n=1}^{\infty} \left( 1 + \frac{3}{n} \right)^n \]
2. \[ \sum_{n=1}^{\infty} (-1)^n \int_n^{n+1} 3^{-x} \, dx \]
3. \[ \sum_{n=5}^{\infty} \frac{(3-1) \cdot (2) \cdot 3 - 1 \cdot (n-1) \cdot 3 - 1}{3^n (n!) \sqrt{n}} \]
4. \[ \sum_{n=1}^{\infty} \frac{n^6}{6^n} \]
5. \[ \sum_{n=1}^{\infty} \frac{1}{n + 3^n} \]
6. \[ \sum_{n=1}^{\infty} (\log(n + 1) - \log(n)) \]

21. Select the FIRST correct reason why the given series diverges.

A. Diverges because the terms don’t have limit zero
B. Divergent geometric series
C. Divergent p series
D. Integral test
E. Comparison with a divergent p series
F. Diverges by limit comparison test
G. Cannot apply any test done so far in class

1. \[ \sum_{n=1}^{\infty} \frac{\ln(n)}{n} \]
2. \[ \sum_{n=1}^{\infty} (n)^{-\frac{1}{2}} \]
3. \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]
4. \[ \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{(n!)^2} \]

5. \[ \sum_{n=1}^{\infty} \frac{(n+1)(5^2 + 1)^n}{5^{2n}} \]
6. \[ \sum_{n=1}^{\infty} \frac{1}{n \ln(n)} \]

Here is a short review of numerical series which you may find helpful.

**REVIEW OF NUMERICAL SERIES**

**SEQUENCES**

A sequence is a list of real numbers. It is called convergent if it has a limit. An increasing sequence has a limit when it has an upper bound.

**SERIES**

(Geometric series, rational numbers as decimals, harmonic series, divergence test)

Given numbers forming a sequence \(a_1, a_2, \ldots\), let us define the \(n\)th partial sum as sum of the first \(n\) of them \(s_n = a_1 + \ldots + a_n\). The SERIES is convergent if the SEQUENCE \(s_1, s_2, s_3, \ldots\) is. In other words it converges if the partial sums of the series approach a limit.

A necessary condition for the convergence of this SERIES is that \(a\)’s have limit 0. If this fails, the series diverges.

The harmonic series \(1 + (1/2) + (1/3) + \ldots\) diverges.

This illustrates that the terms \(a_n\) having limit zero does not guarantee the convergence of a series.

A series with positive terms, i.e. \(a_n > 0\) for all \(n\), converges exactly when its partial sums have an upper bound.

The geometric series \(\sum_{n=1}^{\infty} r^n\) converges exactly when \(-1 < r < 1\).

**INTEGRAL AND COMPARISON TESTS**

(Integral test, p-series, comparison tests for convergence and divergence, limit comparison test)

Integral test: Suppose \(f(x)\) is positive and DECREASING for all large enough \(x\). Then the following are equivalent:

1. \(\int_1^{\infty} f(x) \, dx\) is finite, i.e. converges.
2. \(\sum_{n=1}^{\infty} f(n)\) is finite, i.e. converges.

This gives the p-test: \(\sum_{n=1}^{\infty} \frac{1}{n^p}\) converges exactly when \(p > 1\).

Comparison test: Suppose there is a fixed number \(K\) such that for all sufficiently large \(n\): \(0 < a_n < K b_n\).

Convergence. If \(\sum_{n=1}^{\infty} b_n\) converges then so does \(\sum_{n=1}^{\infty} a_n\).

Divergence. If \(\sum_{n=1}^{\infty} a_n\) diverges then so does \(\sum_{n=1}^{\infty} b_n\).

(Positive series having smaller terms are more likely to converge.)

Limit comparison test: SUPPOSE: \(a_n > 0\), \(b_n > 0\) and \(\lim_{n \to \infty} \frac{a_n}{b_n} = R\) exists. Moreover, \(R\) is not zero.

THEN \(\sum_{n=1}^{\infty} a_n\) and \(\sum_{n=1}^{\infty} b_n\) both converge or both diverge.
OTHER CONVERGENCE TESTS FOR SERIES
(Alternating series test, absolute convergence, RATIO TEST)

Alternating series test: Suppose the sequence $a_1, a_2, a_3, \ldots$ is decreasing and has limit zero. Then \( \sum_{n=1}^{\infty} (-1)^n a_n \) converges.

This applies to $(1)-(1/2)+(1/3)-(1/4)+\ldots$

Absolute Convergence Test: IF \( \sum_{n=1}^{\infty} |a_n| \) converges,

THEN \( \sum_{n=1}^{\infty} a_n \) converges.

Ratio test:
SUPPOSE \( \left| \frac{a_{n+1}}{a_n} \right| \) has limit equal to $r$.

IF $r < 1$ then \( \sum_{n=1}^{\infty} a_n \) CONVERGES.

IF $r > 1$ the \( \sum_{n=1}^{\infty} a_n \) DIVERGES.
1. (1 pt) setSeries7AbsolutelyConvergent/eva8_3_2.pg
Match each of the following with the correct statement.
A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.
D. The series diverges.

1. \[ \sum_{n=1}^{\infty} \frac{(n+1)(6^2 - 1)^n}{6^{2n}} \]
2. \[ \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+4} \]
3. \[ \sum_{n=1}^{\infty} \frac{(-3)^n}{n^4} \]
4. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{7n+7} \]
5. \[ \sum_{n=1}^{\infty} \frac{(n+1)(5^2 - 1)^n}{5^{2n}} \]

2. (1 pt) setSeries7AbsolutelyConvergent/eva8_3_2BB.pg
Match each of the following with the correct statement.
A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.
D. The series diverges.

1. \[ \sum_{n=1}^{\infty} \frac{\sin(3n)}{n^2} \]
2. \[ \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1} \]
3. \[ \sum_{n=1}^{\infty} \frac{(-4)^n}{n^4} \]
4. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{3n+2} \]
5. \[ \sum_{n=1}^{\infty} \frac{(n+1)(5^2 - 1)^n}{5^{2n}} \]

3. (1 pt) setSeries7AbsolutelyConvergent/eva8_4a.pg
Match each of the following with the correct statement.
A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.
D. The series diverges.

1. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!} \]
2. \[ \sum_{n=1}^{\infty} \frac{(-1)^n 2^{n-1}}{(2)^{n+1} n^3} \]
3. \[ \sum_{n=1}^{\infty} \frac{n^5}{3^n} \]
4. \[ \sum_{n=1}^{\infty} (-1)^n \frac{n!}{10^n} \]
5. \[ \sum_{n=1}^{\infty} \frac{(n+2)!}{6^n n!} \]

4. (1 pt) setSeries7AbsolutelyConvergent/eva8_4aBB.pg
Match each of the following with the correct statement.
A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.
D. The series diverges.

1. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (4 + n) 2^n}{(n^2) 4^{2n}} \]
2. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1} \]
3. \[ \sum_{n=1}^{\infty} (n+5)! \]
4. \[ \sum_{n=1}^{\infty} \frac{(-5)^n}{n^5} \]
5. \[ \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2} \]

5. (1 pt) setSeries7AbsolutelyConvergent/ns8_4_20eva.pg
Match each of the following with the correct statement.
A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.
D. The series diverges.

1. \[ \sum_{n=1}^{\infty} \frac{\sin^2(n \pi)}{n^6} \]
2. \[ \sum_{n=1}^{\infty} \frac{(-2)^n}{n^4} \]
3. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (9 + n) 2^n}{(n^2) 3^{2n}} \]
4. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{7n+2} \]
5. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{7n+2} \]

6. (1 pt) setSeries7AbsolutelyConvergent/ns8_4_20BB.pg
Match each of the following with the correct statement.
A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.
D. The series diverges.

1. \[ \sum_{n=1}^{\infty} \frac{(n+2)!}{n! 5^n} \]
2. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (9 + n) 2^n}{(n^2) 3^{2n}} \]
3. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{7n+2} \]
4. \[ \sum_{n=1}^{\infty} \frac{(-5)^n}{n^3} \]
5. \[ \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2} \]
Match each of the following with the correct statement.

A. The series is absolutely convergent.
B. The series converges, but is not absolutely convergent.
C. The series diverges.

In this problem you must attempt to use the Ratio Test to decide whether the series converges.

Enter the numerical value of the limit \( L \) if it converges, INF if it diverges to infinity, MINF if it diverges to negative infinity, or DIV if it diverges but not to infinity or negative infinity.

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
\]

Enter the letter for your choice here:

- A. The Ratio Test says that the series converges absolutely.
- B. The Ratio Test says that the series diverges.
- C. The Ratio Test says that the series converges conditionally.
- D. The Ratio Test is inconclusive, but the series converges absolutely by another test or tests.
- E. The Ratio Test is inconclusive, but the series diverges by another test or tests.
- F. The Ratio Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here:

Consider the series \( \sum_{n=1}^{\infty} a_n \) where

\[
a_n = \frac{(2n+2)(-5)^{n+5}}{7^n}
\]

In this problem you must attempt to use the Ratio Test to decide whether the series converges.

Compute

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
\]

Enter the numerical value of the limit \( L \) if it converges, INF if it diverges to infinity, MINF if it diverges to negative infinity, or DIV if it diverges but not to infinity or negative infinity.

Which of the following statements is true?

A. The Ratio Test says that the series converges absolutely.
B. The Ratio Test says that the series diverges.
C. The Ratio Test says that the series converges conditionally.
D. The Ratio Test is inconclusive, but the series converges absolutely by another test or tests.
E. The Ratio Test is inconclusive, but the series diverges by another test or tests.
F. The Ratio Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here:

Consider the series \( \sum_{n=1}^{\infty} a_n \) where

\[
a_n = \frac{n}{n^2 - 4n - 3}
\]

In this problem you must attempt to use the Ratio Test to decide whether the series converges.

Compute

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
\]

Enter the numerical value of the limit \( L \) if it converges, INF if it diverges to infinity, MINF if it diverges to negative infinity, or DIV if it diverges but not to infinity or negative infinity.

Which of the following statements is true?

A. The Ratio Test says that the series converges absolutely.
B. The Ratio Test says that the series diverges.
C. The Ratio Test says that the series converges conditionally.
D. The Ratio Test is inconclusive, but the series converges absolutely by another test or tests.
E. The Ratio Test is inconclusive, but the series diverges by another test or tests.
F. The Ratio Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here:

Consider the series \( \sum_{n=1}^{\infty} a_n \) where

\[
a_n = \frac{(-n+6)^n}{(-2n-3)^n}
\]

In this problem you must attempt to use the Root Test to decide whether the series converges.

Compute

\[
L = \lim_{n \to \infty} \sqrt[n]{|a_n|}
\]
Enter the numerical value of the limit L if it converges, INF if it diverges to infinity, MINF if it diverges to negative infinity, or DIV if it diverges but not to infinity or negative infinity. 

\[ L = \] 

Which of the following statements is true?
A. The Root Test says that the series converges absolutely.
B. The Root Test says that the series diverges.
C. The Root Test says that the series converges conditionally.
D. The Root Test is inconclusive, but the series converges absolutely by another test or tests.
E. The Root Test is inconclusive, but the series diverges by another test or tests.
F. The Root Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here: 

Consider the series \[ \sum_{n=1}^{\infty} a_n \] where 

\[ a_n = (-1)^n \left( \frac{\ln(n)}{n} \right)^n \]

In this problem you must attempt to use the Root Test to decide whether the series converges.

Compute 

\[ L = \lim_{n \to \infty} \sqrt[n]{|a_n|} \]

Enter the numerical value of the limit L if it converges, INF if it diverges to infinity, MINF if it diverges to negative infinity, or DIV if it diverges but not to infinity or negative infinity. 

\[ L = \] 

Which of the following statements is true?
A. The Root Test says that the series converges absolutely.
B. The Root Test says that the series diverges.
C. The Root Test says that the series converges conditionally.
D. The Root Test is inconclusive, but the series converges absolutely by another test or tests.
E. The Root Test is inconclusive, but the series diverges by another test or tests.
F. The Root Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here: 

Find the interval of convergence for the given power series.

\[ \sum_{n=1}^{\infty} \frac{(x - 1)^n}{n^4} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{6x^n}{n^4} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{(x - 11)^n}{11^n} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{9^n(n^2 + 2)} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n 8^n x^n}{(\sqrt{n} + 2)} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{x^n}{(8^n(\sqrt{n} + 2))} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n (x^n)(n + 4)}{(4)^n} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find all the values of \( x \) such that the given series would converge.

\[ \sum_{n=1}^{\infty} \frac{11^n(x^n)(n + 1)}{(n + 3)} \]

The series is convergent from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Find the interval of convergence for the given power series.

\[ \sum_{n=1}^{\infty} \frac{n^6(x + 3)^n}{(7^n(n + 2))} \]

The series is convergent:

from \( x = \) left end included (enter Y or N): \( \),

to \( x = \) right end included (enter Y or N): \( \).

Match each of the power series with its interval of convergence.

\[ \sum_{n=1}^{\infty} \frac{(6x)^n}{n^6} \]

\[ \sum_{n=1}^{\infty} \frac{(x - 6)^n}{(6)^n} \]

\[ \sum_{n=1}^{\infty} \frac{n!(6x - 6)^n}{6^n} \]

\[ \sum_{n=1}^{\infty} \frac{(x - 6)^n}{(n!)6^n} \]

A. \((-\infty, \infty)\)

B. \([-\frac{1}{6}, \frac{1}{6}]\)

C. \{6/6\}

D. \((0, 12)\)

Suppose that \( \frac{10x}{(17 + x)} = \sum_{n=0}^{\infty} c_n x^n \).

Find the first few coefficients.

\( c_0 = \) 

\( c_1 = \) 

\( c_2 = \) 

\( c_3 = \)
Find the radius of convergence \( R \) of the power series.
\[ R = \text{__________} \]

12. (1 pt) setSeries8Power/eva8_6b.pg

The function \( f(x) = \frac{7}{1-6x} \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the first few coefficients in the power series.
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

13. (1 pt) setSeries8Power/eva8_6b_a.pg

The function \( f(x) = \frac{9}{(1+9x)^2} \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the first few coefficients in the power series.
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

14. (1 pt) setSeries8Power/eva8_6c.pg

The function \( f(x) = \frac{2}{1+8x^2} \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the first few coefficients in the power series.
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

15. (1 pt) setSeries8Power/eva8_6d.pg

The function \( f(x) = 5x^2 \arctan(x^3) \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

What is the lowest term with a nonzero coefficient.

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

16. (1 pt) setSeries8Power/eva8_6e.pg

The function \( f(x) = 8x \arctan(2x) \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the first few coefficients in the power series.
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

17. (1 pt) setSeries8Power/eva8_6f.pg

The function \( f(x) = \ln(2-x) \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the first few coefficients in the power series.
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

18. (1 pt) setSeries8Power/eva8_6g.pg

The function \( f(x) = 3x \ln(1+x) \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the FOLLOWING coefficients in the power series.
\[
\begin{align*}
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
c_5 &= \text{__________} \\
c_6 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

19. (1 pt) setSeries8Power/eva8_6g_a.pg

The function \( f(x) = 6x \ln(1+2x) \) is represented as a power series
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n. \]

Find the FOLLOWING coefficients in the power series.
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
c_3 &= \text{__________} \\
c_4 &= \text{__________} \\
\end{align*}
\]

Find the radius of convergence \( R \) of the series.
\[ R = \text{__________} \]

20. (1 pt) setSeries8Power/eva8_6h.pg

Represent the function \( \frac{7}{1+4x} \) as a power series \( f(x) = \sum_{n=0}^{\infty} c_n x^n \)
\[
\begin{align*}
c_0 &= \text{__________} \\
c_1 &= \text{__________} \\
c_2 &= \text{__________} \\
\end{align*}
\]
21. (1 pt) setSeries8Power/eva8_6.pg

The function \( f(x) = \ln(1 - x^2) \) is represented as a power series
\( f(x) = \sum_{n=0}^{\infty} c_n x^n. \)

Find the radius of convergence \( R = \ldots \).

\[ c_3 = \ldots \]
\[ c_4 = \ldots \]

Find the radius of convergence \( R = \ldots \).

22. (1 pt) setSeries8Power/eva8_7.pg

Evaluate the integral
\[ \int_0^2 \frac{32}{x^2 + 4} \, dx. \]

Your answer should be in the form \( k\pi \), where \( k \) is an integer.

What is the value of \( k \)?

Hint: \( \arctan(x) = \frac{1}{x^2 + 1} \)

23. (1 pt) setSeries8Power/eva8_8.pg

Define the double factorial of \( n \), denoted \( n!! \), as follows:
\[ n!! = \begin{cases} 1 \cdot 3 \cdot 5 \cdots (n-2) \cdot n & \text{if } n \text{ is odd} \\ 2 \cdot 4 \cdot 6 \cdots (n-2) \cdot n & \text{if } n \text{ is even} \end{cases} \]

where \((-1)!! = 0!! = 1!! = 1, 2!! = 2, \) and \(3!! = 3\).

Thus, if \( n \) is even, then \( n!! \) is the product of all the even integers, between 1 and \( n \) and, if \( n \) is odd, then \( n!! \) is the product of all the odd integers, between 1 and \( n \).

Find the radius of convergence for the given power series.
\[ \sum_{n=1}^{\infty} \frac{5^n \cdot n! \cdot (5n + 6)! \cdot (2n)!}{5^n \cdot [(n + 9)!]^3 \cdot (4n - 3)!} \left(-2x + 3\right)^n \]

24. (1 pt) setSeries8Power/powerseries.pg

POWER SERIES AND TAYLOR POLYNOMIALS

Power Series

A power series \( \sum_{n=0}^{\infty} a_n x^n \) has a RADIUS OF CONVERGENCE \( R \).

If the series converges for \( |x| < r \) and diverges for \( |x| > r \).

The radius of convergence is usually calculated by the ratio test, applied to the terms of the power series.

Suppose that \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L \) exists. Then the power series converges if
\[ L < 1 \] and diverges if \( L > 1 \). The radius of convergence is \( R = \frac{1}{L} \).

To determine whether the power series converges when \( x = r \), replace \( x \) by \( r \) in the power series and decide whether the resulting numerical series \( \sum_{n=0}^{\infty} a_n r^n \) converges. The ratio test will not help in deciding this. Use some other convergence test.

To determine whether the power series converges when \( x = -r \), proceed analogously.

Taylor and MacLaurin series

If \( f(x) = \sum_{n=0}^{\infty} a_n x^n \) converges in some interval \((-s, s)\) containing the point zero, then for each \( n \):
\[ a_n = \frac{f^{(n)}(0)}{n!}. \]

Power series may be integrated or differentiated term by term.

That is:
\[ \frac{df}{dx} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n. \]

The integral of \( f(t) \) from \( 0 \) to \( x \) is
\[ \int_0^x f(t) \, dt = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} a_{n-1} x^n. \]

The \( n \)th degree MacLaurin polynomial for \( f(x) \) is
\[ T_n(x) = \sum_{j=0}^{n} \frac{f^{(j)}(0)}{j!} x^j. \]

It approximates \( f(x) \) with error \( R_n(x) \).

That is, \( f(x) = T_n(x) + R_n(x) \). The size of the error is estimated by
\[ |R_n(x)| \leq M|x|^{n+1} \frac{1}{(n+1)!}. \]

Here, \( M \) is an upper bound for the \( (n+1) \)-st derivative of \( f \) between 0 and \( x \). It is enough that \( |f^{(n+1)}(t)| < M \) for all \( t \) such that \( |t| < |x| \).

For every statement above you should know the analogous statement for a power series in powers of \((x - c)\) which has the form
\[ \sum_{n=0}^{\infty} a_n (x - c)^n. \]

To receive a point enter the letter \( y \) answer \( \ldots \).
1. Compute the 9th derivative of

\[ f(x) = \arctan \left( \frac{x^3}{3} \right) \]

at \( x = 0 \).

\[ f^{(9)}(0) = \quad \text{Hint: Use the MacLaurin series for } f(x). \]

2. Compute the 9th derivative of

\[ f(x) = \cos \left( 3x^4 \right) - 1 \]

at \( x = 0 \).

\[ f^{(9)}(0) = \quad \text{Hint: Use the MacLaurin series for } f(x). \]

3. Find the degree 3 Taylor polynomial \( T_3(x) \) of function

\[ f(x) = (-5x + 111)^{5/4} \]

at \( a = 6 \).

\[ T_3(x) = \]

4. The Taylor series for \( f(x) = x^3 \) at 4 is \( \sum_{n=0}^{\infty} c_n (x - 4)^n \).

Find the first few coefficients.

\[ c_0 = \quad \text{ } \]
\[ c_1 = \quad \text{ } \]
\[ c_2 = \quad \text{ } \]
\[ c_3 = \quad \text{ } \]
\[ c_4 = \quad \text{ } \]

5. The Taylor series for \( f(x) = \ln(\sec(x)) \) at \( a = 0 \) is \( \sum_{n=0}^{\infty} c_n (x - a)^n \).

Find the first few coefficients.

\[ c_0 = \quad \text{ } \]
\[ c_1 = \quad \text{ } \]
\[ c_2 = \quad \text{ } \]
\[ c_3 = \quad \text{ } \]
\[ c_4 = \quad \text{ } \]

6. Find the exact error in approximating \( \ln(\sec(0.2)) \) by its fourth degree Taylor polynomial at \( a = 0 \).

The error is

7. Find \( T_4(x) \) the Taylor polynomial of degree 4 of the function

\[ f(x) = \arctan(5x) \]

at \( a = 0 \).

\[ T_4(x) = \]

8. Find \( T_{10}(x) \) the Taylor polynomial of degree 10 of the function

\[ f(x) = \arctan^3(x) \]

at \( a = 0 \).

\[ T_{10}(x) = \]

9. Let \( T_5(x) \) be the fifth degree Taylor polynomial of the function

\[ f(x) = \cos(0.6x) \]

at \( a = 0 \).

A. Find \( T_5(x) \). (Enter a function.)

\[ T_5(x) = \]

B. Find the largest integer \( k \) such that for all \( x \) for which \( |x| < \frac{1}{5} \) the Taylor polynomial \( T_k(x) \) approximates \( f(x) \) with error less than \( \frac{1}{10^5} \).

\[ k = \]

10. Match each of the Maclaurin series with right function.

\[ \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

1. \( A. \arctan(x) \)

2. \( B. \cos(x) \)

3. \( C. e^x \)

4. \( D. \sin(x) \)

11. Match each of the Maclaurin series with right function.

\[ \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{(2n)!} \]

1. \( A. 2 \sin(x) \)

2. \( B. \sin^2(x) \)

3. \( C. \cos^2(x) \)

4. \( D. \cos(x)^2 \)
12. (1 pt) setSeries9Taylor/e8_7_4_b.pg
Select the FIRST correct reason why the given series diverges.
A. \( \sin(x) \)
B. \( \exp(x) \)
C. \( \cos(x) \)
D. \( \arctan(x) \)

1. \( \sum_{n=0}^{\infty} (-1)^n x^{2n+1} \frac{1}{(2n+1)!} \)
2. \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \)
3. \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \)
4. \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \)

13. (1 pt) setSeries9Taylor/e8_7_4_c.pg
Select the FIRST correct reason why the given series diverges.
A. \( \sin(2x) \)
B. \( \exp(2x) \)
C. \( \cos(2x) \)
D. \( \arctan(2x) \)

1. \( \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} \)
2. \( \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n}}{(2n)!} \)
3. \( \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{2n+1} \)
4. \( \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n)!} \)

14. (1 pt) setSeries9Taylor/e8_7_5.pg
Let \( F(x) = \int_0^x \sin(2t^2) \, dt \).
Find the MacLaurin polynomial of degree 7 for \( F(x) \).

Use this polynomial to estimate the value of \( \int_0^{0.65} \sin(2x^2) \, dx \).

15. (1 pt) setSeries9Taylor/e8_7_6.pg
Let \( F(x) = \int_0^x e^{-2t^4} \, dt \).
Find the MacLaurin polynomial of degree 5 for \( F(x) \).

Use this polynomial to estimate the value of \( \int_0^{0.13} e^{-2x^4} \, dx \).

16. (1 pt) setSeries9Taylor/e8_7_7.pg
Find the Maclaurin series of the function \( f(x) = 9x^3 - 10x^2 - 4x + 10 \)

17. (1 pt) setSeries9Taylor/e8_7_8.pg
Represent the function \( x^{0.5} \) as a power series \( \sum_{n=0}^{\infty} c_n (x - 2)^n \).

18. (1 pt) setSeries9Taylor/e8_7_9.pg
Find the MacLaurin polynomial of degree 7 for \( f(x) = \ln(x) \) at \( a = 2 \).

19. (1 pt) setSeries9Taylor/ur_sr_9_1.pg
Evaluate \( \lim_{x \to 0} \frac{\ln(1 - x) + x + x^2}{15x^3} \).
Hint: Using power series.

20. (1 pt) setSeries9Taylor/ur_sr_9_2.pg
Evaluate \( \lim_{x \to 0} \frac{e^{3x^3} - 1 + 3x^3 - \frac{9}{4}x^6}{2x^9} \).
Hint: Using power series.

21. (1 pt) setSeries9Taylor/ur_sr_9_3.pg
Assume that \( \sin(x) \) equals its Maclaurin series for all \( x \).
Use the Maclaurin series for \( \sin(3x^2) \) to evaluate the integral \( \int_0^{0.74} \sin(3x^2) \, dx \).
Your answer will be an infinite series. Use the first two terms to estimate its value.
Find the first few coefficients.

The Taylor series for \( f(x) = e^x \) at \( a = 2 \) is \( \sum_{n=0}^{\infty} c_n (x - 2)^n \).

Find the first few coefficients.

\[
c_0 = \\
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
\]

Assume that \( e^x \) equals its Maclaurin series for all \( x \).

Use the Maclaurin series for \( e^{-4x^2} \) to evaluate the integral

\[ \int_0^{0.17} e^{-4x^2} \, dx \]

Your answer will be an infinite series. Use the first two terms to estimate its value.

The Taylor series for \( f(x) = \sin(x) \) at \( a = \frac{\pi}{2} \) is \( \sum_{n=0}^{\infty} c_n (x - \frac{\pi}{2})^n \).

Find the first few coefficients.

\[
c_0 = \\
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
\]

The Taylor series for \( f(x) = \cos(x) \) at \( a = \frac{\pi}{4} \) is \( \sum_{n=0}^{\infty} c_n (x - \frac{\pi}{4})^n \).

Find the first few coefficients.

\[
c_0 = \\
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
\]

Find the Maclaurin series of the function \( f(x) = \arctan(2x^2) \).

\[
(f(x) = \sum_{n=0}^{\infty} c_n x^n)
\]

\[
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
c_5 = \\
\]

Find the Maclaurin series of the function \( f(x) = (9x^2) e^{-4x} \).

\[
(f(x) = \sum_{n=0}^{\infty} c_n x^n)
\]

\[
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
c_5 = \\
\]

Find the Maclaurin series of the function \( f(x) = (2x^2) \sin(3x) \).

\[
(f(x) = \sum_{n=0}^{\infty} c_n x^n)
\]

\[
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
c_5 = \\
\]

Find the Maclaurin series of the function \( f(x) = 6 \cos(9x^2) \).

\[
(f(x) = \sum_{n=0}^{\infty} c_n x^n)
\]

\[
c_0 = \\
c_1 = \\
c_2 = \\
c_3 = \\
c_4 = \\
\]

Match the series with the right expression. (Use the Maclaurin series.)

1. \( \sum_{n=0}^{\infty} \frac{3^n}{n!} \)
2. \( \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n}}{(2n)!} \)
3. \( \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{2n + 1} \)
4. \( \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1}}{(2n + 1)!} \)
33. (1 pt) setSeries9Taylor/e8_7_10.pg

Find $T_5(x)$: Taylor polynomial of degree 5 of the function $f(x) = \cos(x)$ at $a = 0$.
(You need to enter function.)

$T_5(x) =$

Find all values of $x$ for which this approximation is within 0.001 of the right answer. Assume for simplicity that we limit ourselves to $|x| \leq 1$.

$|x| \leq ____$

34. (1 pt) setSeries9Taylor/e8_7_10_2.pg

Let $T_4(x)$: be the Taylor polynomial of degree 4 of the function $f(x) = \cos(x)$ at $a = 0$.

Suppose you approximate $f(x)$ by $T_4(x)$, and if $|x| \leq 1$, what is the bound for your error of your estimate? (Hint: use the alternating series approximation.)

$____$

35. (1 pt) setSeries9Taylor/e8_7_10_3.pg

Let $T_k(x)$: be the Taylor polynomial of degree $k$ of the function $f(x) = \sin(x)$ at $a = 0$.

Suppose you approximate $f(x)$ by $T_k(x)$, and if $|x| \leq 1$, how many terms do you need (that is, what is $k$) for you to have your error to be less than $\frac{1}{5040}$? (Hint: use the alternating series approximation.)

$____$

36. (1 pt) setSeries9Taylor/e8_7_10_4.pg

Let $T_4(x)$: be the Taylor polynomial of degree 4 of the function $f(x) = \ln(1 + x)$ at $a = 0$.

Suppose you approximate $f(x)$ by $T_4(x)$, find all positive values of $x$ for which this approximation is within 0.001 of the right answer. (Hint: use the alternating series approximation.)

$0 < x \leq ____$

37. (1 pt) setSeries9Taylor/e8_7_11.pg

Represent the function $4\ln(3 - x)$ as a power series (Maclaurin series) $f(x) = \sum_{n=0}^{\infty} c_n x^n$

$c_0 =$

c_1 =$

c_2 =$

c_3 =$

c_4 =$

Find the radius of convergence $R =$

38. (1 pt) setSeries9Taylor/e8_7_15.pg

Evaluate $\lim_{x \to 0} \frac{\cos(x) - 1 + \frac{x^2}{2}}{4x^4}$

Hint: Using power series.
1. (1 pt) setDiffEQ1/e7_j_1.png
Match each of the following differential equations with a solution from the list below.

   1. \( y'' - 12y' + 32y = 0 \)
   2. \( y'' + y = 0 \)
   3. \( y'' + 12y' + 32y = 0 \)
   4. \( 2x^2y'' + 3xy' = y \)

   A. \( y = e^{-8x} \)
   B. \( y = e^{4x} \)
   C. \( y = \frac{1}{x} \)
   D. \( y = \cos(x) \)

2. (1 pt) setDiffEQ1/e7_j_1a.png
Match each of the differential equation with its solution.

   1. \( y'' + y = 0 \)
   2. \( xy' - y = x^2 \)
   3. \( y'' + 7y' + 12y = 0 \)
   4. \( 2x^2y'' + 3xy' = y \)

   A. \( y = \frac{x^2}{2} \)
   B. \( y = e^{-3x} \)
   C. \( y = 3x + x^2 \)
   D. \( y = \sin(x) \)

3. (1 pt) setDiffEQ1/e7_j_1b.png
Match each differential equation to a function which is a solution.

**FUNCTIONS**
A. \( y = 3x + x^2 \)
B. \( y = e^{-3x} \)
C. \( y = \sin(x) \)
D. \( y = x^\frac{1}{2} \)
E. \( y = 3 \exp(8x) \)

**DIFFERENTIAL EQUATIONS**

   1. \( 2x^2y'' + 3xy' = y \)
   2. \( xy' - y = x^2 \)
   3. \( y' = 8y \)
   4. \( y'' + y = 0 \)

4. (1 pt) setDiffEQ1/osu_de_1.png
Match the following differential equations with their solutions. The symbols \( A, B, C \) in the solutions stand for arbitrary constants.

You must get all of the answers correct to receive credit.

   1. \( \frac{d^2y}{dx^2} + 81y = 0 \)
   2. \( \frac{dy}{dx} = -2xy \)
   3. \( \frac{d^2y}{dx^2} + 8 \frac{dy}{dx} + 16y = 0 \)

---

5. (1 pt) setDiffEQ1/ur_de_1.png
Just as there are simultaneous algebraic equations (where a pair of numbers have to satisfy a pair of equations) there are systems of differential equations, (where a pair of functions have to satisfy a pair of differential equations). Indicate which pairs of functions satisfy this system. It will take some time to make all of the calculations.

\[ y_1' = y_1 - 2y_2 \quad y_2' = 3y_1 - 4y_2 \]

- A. \( y_1 = \cos(x) \quad y_2 = -\sin(x) \)
- B. \( y_1 = \sin(x) \quad y_2 = \cos(x) \)
- C. \( y_1 = e^x \quad y_2 = e^x \)
- D. \( y_1 = e^{-x} \quad y_2 = e^{-x} \)
- E. \( y_1 = 2e^{-2x} \quad y_2 = 3e^{-2x} \)
- F. \( y_1 = e^{4x} \quad y_2 = e^{4x} \)
- G. \( y_1 = \sin(x) + \cos(x) \quad y_2 = \cos(x) - \sin(x) \)

As you can see, finding all of the solutions, particularly of a system of equations, can be complicated and time consuming. It helps greatly if we study the structure of the family of solutions to the equations. Then if we find a few solutions we will be able to predict the rest of the solutions using the structure of the family of solutions.

6. (1 pt) setDiffEQ1/ur_de_1.png
It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the order of the equation – (what is the highest number of derivatives involved) and whether or not the equation is linear.

Linearity is important because the structure of the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

   1. \( y'' - y + y^2 = 0 \)
   2. \( r^2 \frac{d^2y}{dt^2} + r \frac{dy}{dt} + 2y = \sin t \)
   3. \( y'' - y + r^2 = 0 \)
   4. \( \frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} = 1 \)
Find the constant value of $c$ for which the solution satisfies the initial condition $y(6) = 5$. 

$c = \underline{\hspace{2cm}}$

The functions $y = x^2 + \frac{c}{x^2}$ are all solutions of the equation $xy' + 2y = 4x^2$, $(x > 0)$. Find the value of $c$ for which the solution satisfies the initial condition $y(2) = 4$. 

$c = \underline{\hspace{2cm}}$

The family of functions $y = ce^{-2x} + e^{-x}$ is solution of equation $y' + 2y = e^{-x}$. Find the value of $c$ for which the solution satisfies the initial condition $y(-4) = 8$. 

$c = \underline{\hspace{2cm}}$

Find the two values of $k$ for which $y(x) = e^{ks}$ is a solution of the differential equation $y'' - 11y' + 28y = 0$. 

smaller value = \underline{\hspace{2cm}}

larger value = \underline{\hspace{2cm}}

Find all values of $k$ for which the function $y = \sin(kt)$ satisfies the differential equation $y'' + 10y = 0$. Separate your answers by commas.

Find the value of $k$ for which the constant function $x(t) = k$ is a solution of the differential equation $4t^2 \frac{dx}{dt} + 2x + 8 = 0$. 

Which of the following functions are solutions of the differential equation $y'' - 4y' - 12y = 0$?

- A. $y(x) = 0$
- B. $y(x) = -2x$
- C. $y(x) = e^x$
- D. $y(x) = e^{-2x}$
- E. $y(x) = e^{6x}$
- F. $y(x) = 6x$
- G. $y(x) = e^{-x}$

Consider the curves in the first quadrant that have equations $y = A \exp(5x)$, where $A$ is a positive constant. Different values of $A$ give different curves. The curves form a family, $F$. Let $P = (4, 5)$. Let $C$ be the member of the family $F$ that goes through $P$.

A. Let $y = f(x)$ be the equation of $C$. Find $f(x)$. $f(x) = \underline{\hspace{2cm}}$

B. Find the slope at $P$ of the tangent to $C$. slope = \underline{\hspace{2cm}}

C. A curve $D$ is perpendicular to $C$ at $P$. What is the slope of the tangent to $D$ at the point $P$? slope = \underline{\hspace{2cm}}

D. Give a formula $g(y)$ for the slope at $(x, y)$ of the member of $F$ that goes through $(x, y)$. The formula should not involve $A$ or $x$. $g(y) = \underline{\hspace{2cm}}$

E. A curve which at each of its points is perpendicular to the member of the family $F$ that goes through that point is called an orthogonal trajectory to $F$. Each orthogonal trajectory to $F$ satisfies the differential equation $\frac{dy}{dx} = -\frac{1}{g(y)}$. where $g(y)$ is the answer to part D. Find a function $h(y)$ such that $x = h(y)$ is the equation of the orthogonal trajectory to $F$ that passes through the point $P$. $h(y) = \underline{\hspace{2cm}}$

The solution of a certain differential equation is of the form $y(t) = a \exp(2t) + b \exp(4t)$, where $a$ and $b$ are constants. The solution has initial conditions $y(0) = 2$ and $y'(0) = 3$. Find the solution by using the initial conditions to get linear equations for $a$ and $b$. 

$y(t) = \underline{\hspace{2cm}}$
Match the following equations with their direction field. Clicking on each picture will give you an enlarged view. While you can probably solve this problem by guessing, it is useful to try to predict characteristics of the direction field and then match them to the picture. Here are some handy characteristics to start with – you will develop more as you practice.

A. Set \( y \) equal to zero and look at how the derivative behaves along the \( x \)-axis.
B. Do the same for the \( y \)-axis by setting \( x \) equal to 0
C. Consider the curve in the plane defined by setting \( y' = 0 \) – this should correspond to the points in the picture where the slope is zero.
D. Setting \( y' \) equal to a constant other than zero gives the curve of points where the slope is that constant. These are called isoclines, and can be used to construct the direction field picture by hand.

Go to this page to launch the phase plane plotter to check your answers. (Choose the "integral curves utility" from the applet menu, enter \( dx/dt = 1 \) to identify the variables \( x \) and \( t \) and then enter the function you want for \( dy/dx = dy/dt = \ldots \)).

\[ 1. \quad y' = -1 - 2y \]
\[ 2. \quad y' = y + 2 \]
\[ 3. \quad y' = -\frac{(2x + y)}{(2y)} \]

This problem is harder, and doesn’t give you clues as to which matches you have right. Study the previous problem, if you are having trouble.

Go to this page to launch the phase plane plotter to check your answers.

Match the following equations with their direction field. Clicking on each picture will give you an enlarged view.

\[ 1. \quad y' = -y(5 - y) \]
\[ 2. \quad y' = xe^{-2x} - 2y \]
\[ 3. \quad y' = x + 2y \]
\[ 4. \quad y' = \frac{y^3}{6} - y - \frac{x^3}{6} \]

Match the following equations with their direction field. Clicking on each picture will give you an enlarged view.

\[ 1. \quad y' = y + xe^{-x} + 1 \]
\[ 2. \quad y' = 2y - 2 \]
\[ 3. \quad y' = 2xy + 2xe^{-x^2} \]
\[ 4. \quad y' = -\frac{(2x + y)}{(2y)} \]
Match the following equations with their direction field. Clicking on each picture will give you an enlarged view. While you can probably solve this problem by guessing, it is useful to try to predict characteristics of the direction field and then match them to the picture. Here are some handy characteristics to start with – you will develop more as you practice.

1. Set \( y \) equal to zero and look at how the derivative behaves along the \( x \)-axis.
2. Do the same for the \( y \)-axis by setting \( x \) equal to 0.
3. Consider the curve in the plane defined by setting \( y' = 0 \) – this should correspond to the points in the picture where the slope is zero.
4. Setting \( y' \) equal to a constant other than zero gives the curve of points where the slope is that constant. These are called isoclines, and can be used to construct the direction field picture by hand.

1. \( y' = -\frac{2x+y}{2y} \)
2. \( y' = -2 + x - y \)
3. \( y' = 2y + x^2 e^{2x} \)
4. \( y' = e^{-x} + 2y \)

5. Use Euler’s method with step size 0.5 to compute the approximate \( y \)-values at \( x = 1 \) for the solution of the initial-value problem

\[ y' = -2 - 3x - 4y, \quad y(1) = -2. \]

\[ y_1 = \quad \quad y_2 = \quad \quad y_3 = \quad \quad y_4 = \]

6. Use Euler’s method with step size 0.4 to estimate \( y(2) \), where \( y(x) \) is the solution of the initial-value problem

\[ y' = -3x + y^2, \quad y(0) = 0. \]

\[ y(2) = \]

7. Suppose you have just poured a cup of freshly brewed coffee with temperature 90°C in a room where the temperature is 25°C. Newton’s Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Therefore, the temperature of the coffee, \( T(t) \), satisfies the differential equation

\[ \frac{dT}{dt} = k(T - T_{room}) \]

where \( T_{room} = 25 \) is the room temperature, and \( k \) is some constant.

Suppose it is known that the coffee cools at a rate of 1°C per minute when its temperature is 65°C.

\[ \text{lim}_{t \to \infty} T(t) = \quad \quad \text{lim}_{t \to \infty} \frac{dT}{dt} = \quad \quad \text{lim}_{t \to \infty} T(t) = \quad \quad k = \quad \quad \text{D.} \quad \text{Use Euler’s method with step size} \quad h = 2 \text{ minutes to estimate the temperature of the coffee after 10 minutes.} \]

\[ T(10) = \]
1. Solve the separable differential equation
   \[ \frac{dy}{dx} = -0.4y, \]
   and find the particular solution satisfying the initial condition
   \[ y(0) = 3. \]
   \[ y(x) = \] 

2. Solve the separable differential equation
   \[ \frac{dx}{dt} = 6 \frac{y}{x}, \]
   and find the particular solution satisfying the initial condition
   \[ x(0) = 1. \]
   \[ x(t) = \] 

3. Solve the separable differential equation
   \[ \frac{dy}{dt} = 9y^5, \]
   and find the particular solution satisfying the initial condition
   \[ y(0) = -1. \]
   \[ y(t) = \] 

4. Solve the separable differential equation
   \[ y'(x) = \sqrt{-6y(x) + 13}, \]
   and find the particular solution satisfying the initial condition
   \[ y(-4) = -2. \]
   \[ y(x) = \] 

5. Solve the separable differential equation
   \[ \frac{dy}{dx} = -\frac{0.9}{\cos(y)}, \]
   and find the particular solution satisfying the initial condition
   \[ y(0) = \frac{\pi}{3}. \]
   \[ y(x) = \] 

6. Solve the separable differential equation
   \[ \frac{dx}{dt} = x^2 + \frac{1}{36}, \]
   and find the particular solution satisfying the initial condition
   \[ x(0) = -5. \]
   \[ x(t) = \] 

7. Find the particular solution of the differential equation
   \[ \frac{dy}{dx} = (x - 3)e^{-2y}, \]
   satisfying the initial condition \[ y(3) = \ln(3). \]
   Answer: \( y = \) 
   Your answer should be a function of \( x \).

8. Find the particular solution of the differential equation
   \[ \frac{x^2}{y^2 - 8} \frac{dy}{dx} = \frac{1}{2y}, \]
   satisfying the initial condition \( y(1) = \sqrt{9} \).
   Answer: \( y = \) 
   Your answer should be a function of \( x \).

9. Find \( u \) from the differential equation and initial condition.
   \[ \frac{du}{dt} = e^{1.6t - 2.9u}, \quad u(0) = 3.9. \]
   \[ u = \] 

10. Solve the separable differential equation for \( u \)
    \[ \frac{du}{dt} = e^{3u+5t}. \]
    Use the following initial condition: \( u(0) = 4. \)
    \[ u = \] 

11. Solve the separable differential equation for \( u \)
    \[ \frac{du}{dt} = e^{2u+5t}. \]
    Use the following initial condition: \( u(0) = -15. \)
    \[ u = \] 

12. Solve the separable differential equation
    \[ 4x - 3y\sqrt{x^2 + 1} \frac{dy}{dx} = 0. \]
    Subject to the initial condition: \( y(0) = -3. \)
    \[ y = \] 

13. Find \( f(x) \) if \( y = f(x) \) satisfies
    \[ \frac{dy}{dx} = 36yx^{17}. \]
    and the \( y \)-intercept of the curve \( y = f(x) \) is 2.
    \[ f(x) = \]
14. Find an equation of the curve that satisfies
\[ \frac{dy}{dx} = 65yx^{12} \]
and whose y-intercept is 3.
\[ y(x) = \text{_____} \]

15. Find the solution of the differential equation
\[ 3e^{6x} \frac{dy}{dx} = -36 \frac{x}{y^2} \]
which satisfies the initial condition \( y(0) = 1 \).
\[ y = \text{_____} \]

16. Find a function \( y \) of \( x \) such that
\[ 5yy' = x \text{ and } y(5) = 6. \]
\[ y = \text{_____} \]

17. Find \( k \) such that \( x(t) = 14t^4 \) is a solution of the differential equation
\[ \frac{dx}{dt} = kx. \]
\[ k = \text{_____} \]

18. Solve the separable differential equation
\[ 3yy' = x. \]
Use the following initial condition: \( y(3) = 2 \).
Express \( x^2 \) in terms of \( y \).
\[ x^2 = \text{_____} \text{ (function of } y \text{)} \]

19. Solve the differential equation
\[ (y^9) \frac{dy}{dx} = 1 + x. \]
Use the initial condition \( y(1) = 3 \).
Express \( y^{10} \) in terms of \( x \).
\[ y^{10} = \text{_____} \]

20. Solve the separable differential equation for:
\[ \frac{dy}{dx} = \frac{1 + x}{xy^3}; \quad x > 0 \]
Use the following initial condition: \( y(1) = 3 \).
\[ y = \text{_____} \]

21. Find the function \( y = y(x) \) (for \( x > 0 \)) which satisfies the separable differential equation
\[ \frac{dy}{dx} = \frac{7 + 16x}{xy^2}; \quad x > 0 \]
with the initial condition \( y(1) = 3 \).
\[ y = \text{_____} \]

22. Find the solution of the differential equation
\[ (\ln(y))^2 \frac{dy}{dx} = x^2y \]
which satisfies the initial condition \( y(1) = e^2 \).
\[ y = \text{_____} \]

23. Solve the following initial value problem:
\[ (t^2 - 8t + 12) \frac{dy}{dt} = y \]
with \( y(4) = 1 \). (Find \( y \) as a function of \( t \).)
\[ y = \text{_____} \]

24. The differential equation
\[ \frac{dy}{dx} = \frac{\cos(x) y^2 + 14y + 48}{6y + 46} \]
has an implicit general solution of the form \( F(x, y) = K \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = K. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \text{_____} \]

25. The differential equation
\[ 32 \frac{dy}{dx} = (9 - x^2)^{-1/2} \exp(-4y) \]
has an implicit general solution of the form \( F(x, y) = K \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = K. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \text{_____} \]
26. (1 pt) setDiffEQ3Separable/ur_de_3.4.png
The differential equation
\[ \frac{dy}{dx} = \frac{18}{y^{1/3} + 81x^2y^{1/3}} \]
has an implicit general solution of the form \( F(x, y) = k \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = k. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \]

27. (1 pt) setDiffEQ3Separable/ur_de_3.5.png
The differential equation
\[ (10 + 2\cos(x)) \frac{dy}{dx} = \sin(x) \cos(y) \]
has an implicit general solution of the form \( F(x, y) = k \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = k. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \]

28. (1 pt) setDiffEQ3Separable/ur_de_3.6.png
A. Find \( y \) in terms of \( x \) if
\[ \frac{dy}{dx} = x^8y^{-5} \]
and \( y(0) = 7 \).
\[ y(x) = \]
B. For what \( x \)-interval is the solution defined?
(Your answers should be numbers or plus or minus infinity. For plus infinity enter "PINF"; for minus infinity enter "MINF".)
The solution is defined on the interval:
\[ x < \]

29. (1 pt) setDiffEQ3Separable/ur_de_3.7.png
The differential equation
\[ \frac{dy}{dx} = \frac{2x + 6}{12y^2 + 14y + 4} \]
has an implicit general solution of the form \( F(x, y) = k \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = k. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \]

30. (1 pt) setDiffEQ3Separable/ur_de_3.8.png
The differential equation
\[ \exp(y) \frac{dy}{dx} = \frac{8x + 4}{-4 \sin(y) + 10 \cos(y)} \]
has an implicit general solution of the form \( F(x, y) = k \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = k. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \]

31. (1 pt) setDiffEQ3Separable/ur_de_3.9.png
A. Solve the following initial value problem:
\[ \cos(t)^2 \frac{dy}{dt} = 1 \]
with \( y(27) = \tan(27) \).
(Your answer should involve \( \pi \).
\[ y = \]
B. On what interval is the solution valid?
(Your answer should involve \( \pi \).
Answer: It is valid for \( \pi < t < \pi \).
C. Find the limit of the solution as \( t \) approaches the left end of the interval. (Your answer should be a number or "PINF" or "MINF".
"PINF" stands for plus infinity and "MINF" stands for minus infinity.)
Answer: 
D. Similar to C, but for the right end.
Answer: 

32. (1 pt) setDiffEQ3Separable/ur_de_3.10.png
The differential equation
\[ \frac{dy}{dx} = 21 + 14x + 24y + 16xy \]
has an implicit general solution of the form \( F(x, y) = k \).
In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form
\[ F(x, y) = G(x) + H(y) = k. \]
Find such a solution and then give the related functions requested.
\[ F(x, y) = G(x) + H(y) = \]
Find the particular solution of the differential equation \( \frac{dy}{dx} + 8y = 9 \)
satisfying the initial condition \( y(0) = 0 \).  
Answer: \( y = \) ____________

Your answer should be a function of \( x \).

Find the function satisfying the differential equation \( f'(t) - f(t) = 5t \)
and the condition \( f(2) = 8 \).
\( f(t) = \) ____________

GUESS one function \( y(t) \) which solves the problem below, by determining the general form the function might take and then evaluating some coefficients.
\( 6t \frac{dy}{dt} + y = t^2 \)
Find \( y(t) \).
\( y(t) = \) ____________

GUESS one function \( y(t) \) which solves the problem below, by determining the general form the function might take and then evaluating some coefficients.
\( \frac{dy}{dt} + 3y = \exp(1t) \)
Find \( y(t) \).
\( y(t) = \) ____________

Find the function satisfying the differential equation \( y' - 2y = 6e^{4t} \)
and \( y(0) = -2 \).
\( y = \) ____________

Solve the following initial value problem:
\( \frac{dy}{dt} + 8y = 3t \)
with \( y(1) = 3 \).
\( y = \) ____________

Solve the following initial value problem:
\( \frac{dy}{dt} + 0.5ty = 7t \)
with \( y(0) = 3 \).
\( y = \) ____________

Find the function satisfying the differential equation \( \frac{dy}{dt} - 8y = 32t \),
for \( t > -1 \) with \( y(0) = 8 \).
\( y = \) ____________

Solve the initial value problem
\( \frac{dx}{dt} + 4x = \cos(3t) \)
with \( x(0) = -2 \).
\( x(t) = \) ____________

Find the particular solution of the differential equation
\( \frac{dy}{dx} + y\cos(x) = 3\cos(x) \)
satisfying the initial condition \( y(0) = 5 \).
Answer: \( y(x) = \) ____________

Solve the initial value problem
\( \frac{dy}{dt} - y = 8\exp(t) + 8\exp(9t) \)
with \( y(0) = 2 \).
\( y = \) ____________

Solve the initial value problem
\( \frac{dy}{dt} + 2y = 15\sin(t) + 20\cos(t) \)
with \( y(0) = 6 \).
\( y = \) ____________

Solve the following initial value problem:
\( 9\frac{dy}{dt} + y = 63t \)
with \( y(0) = 1 \).
(Find \( y \) as a function of \( t \).)
\( y = \) ____________

Solve the initial value problem
\( 4(\sin(t)\frac{dy}{dt} + (\cos(t))y) = (\cos(t))(\sin(t))^7 \),
for \( 0 < t < \pi \) and \( y(\pi/2) = 4 \).
\( y = \) ____________

Find the function \( y(t) \) that satisfies the differential equation
\( \frac{dy}{dt} - 2ty = 15t^2 e^{t^2} \)
and the condition \( y(0) = -2 \).
\( y(t) = \) ____________
16. Let $g(t)$ be the solution of the initial value problem
\[ 6t \frac{dy}{dt} + y = 0, \quad t > 0, \]
with $g(1) = 1$.
Find $g(t)$.

B. Let $f(t)$ be the solution of the initial value problem
\[ 6t \frac{dy}{dt} + y = t^2 \]
with $f(0) = 0$.
Find $f(t)$.

C. Find a constant $c$ so that
\[ k(t) = f(t) + cg(t) \]
solves the differential equation in part B and $k(1) = 9$.
$c = \underline{\phantom{0}}$

17. Let $g(t)$ be the solution of the initial value problem
\[ \frac{dy}{dt} + 2y = 0, \]
with $y(0) = 1$.
Find $g(t)$.

B. Let $f(t)$ be the solution of the initial value problem
\[ \frac{dy}{dt} + 2y = \exp(2t) \]
with $y(0) = 1/4$.
Find $f(t)$.

C. Find a constant $c$ so that
\[ k(t) = f(t) + cg(t) \]
solves the differential equation in part B and $k(0) = 5$.
$c = \underline{\phantom{0}}$

18. Find a family of solutions to the differential equation
\[ (x^2 + 4xy)dx + xdy = 0 \]
(To enter the answer in the form below you may have to rearrange the equation so that the constant is by itself on one side of the equation.) Then the solution in implicit form is:
the set of points $(x, y)$ where $F(x, y) =$

19. A function $y(t)$ satisfies the differential equation
\[ \frac{dy}{dt} = -y^4 - 4y^3 + 45y^2. \]
(a) What are the constant solutions of this equation? Separate your answers by commas.
(b) For what values of $y$ is $y$ increasing?

20. A Bernoulli differential equation is one of the form
\[ \frac{dy}{dx} + P(x)y = Q(x)y^n. \]
Observe that, if $n = 0$ or $1$, the Bernoulli equation is linear.
For other values of $n$, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation
\[ \frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x). \]
Use an appropriate substitution to solve the equation
\[ xy' + y = 9xy^2, \]
and find the solution that satisfies $y(1) = -9$.
$y(x) = \underline{\phantom{0}}$

21. A Bernoulli differential equation is one of the form
\[ \frac{dy}{dx} + P(x)y = Q(x)y^n \quad (\ast) \]
Observe that, if $n = 0$ or $1$, the Bernoulli equation is linear.
For other values of $n$, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation
\[ \frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x). \]
Consider the initial value problem
\[ xy' + y = 5xy^2, \quad y(1) = -8. \]
(a) This differential equation can be written in the form $(\ast)$ with
$P(x) =$
$Q(x) =$
$n =$
(b) The substitution $u =$ will transform it into the linear equation
\[ \frac{du}{dx} + u = \underline{\phantom{0}} \]
(c) Using the substitution in part (b), we rewrite the initial condition in terms of $x$ and $u$:
$u(1) =$
(d) Now solve the linear equation in part (b), and find the solution that satisfies the initial condition in part (c):
$u(x) =$
(e) Finally, solve for $y$.
$y(x) =$

22. A Bernoulli differential equation is one of the form
\[ \frac{dy}{dx} + P(x)y = Q(x)y^n. \]
Observe that, if $n = 0$ or $1$, the Bernoulli equation is linear.
For other values of $n$, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation
\[ \frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x). \]
Use an appropriate substitution to solve the equation

\[ y' - \frac{2}{x} y = \frac{y^5}{x^4}, \]

and find the solution that satisfies \( y(1) = 1 \).

\[ y(x) = \underline{\text{__________}} \]
1. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31a.pg
A curve passes through the point (0, 6) and has the property that the slope of the curve at every point \( P \) is four times the \( y \)-coordinate of \( P \). What is the equation of the curve? 
\[ y(x) = \] 

2. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31c.png
A tank contains 2240 L of pure water. A solution that contains 0.02 kg of sugar per liter enters tank at the rate 8 L/min. The solution is mixed and drains from the tank at the same rate.
(a) How much sugar is in the tank initially? 
\[ \text{amount} = \] (function of \( t \))
(b) Find the amount of sugar in the tank after \( t \) minutes.
\[ \text{amount} = \] (function of \( t \))
(c) Find the concentration of sugar in the solution in the tank after 57 minutes.
\[ \text{concentration} = \] 

3. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31a.png
A tank contains 1240 L of pure water. A solution that contains 0.04 kg of sugar per liter enters tank at the rate 5 L/min. The solution is mixed and drains from the tank at the same rate.
(a) How much sugar is in the tank at the beginning.
\[ y(0) = \] (include units)
(b) With \( S \) representing the amount of sugar (in kg) at time \( t \) (in minutes) write a differential equation which models this situation.
\[ S' = f(t, S) = \] 

Note: Make sure you use a capital \( S \), (and don’t use \( S(t) \), it confuses the computer). Don’t enter units for this function.
(c) Find the amount of sugar (in kg) after \( t \) minutes.
\[ S(t) = \] (function of \( t \))
(d) Find the amount of the sugar after 69 minutes.
\[ S(69) = \] (include units)

Click here for help with units

4. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31b.png
A tank contains 2480 L of pure water. Solution that contains 0.05 kg of sugar per liter enters the tank at the rate 4 L/min, and is thoroughly mixed into it. The new solution drains out of the tank at the same rate.
(a) How much sugar is in the tank at the beginning?
\[ y(0) = \] (kg)
(b) Find the amount of sugar after \( t \) minutes.
\[ y(t) = \] (kg)
(c) As \( t \) becomes large, what value is \( y(t) \) approaching? In other words, calculate the following limit. 
\[ \lim_{t \to \infty} y(t) = \] (kg)

5. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31c.png
A tank contains 90 kg of salt and 2000 L of water. A solution of a concentration 0.045 kg of salt per liter enters a tank at the rate 10 L/min. The solution is mixed and drains from the tank at the same rate.
(a) What is the concentration of our solution in the tank initially? 
\[ \text{concentration} = \] (kg/L)
(b) Find the amount of salt in the tank after 1 hours.
\[ \text{amount} = \] (kg)
(c) Find the concentration of salt in the solution in the tank as time approaches infinity.
\[ \text{concentration} = \] (kg/L)

6. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31d.png
A tank contains 50 kg of salt and 2000 L of water. Pure water enters a tank at the rate 8 L/min. The solution is mixed and drains from the tank at the rate 4 L/min.
(a) What is the amount of salt in the tank initially?
\[ \text{amount} = \] (kg)
(b) Find the amount of salt in the tank after 4.5 hours.
\[ \text{amount} = \] (kg)
(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)
\[ \text{concentration} = \] (kg/L)

7. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_31c.png
A tank contains 2480 L of pure water. A solution that contains 0.08 kg of sugar per liter enters tank at the rate 4 L/min. The solution is mixed and drains from the tank at the same rate.
(a) How much sugar is in the tank at the beginning.
\[ y(0) = \] (include units)
(b) Find the amount of sugar (in kg) after \( t \) minutes.
\[ y(t) = \] (function of \( t \))
(b) Find the amount of the sugar after 54 minutes.
\[ y(54) = \] (include units)

8. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_5_2.png
A cell of some bacteria divides into two cells every 50 minutes. The initial population is 6 bacteria.
(a) Find the size of the population after \( t \) hours
\[ y(t) = \] (function of \( t \))
(b) Find the size of the population after 6 hours.
\[ y(6) = \] 
(c) When will the population reach 36?
\[ T = \] 

9. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_a_5_7.png
A cell of some bacteria divides into two cells every 40 minutes. The initial population is 200 bacteria.
(a) Find the population after \( t \) hours
\[ y(t) = \] (function of \( t \))
(b) Find the population after 2 hours.
10. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_5_3.png
A bacteria culture starts with 840 bacteria and grows at a rate proportional to its size. After 2 hours there will be 1680 bacteria.
(a) Express the population after $t$ hours as a function of $t$. 
(b) What will be the population after 8 hours?
(c) How long will it take for the population to reach 1810?

11. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_6_1.png
A population obeys the logistic model. It satisfies the equation \[
\frac{dP}{dt} = \frac{2}{900}P(9-P) \text{ for } P > 0.
\]
(a) The population is increasing when $\_\_\_ < P < \_\_\_$. 
(b) The population is decreasing when $P > \_\_\_$. 
(c) Assume that $P(0) = 2$. Find $P(65)$.

12. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_10.png
Suppose that a population develops according to the logistic equation
\[
\frac{dP}{dt} = 0.1P - 0.001P^2
\]
where $t$ is measured in weeks.
(a) What is the carrying capacity? 
(b) Is the solution increasing or decreasing when $P$ is between 0 and the carrying capacity? 
(c) Is the solution increasing or decreasing when $P$ is greater than the carrying capacity?

13. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_11.png
Biologists stocked a lake with 500 fish and estimated the carrying capacity (the maximal population for the fish of that species in that lake) to be 6000. The number of fish tripled in the first year.
(a) Assuming that the size of the fish population satisfies the logistic equation
\[
\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right),
\]
determine the constant $k$, and then solve the equation to find an expression for the size of the population after $t$ years.

14. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_12.png
Another model for a growth function for a limited population is given by the Gompertz function, which is a solution of the differential equation
\[
\frac{dP}{dt} = c \ln \left(\frac{K}{P}\right) P
\]
where $c$ is a constant and $K$ is the carrying capacity.
(a) Solve this differential equation for $c = 0.05, K = 3000$, and initial population $P_0 = 100$.
(b) Compute the limiting value of the size of the population. 
(c) At what value of $P$ does $P$ grow fastest?

15. (1 pt) setDiffEQ5ModelingWith1stOrder/ns7_5_10.png
An unknown radioactive element decays into non-radioactive substances. In 500 days the radioactivity of a sample decreases by 32 percent.
(a) What is the half-life of the element? 
(b) How long will it take for a sample of 100 mg to decay to 72 mg? 
(c) When will the population reach 400?

16. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_2.png
A body of mass 4 kg is projected vertically upward with an initial velocity 65 meters per second.
The gravitational constant is $g = 9.8 m/s^2$. The air resistance is equal to $k|v|$ where $k$ is a constant.
Find a formula for the velocity at any time (in terms of $k$): 
\[
v(t) = \_\_\_.
\]
Find the limit of this velocity for a fixed time $t_0$ as the air resistance coefficient $k$ goes to 0. (Enter $t_0$ as $t_0$.) 
\[
v(t_0) = \_\_\_.
\]
How does this compare with the solution to the equation for velocity when there is no air resistance?
This illustrates an important fact, related to the fundamental theorem of ODE and called 'continuous dependence' on parameters and initial conditions. What this means is that, for a fixed time, changing the initial conditions slightly, or changing the parameters slightly, only slightly changes the value at time $t$.
The fact that the terminal time $t$ under consideration is a fixed, finite number is important. If you consider 'infinite' $t$, or the 'final' result you may get very different answers. Consider for example a solution to $y' = y$, whose initial condition is essentially zero, but which might vary a bit positive or negative. If the initial condition is positive the "final" result is plus infinity, but if the initial condition is negative the final condition is negative infinity.

17. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_8.png
You have 700 dollars in your bank account. Suppose your money is compounded every month at a rate of 0.4 percent per month.
(a) How much do you have after $t$ years?
\[
y(t) = \_\_\_.
\]
(b) How much do you have after 70 months?

\[ y(70) = \] 

18. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_1.pg

A young person with no initial capital invests \( k \) dollars per year in a retirement account at an annual rate of return 0.08. Assume that investments are made continuously and that the return is compounded continuously.

Determine a formula for the sum \( S(t) \) – (this will involve the parameter \( k \)):

\[ S(t) = \] 

What value of \( k \) will provide 2814000 dollars in 49 years?

\[ k = \] 

19. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_3.pg

Here is a somewhat realistic example which combines the work on earlier problems. You should use the phase plane plotter to look at some solutions graphically before you start solving this problem and to compare with your analytic answers to help you find errors. You will probably be surprised to find how long it takes to get all of the details of solution of a realistic problem right, even when you know how to do each of the steps.

There are 2660 dollars in the bank account at the beginning of January 1990, and money is added and withdrawn from the account at a rate which follows a sinusoidal pattern, peaking in January and in July with money being added at a rate corresponding to 2070 dollars per year, while maximum withdrawals take place at the rate of 510 dollars per year in April and October.

The interest rate remains constant at the rate of 8 percent per year, compounded continuously.

Let \( y(t) \) represents the amount of money at time \( t \) (\( t \) is in years).

\[ y(0) = \] (dollars)

Write a formula for the rate of deposits and withdrawals (using the functions \( \sin(t) \), \( \cos(t) \) and constants):

\[ g(t) = \] 

The interest rate remains constant at 8 percent per year over this period of time.

With \( y \) representing the amount of money in dollars at time \( t \) (in years) write a differential equation which models this situation.

\[ y' = f(t; y) = \] 

**Note:** Use \( y \) rather than \( y(t) \) since the latter confuses the computer. Don’t enter units for this equation.

Find an equation for the amount of money in the account at time \( t \) where \( t \) is the number of years since January 1990.

\[ y(t) = \] 

Find the amount of money in the bank at the beginning of January 2000 (10 years later): 

\[ y(t) = \] 

Find a solution to the equation which does not become infinite (either positive or negative) over time:

\[ y(t) = \] 

During which months of the year does this non-growing solution have the highest values? 

20. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_13.pg

How long will it take an investment to triple in value if the interest rate is 5% compounded continuously?

Answer: \[ \] years.

21. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_4.pg

Newton’s law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton’s law of cooling. If the coffee has a temperature of 195 degrees Fahrenheit when freshly poured, and 2 minutes later has cooled to 180 degrees in a room at 70 degrees, determine when the coffee reaches a temperature of 155 degrees.

The coffee will reach a temperature of 155 degrees in \[ \] minutes.

22. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_14.pg

A thermometer is taken from a room where the temperature is 18°C to the outdoors, where the temperature is 2°C. After one minute the thermometer reads 13°C.

(a) What will the reading on the thermometer be after 4 more minutes?

(b) When will the thermometer read 3°C?

\[ \] minutes after it was taken to the outdoors.

23. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_5.pg

Susan finds an alien artifact in the desert, where there are temperature variations from a low in the 30s at night to a high in the 100s in the day. She is interested in how the artifact will respond to faster variations in temperature, so she kidnaps the artifact, takes it back to her lab (hotly pursued by the military police who patrol Area 51), and sticks it in an "oven" – that is, a closed box whose temperature she can control precisely.

Let \( T(t) \) be the temperature of the artifact. Newton’s law of cooling says that \( T(t) \) changes at a rate proportional to the difference between the temperature of the environment and the temperature of the artifact. This says that there is a constant \( k \), not dependent on time, such that \( T' = k(E - T) \), where \( E \) is the temperature of the environment (the oven).

Before collecting the artifact from the desert, Susan measured its temperature at a couple of times, and she has determined that for the alien artifact, \( k = 0.85 \).

Susan preheats her oven to 85 degrees Fahrenheit (she has stubbornly refused to join the metric world). At time \( t = 0 \) the oven is at exactly 85 degrees and is heating up, and the oven runs through a temperature cycle every 2\( \pi \) minutes, in which its temperature varies by 25 degrees above and 25 degrees below 85 degrees.

Let \( E(t) \) be the temperature of the oven after \( t \) minutes.

\[ E(t) = \] 

At time \( t = 0 \), when the artifact is at a temperature of 60 degrees, she puts it in the oven. Let \( T(t) \) be the temperature of the artifact at time \( t \). Then \( T(0) = \) (degrees)
Write a differential equation which models the temperature of the artifact.

\[ T' = f(t, T) = \] 

Note: Use \( T \) rather than \( T(t) \) since the latter confuses the computer. Don't enter units for this equation.

Solve the differential equation. To do this, you may find it helpful to know that if \( a \) is a constant, then

\[
\int \sin(t)e^{at} \, dt = \frac{1}{a^2+1} e^{at}(a \sin(t) - \cos(t)) + C.
\]

After Susan puts in the artifact in the oven, the military police break in and take her away. Think about what happens to her artifact as \( t \to \infty \) and fill in the following sentence:

For large values of \( t \), even though the oven temperature varies between 60 and 110 degrees, the artifact varies from ________ to ________ degrees.

24. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_6.pg

Here is a multipart example on finance. Be patient and careful as you work on this problem. You will probably be surprised to find how long it takes to get all of the details of solution of a realistic problem right, even when you know how to do each of the steps. Use the computer to check the steps for you as you go along. There is partial credit on this problem.

A recent college graduate borrows 95000 dollars at an (annual) interest rate of 6 per cent. Anticipating steady salary increases, the buyer expects to make payments at a monthly rate of \( 650(1 + t/130) \) dollars per month, where \( t \) is the number of months since the loan was made.

Let \( y(t) \) be the amount of money that the graduate owes \( t \) months after the loan is made.

\[ y(0) = \] (dollars)

With \( y \) representing the amount of money in dollars at time \( t \) (in months) write a differential equation which models this situation.

\[ y' = f(t, y) = \]

Note: Use \( y \) rather than \( y(t) \) since the latter confuses the computer. Don’t enter units for this equation.

Find an equation for the amount of money owed after \( t \) months.

\[ y(t) = \]

Next we are going to think about how many months it will take until the loan is paid off. Remember that \( y(t) \) is the amount that is owed after \( t \) months. The loan is paid off when \( y(t) = \) ________

Once you have calculated how many months it will take to pay off the loan, give your answer as a decimal, ignoring the fact that in real life there would be a whole number of months. To do this, you should use a graphing calculator or use a plotter on this page to estimate the root. If you use the

The loan will be paid off in ________ months.

If the borrower wanted the loan to be paid off in exactly 20 years, with the same payment plan as above, how much could be borrowed?

Borrowed amount =

25. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_15.pg

In the circuit shown in the figure above a battery supplies a constant voltage of \( E = 60 \text{V} \), the inductance is \( L = 2 \text{H} \), the resistance is \( R = 30 \Omega \), and \( I(0) = 0 \). Find the current after \( t \) seconds.

\[ I(t) = \]

26. (1 pt) setDiffEQ5ModelingWith1stOrder/ur_de_5_16.pg

In the circuit shown in the figure above a generator supplies a voltage of \( E(t) = 50 \sin(30t) \text{V} \), the inductance is \( L = 2 \text{H} \), the resistance is \( R = 10 \Omega \), and \( I(0) = 0 \). Find the current after \( t \) seconds.

\[ I(t) = \]
The figure above shows a circuit containing an electromotive force, a capacitor with a capacitance of $C$ farads (F), and a resistor with a resistance of $R$ ohms $\Omega$. The voltage drop across the capacitor is $Q/C$, where $Q$ is the charge (in coulombs), so in this case Kirchhoff’s Law gives

$$RI + \frac{Q}{C} = E(t).$$

Since $I = \frac{dQ}{dt}$, we have

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E(t).$$

Suppose the resistance is 10$\Omega$, the capacitance is 0.2F, a battery gives a constant voltage of 40V, and the initial charge is $Q(0) = 0$C. Find the charge and the current at time $t$.

$Q(t) = \underline{\phantom{0.000}}$

$I(t) = \underline{\phantom{0.000}}$

Let $P(t)$ be the performance level of someone learning a skill as a function of the training time $t$. The derivative $\frac{dP}{dt}$ represents the rate at which performance improves. If $M$ is the maximum level of performance of which the learner is capable, then a model for learning is given by the differential equation

$$\frac{dP}{dt} = k(M - P(t))$$

where $k$ is a positive constant.

Two new workers, Bill and Bob, were hired for an assembly line. Bill could process 11 units per minute after one hour and 15 units per minute after two hours. Bob could process 10 units per minute after one hour and 16 units per minute after two hours. Using the above model and assuming that $P(0) = 0$, estimate the maximum number of units per minute that each worker is capable of processing.

Bill: \underline{\phantom{0.000}}

Bob: \underline{\phantom{0.000}}
1. (1 pt) setDiffEQ6AutonomousStability/ur_de_6_1.pg

The graph of the function $f(x)$ is

[Graph of function $f(x)$]

Consider the differential equation $x'(t) = f(x(t))$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable.

2. (1 pt) setDiffEQ6AutonomousStability/ur_de_6_2.pg

The graph of the function $f(x)$ is

[Graph of function $f(x)$]

Given the differential equation $x'(t) = f(x(t))$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable.

3. (1 pt) setDiffEQ6AutonomousStability/ur_de_6_3.png

Given the differential equation $x' = -(x + 2.5) \cdot (x + 1)^3 \cdot (x - 0)^2 \cdot (x - 1)$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable. (It helps to sketch the graph. xFunctions will plot functions as well as phase planes.)

4. (1 pt) setDiffEQ6AutonomousStability/ur_de_6_4.png

Given the differential equation $x'(t) = x^4 + 1x^3 - 13x^2 - 1x + 12$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable. (It helps to sketch the graph. xFunctions will plot functions as well as phase planes.)
1. (1 pt) setDiffEQ7Exact/ur_de_7_1.pg
The following differential equation is exact. Find a function \( F(x,y) \) whose level curves are solutions to the differential equation
\[ ydy - xdx = 0 \]
\[ F(x,y) = \]

2. (1 pt) setDiffEQ7Exact/ur_de_7_2.pg
Use the "mixed partials" check to see if the following differential equation is exact. If it is exact find a function \( F(x,y) \) whose level curves are solutions to the differential equation
\[ (2x^2 - 1)ydx + (-1x - 2y^1)dy = 0 \]
\[ F(x,y) = \]

3. (1 pt) setDiffEQ7Exact/ur_de_7_3.pg
Use the "mixed partials" check to see if the following differential equation is exact. If it is exact find a function \( F(x,y) \) whose level curves are solutions to the differential equation
\[ (-4xy^2 - 2y)dx + (-4x^2y - 2x)dy = 0 \]
\[ F(x,y) = \]

4. (1 pt) setDiffEQ7Exact/ur_de_7_4.pg
Use the "mixed partials" check to see if the following differential equation is exact.
\[ \frac{dy}{dx} = +2x^2 - 4y \]
\[ \frac{dy}{dx} = \frac{4x - 3y^3}{4x^2 - 4y} \]
\[ F(x,y) = \]

5. (1 pt) setDiffEQ7Exact/ur_de_7_5.pg
Use the "mixed partials" check to see if the following differential equation is exact. If it is exact find a function \( F(x,y) \) whose level curves are solutions to the differential equation
\[ (1e^x\sin(y) - 3y)dx + (-3x + 1e^x\cos(y))dy = 0 \]
\[ F(x,y) = \]

6. (1 pt) setDiffEQ7Exact/ur_de_7_6.pg
Check that the equation below is not exact but becomes exact when multiplied by the integrating factor.
\[ x^2y^3 + x(1 + y^2)y' = 0 \]
Integrating factor: \( \mu(x,y) = 1/(xy^3) \).
Solve the differential equation. You can define the solution curve implicitly by a function in the form
\[ F(x,y) = G(x) + H(y) = K \]
\[ F(x,y) = \]

7. (1 pt) setDiffEQ7Exact/ur_de_7_7.pg
Find an explicit or implicit solutions to the differential equation
\[ (x^2 + 4xy)dx + xdy = 0 \]
\[ F(x,y) = \]
1. (1 pt) setDiffEQ8FundTheorem/ur_de_8_1.pg

This problem involves using the uniqueness property (from the Fundamental Theorem of ordinary differential equations.) It can’t be graded by WeBWorK, but is to be handed in at the first class after the due date.

A. State the uniqueness property of the fundamental theorem.

B. Show directly using the differential equation, that if \( y_1(t) \) is a solution to the differential equation \( y'(t) = y(t) \), then \( y_2(t) = y_1(t + a) \) is also a solution to the differential equation. (You will need to use the known facts about \( y_1 \) to calculate that \( y'_2(t) = y_2(t) \). (We know that the solution is the exponential function, but you will not need to use this fact.)

C. Describe the relationship between the graphs of \( y_1 \) and \( y_2 \) and using a sketch of the direction field explain why it is obvious that if \( y_1 \) is a solution then \( y_2 \) has to be a solution also.

D. Describe in words why if \( y_1(t) \) is any solution to the differential equation \( y' = f(y) \) then \( y_2(t) = y_1(t + a) \) is also a solution.

E. Show that if \( y_1(t) \) solves \( y'(t) = y(t) \), then \( y_2(t) = Ay_1(t) \) also solves the same equation.

F. Suppose that \( y_1(t) \) solves \( y'(t) = y(t) \) and \( y(0) = 1 \). (Such a solution is guaranteed by the fundamental theorem.) Let \( y_2(t) = y_1(t + a) \) and let \( y_3(t) = y_1(a)y_1(t) \). Calculate the values \( y_2(0) \) and \( y_3(0) \). Use the uniqueness property to show that \( y_2(t) = y_3(t) \) for all \( t \).

G. Explain how this proves that any solution to \( y' = y \) must be a function which obeys the law of exponents.

H. Let \( z = x + iy \). Define \( \exp(z) \) (or \( e^z \)) using a Taylor series. Show that if \( z = x + iy \) is a constant, then

\[
\frac{d}{dt} \exp(tz) = z \exp(tz)
\]

by differentiating the power series.

I. Use your earlier results to show that \( \exp(z + w) = \exp(z) \exp(w) \). This method of checking the law of exponents is MUCH easier than expanding the power series.

You can find a direction field plotter [here](url), or at the [direction field plotter page](url). Choose "integral curves utility" from the "main screen" menu of xFunctions to get to the phaseplane plotter.

2. (1 pt) setDiffEQ8FundTheorem/ur_de_8_2.pg

This problem involves using the uniqueness property (from the Fundamental Theorem of ordinary differential equations.) It can’t be graded by WeBWorK, but is to be handed in at the first class after the due date.

A. Using the same technique as in the previous problem show that if a function \( y_1(t) \) satisfies: (1) \( y_1(0) = 1 \) and (2) \( y'(t) = y(t) \) then

\[
(y_1(t))' = y_1(\pi t)
\]

B. Explain in words how this relates to another law of exponents.

You can find a direction field plotter [here](url), or at the [direction field plotter page](url). Choose "integral curves utility" from the "main screen" menu of xFunctions to get to the phaseplane plotter.

3. (1 pt) setDiffEQ8FundTheorem/ur_de_8_3.pg

This problem involves using the uniqueness property (from the Fundamental Theorem of ordinary differential equations.) It can’t be graded by WeBWorK, but is to be handed in at the first class after the due date.

Use the same ideas as in the previous problems.
A. Suppose that $y_1(t)$ satisfies the equation $y'' + y = 0$ and $y_1(0) = 0$ and $y_1'(0) = 1$. Such a function exists because of the fundamental theorem. (We all know that it is $\sin(t)$, but you should not use that fact in answering the questions below.) Show that $y_2(t) = y_1'(t)$ also satisfies the equation $y'' + y = 0$ and that $y_2(0) = 1$ and $y_2'(0) = 0$.

B. If $y_3(t) = y_2'(t)$ show, using the uniqueness property, that $y_3(t) = -y_1(t)$

C. State the uniqueness property for solutions to second order differential equations (or equivalently to a system of two first order differential equations).

D. Use the uniqueness property to show that $y_1(t + a) = y_1'(a)y_1(t) + y_1(a)y_2(t) = y_2(a)y_1(t) + y_1(a)y_2(t)$

The formulas for the sin of sums of angles can be calculated completely from the one fact that it satisfies a differential equation. This is a general fact. Any solution of a differential equation has the potential for obeying certain "laws" which are dictated by the differential equation.
1. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_1.pg
Find $y$ as a function of $t$ if
$$10000y'' - 729y = 0,$$
y(0) = 8, \quad y'(0) = 3.
y(t) = \_____________________

2. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_6.pg
Find $y$ as a function of $t$ if
$$40000y'' - 9y = 0$$
with $y(0) = 3, \quad y'(0) = 1.$
y(t) = \_____________________

3. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_8.pg
Find $y$ as a function of $t$ if
$$25y'' + 81y = 0,$$
y(0) = 4, \quad y'(0) = 4.
y(t) = \_____________________

4. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_2.pg
Find $y$ as a function of $t$ if
$$y'' - y' = 0,$$
y(0) = 6, \quad y(1) = 9.
y(t) = \_____________________
Remark: The initial conditions involve values at two points.

5. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_7.pg
Find $y$ as a function of $t$ if
$$64y'' - 16y' + 5y = 0,$$
y(0) = 7, \quad y'(0) = 4.
y(t) = \_____________________

6. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_3.pg
Find $y$ as a function of $t$ if
$$8y'' + 31y = 0,$$
y(0) = 8, \quad y'(0) = 9.
y(t) = \_____________________
Note: This particular weBWorK problem can’t handle complex numbers, so write your answer in terms of sines and cosines, rather than using $e$ to a complex power.

7. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_4.pg
Find $y$ as a function of $t$ if
$$49y'' + 42y' + 11y = 0,$$
y(0) = 2, \quad y'(0) = 8.
y(t) = \_____________________
Note: This problem cannot interpret complex numbers. You may need to simplify your answer before submitting it.

8. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_5.pg
Find $y$ as a function of $t$ if
$$100y'' - 13y' + y = 0,$$
y(0) = 8, \quad y'(0) = 7.
y(t) = \_____________________

Note: This problem cannot interpret complex numbers. You may need to simplify your answer before submitting it.

9. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_11.pg
Find $y$ as a function of $t$ if
$$y'' + 14y' + 113y = 0,$$
y(0) = 6, \quad y'(0) = 7.$$
y(t) = \_____________________

10. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_9.pg
Find $y$ as a function of $t$ if
$$81y'' - 90y' + 34y = 0,$$
y(0) = 1, \quad y'(0) = 2.$$
y(t) = \_____________________
Note: This problem cannot interpret complex numbers. You may need to simplify your answer before submitting it.

11. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_10.pg
Find the function $y_1$ of $t$ which is the solution of
$$64y'' + 96y' + 20y = 0$$
with initial conditions $y_1(0) = 1, \quad y_1'(0) = 0.$
y_1 = \_____________________

Find the function $y_2$ of $t$ which is the solution of
$$64y'' + 96y' + 20y = 0$$
with initial conditions $y_2(0) = 0, \quad y_2'(0) = 1.$
y_2 = \_____________________

Find the Wronskian
$$W(t) = W(y_1, y_2).$$
$W(t) =$ \_____________________
Remark: You can find $W$ by direct computation and use Abel’s theorem as a check. You should find that $W$ is not zero and so $y_1$ and $y_2$ form a fundamental set of solutions of
$$64y'' + 96y' + 20y = 0.$$

12. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_12.pg
Find $y$ as a function of $t$ if
$$5184y'' + 1872y' + 169y = 0,$$
y(0) = 9, \quad y'(0) = 6.$$
y(t) = \_____________________

13. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_13.pg
Find $y$ as a function of $t$ if
$$81y'' + 18y' - 80y = 0,$$
y(1) = 5, \quad y'(1) = 2.$$
y(t) = \_____________________

14. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_de_9_14.pg
Determine whether the following pairs of functions are linearly independent or not.
1. \( f(\theta) = 2\cos3\theta \) and \( g(\theta) = 8\cos^3\theta - 6\cos\theta \)
2. \( f(t) = t \) and \( g(t) = |t| \)
3. The Wronskian of two functions is \( W(t) = t \) are the functions linearly independent or dependent?

15. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_15.pg
Suppose that the Wronskian of two functions \( f_1(t) \) and \( f_2(t) \) is given by
\[
W(t) = t^2 - 4 = \det \begin{pmatrix} f_1(t) & f_2(t) \\ f_1'(t) & f_2'(t) \end{pmatrix}
\]
Even though you don’t know the functions \( f_1 \) and \( f_2 \) you can determine whether the following questions are true or false.

1. The vectors \((f_1(4), f_1'(4))\) and \((f_2(4), f_2'(4))\) are linearly independent
2. The vectors \((f_1(-2), f_1'(-2))\) and \((f_2(-2), f_2'(-2))\) are linearly independent
3. The equations
   \[
   \begin{align*}
   af_1(0) + bf_2(0) &= c \\
   af_1'(0) + bf_2'(0) &= d
   \end{align*}
   
   have a unique solution for any \( c \) and \( d \)
4. The equations
   \[
   \begin{align*}
   af_1(2) + bf_2(2) &= 0 \\
   af_1'(2) + bf_2'(2) &= 0
   \end{align*}
   
   have more than one solution.
5. The vectors \((f_1(0), f_1'(0))\) and \((f_2(0), f_2'(0))\) are linearly independent

16. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_16.pg
Determine which of the following pairs of functions are linearly independent.

17. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_17.pg
Match the second order linear equations with the Wronskian of (one of) their fundamental solution sets.

\[
\begin{align*}
1. & \quad y'' - \frac{2}{4}y' + 5y = 0 \\
2. & \quad y'' - \frac{3}{4}y' + 5y = 0, \quad t > 0 \\
3. & \quad y'' - \cos(t)y' + 5y = 0 \\
4. & \quad y'' + \frac{3}{2}y' + 5y = 0 \\
5. & \quad y'' + 4y' + 5y = 0
\end{align*}
\]
   - A. \( W(t) = 7t \)
   - B. \( W(t) = \frac{5}{7} \)
   - C. \( W(t) = t^2 \)
   - D. \( W(t) = 2e^{-3t} \)
   - E. \( W(t) = e^{\sin(t)} \)

18. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_18.pg
Find \( y \) as a function of \( x \) if
\[
x^2y'' - 2xy' - 28y = 0,
\]
\( y(1) = -6, \quad y'(1) = -4 \).
\( y = \quad \)

19. (1 pt) setDiffEQ9Linear2ndOrderHomog/ur_19.pg
Find \( y \) as a function of \( x \) if
\[
x^2y'' - 13xy' + 49y = 0,
\]
\( y(1) = 7, \quad y'(1) = 5 \).
\( y = \quad \)
1. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_4.pg
Find a single solution of $y''$ if $y'' = 2$.

\[ y = \quad \] 

2. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_5.pg
Use the method of undetermined coefficients to find one solution of

\[ y'' - y' - 5y = 3e^{3t}. \]

It doesn’t matter which specific solution you find for this problem.

\[ y = \quad \] 

3. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_13.pg
Use the method of undetermined coefficients to find one solution of

\[ y'' + 4y' - 7y = (-3t^2 + 1t + 6)e^{4t}. \]

Note that the method finds a specific solution, not the general one.

\[ y = \quad \] 

4. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_6.pg
Use the method of undetermined coefficients to find one solution of

\[ y'' - 6y' + 16y = 32e^{3t}\cos(3t) + 48e^{3t}\sin(3t) + 4e^{2t}. \]

(It doesn’t matter which specific solution you find for this problem.)

\[ y = \quad \] 

5. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_7.pg
Use the method of undetermined coefficients to find one solution of

\[ y'' + 2y' + 2y = (10t + 7)e^{-t}\cos(t) + (11t + 25)e^{-t}\sin(t). \]

(It doesn’t matter which specific solution you find for this problem.)

\[ y = \quad \] 

6. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_11.pg
Find a particular solution to the differential equation

\[ y'' - 2y' - 3y = -18t^3. \]

\[ y_p = \quad \] 

7. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_10.pg
Find a particular solution to

\[ y'' + 8y' + 16y = 10.5e^{-4t}. \]

\[ y_p = \quad \] 

8. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_8.pg
Find a particular solution to the differential equation

\[ 3y'' + 2y' - 1y = -2t^2 - 2t - 3e^{4t}. \]

\[ y_p = \quad \] 

9. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_12.pg
Find a particular solution to

\[ y'' + 6y' + 8y = 2te^{5t}. \]

\[ y_p = \quad \] 

10. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_9.pg
Find a particular solution to

\[ y'' + 4y = 16\sin(2t). \]

\[ y_p = \quad \] 

11. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_1.pg
Find the solution of

\[ y'' - 4y' - 5y = 27e^{8t} \]

with \( y(0) = 9 \) and \( y'(0) = 2 \).

\[ y = \quad \] 

12. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_2.pg
Find the solution of

\[ y'' - 10y' + 25y = 81e^{8t} \]

with \( y(0) = 8 \) and \( y'(0) = 3 \).

\[ y = \quad \] 

13. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_3.pg
Find the solution of

\[ y'' + 9y' = 648\sin(9t) + 486\cos(9t) \]

with \( y(0) = 5 \) and \( y'(0) = 5 \).

\[ y = \quad \] 

14. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_14.pg
Find \( y \) as a function of \( x \) if

\[ x^2y'' + 6xy' - 36y = x^7, \]

\( y(1) = 5,\ \ y'(1) = -3 \).

\[ y = \quad \] 

15. (1 pt) setDiffEQ10Linear2ndOrderNonhom/ur_de_10_15.pg
Find \( y \) as a function of \( x \) if

\[ x^2y'' - 15xy' + 64y = x^7, \]

\( y(1) = -2,\ \ y'(1) = 2 \).

\[ y = \quad \]
1. (1 pt) setDiffEQ11ModelingWith2ndOrder/ur_de_11_1.pg

Another "realistic" problem:
The following problem is similar to the problem in an earlier assignment about the bank account growing with periodic deposits. The basic procedure for this problem is not too hard, but getting details of the calculation correct is NOT easy, and may take some time.

A ping-pong ball is caught in a vertical plexiglass column in which the air flow alternates sinusoidally with a period of 60 seconds. The air flow starts with a maximum upward flow at the rate of $5.6 \text{ m/s}$ and at $t = 30$ seconds the flow has a minimum (upward) flow of rate of $-3 \text{ m/s}$. (To make this clear: a flow of $-5 \text{ m/s}$ upward is the same as a flow downward of $5 \text{ m/s}$.

The ping-pong ball is subjected to the forces of gravity ($-mg$) where $g = 9.8 \text{ m/s}^2$ and forces due to air resistance which are equal to $k$ times the apparent velocity of the ball through the air.

What is the average velocity of the air flow? You can average the velocity over one period or over a very long time – the answer should come out about the same – right? (Include units.)

Write a formula for the velocity of the air flow as a function of time.

$A(t) =$ ________________

Write the differential equation satisfied by the velocity of the ping-pong ball (relative to the fixed frame of the plexiglass tube.) The formulas should not have units entered, but use units to troubleshoot your answers. Your answer can include the parameters $m$ - the mass of the ball and $k$ the coefficient of air resistance, as well as time $t$ and the velocity of the ball $v$. (Use just $v$, not $v(t)$ the latter confuses the computer.)

$v'(t) =$ ________________

Calculate the specific solution that has initial conditions $t = 0$ and $w(0) = 2.8$.
$w(t) =$

Think about what effect increasing the mass has on the amplitude, on the phase shift? Does this correspond with your expectations?

2. (1 pt) setDiffEQ11ModelingWith2ndOrder/ur_de_11_2.pg

A steel ball weighing 128 pounds is suspended from a spring. This stretches the spring $\frac{128}{65}$ feet.

The ball is started in motion from the equilibrium position with a downward velocity of 6 feet per second. The air resistance (in pounds) of the moving ball numerically equals 4 times its velocity (in feet per second).

Suppose that after $t$ seconds the ball is $y$ feet below its rest position. Find $y$ in terms of $t$. (Note that this means that the positive direction for $y$ is down.)

$y =$

Take as the gravitational acceleration 32 feet per second per second.

3. (1 pt) setDiffEQ11ModelingWith2ndOrder/ur_de_11_3.pg

A hollow steel ball weighing 4 pounds is suspended from a spring. This stretches the spring $\frac{1}{7}$ feet.

The ball is started in motion from the equilibrium position with a downward velocity of 6 feet per second. The air resistance (in pounds) of the moving ball numerically equals 4 times its velocity (in feet per second).

Suppose that after $t$ seconds the ball is $y$ feet below its rest position. Find $y$ in terms of $t$. (Note that the positive direction is down.)

Take as the gravitational acceleration 32 feet per second per second.

$y =$

4. (1 pt) setDiffEQ11ModelingWith2ndOrder/ur_de_11_4.pg

This problem is an example of critically damped harmonic motion.

A hollow steel ball weighing 4 pounds is suspended from a spring. This stretches the spring $\frac{1}{8}$ feet.

The ball is started in motion from the equilibrium position with a downward velocity of 5 feet per second. The air resistance (in pounds) of the moving ball numerically equals 4 times its velocity (in feet per second). Suppose that after $t$ seconds the ball is $y$ feet below its rest position. Find $y$ in terms of $t$. 

$y =$

Take as the gravitational acceleration 32 feet per second per second.
Take as the gravitational acceleration 32 feet per second per second. (Note that the positive y direction is down in this problem.)

\[ y = \]
Match the third order linear equations with their fundamental solution sets.

1. \( y''' - y'' - y' + y = 0 \)
2. \( y''' - 6y'' + 8y' = 0 \)
3. \( y''' + y' = 0 \)
4. \( y''' + 3y'' + 3y' + y = 0 \)
5. \( y''' - 6y'' + y' - 6y = 0 \)
6. \( ty''' - y'' = 0 \)

- A. 1, \( t, t^3 \)
- B. 1, \( e^{2t}, e^{kt} \)
- C. 1, \( \cos(t), \sin(t) \)
- D. \( e^t, \cos(t), \sin(t) \)
- E. \( e^t, te^t, e^{-t} \)
- F. \( e^{-t}, te^{-t}, t^2e^{-t} \)

Find \( y \) as a function of \( x \) if

\[ y''' - 12y'' + 32y' = 0, \]
\( y(0) = 7, \ y'(0) = 5, \ y''(0) = 6 \)
\( y(x) = \)

Find \( y \) as a function of \( x \) if

\[ y''' + 4y' = 0, \]
\( y(0) = 8, \ y'(0) = 8, \ y''(0) = 4 \)
\( y(x) = \)

Find \( y \) as a function of \( x \) if

\[ y^{(4)} - 10y''' + 25y'' = 0, \]
\( y(0) = 8, \ y'(0) = 12, \ y''(0) = 33, \ y'''(0) = -16 \)
\( y(x) = \)
1. (1 pt) setDiffEQ13Systems1stOrder/ur_de_13_1.pg
Write the given second order equation as its equivalent system of first order equations.

\[ u'' + 3u' + 8u = 0 \]

Use \( v \) to represent the "velocity function", i.e. \( v = u'(t) \).

Use \( v \) and \( u \) for the two functions, rather than \( u(t) \) and \( v(t) \). (The latter confuses webwork. Functions like \( \sin(\cdot) \) are ok.)

\[ \begin{align*}
u' &= \\v' &=
\end{align*} \]

Now write the system using matrices:

\[ \frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 3 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 8 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \]

2. (1 pt) setDiffEQ13Systems1stOrder/ur_de_13_2.pg
Write the given second order equation as its equivalent system of first order equations.

\[ u'' - 7u' - 3.5u = -8 \sin(3t), \quad u(1) = 2, \quad u'(1) = 3.5 \]

Use \( v \) to represent the "velocity function", i.e. \( v = u'(t) \).

Use \( v \) and \( u \) for the two functions, rather than \( u(t) \) and \( v(t) \). (The latter confuses webwork. Functions like \( \sin(\cdot) \) are ok.)

\[ \begin{align*}
u' &= \\v' &=
\end{align*} \]

Now write the system using matrices:

\[ \frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} -7 \\ -3.5 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} -8 \sin(3t) \end{pmatrix} \]

and the initial value for the vector valued function is:

\[ \begin{pmatrix} u(1) \\ v(1) \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \]

3. (1 pt) setDiffEQ13Systems1stOrder/ur_de_13_3.pg
Write the given second order equation as its equivalent system of first order equations.

\[ r^2u'' - 3ru' + (r^2 + 7)u = 7 \sin(3t) \]

Use \( v \) to represent the "velocity function", i.e. \( v = u'(t) \).

Use \( v \) and \( u \) for the two functions, rather than \( u(t) \) and \( v(t) \). (The latter confuses webwork. Functions like \( \sin(\cdot) \) are ok.)

\[ \begin{align*}
u' &= \\v' &=
\end{align*} \]

Now write the system using matrices:

\[ \frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} -3r \\ r^2 + 7 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 7 \sin(3t) \end{pmatrix} \]

4. (1 pt) setDiffEQ13Systems1stOrder/ur_de_13_4.pg
This problem is similar to problem 21 on page 346. Consult that page for the diagram. You will probably want to write the solution out first, before trying to enter the answers into the computer.

Consider two interconnected tanks as shown in Fig 7.1.6 on page 347. Tank 1 initial contains 80 L (liters) of water and 245 g of salt, while tank 2 initially contains 20 L of water and 255 g of salt. Water containing 10 g/L of salt is poured into tank 1 at a rate of 2 L/min while the mixture flowing into tank 2 contains a salt concentration of 40 g/L of salt and is flowing at the rate of 4 L/min. The two connecting tubes have a flow rate of 5.5 L/min from tank 1 to tank 2; and of 3.5 L/min from tank 2 back to tank 1. Tank 2 is drained at the rate of 6 L/min.

You may assume that the solutions in each tank are thoroughly mixed so that the concentration of the mixture leaving any tank along any of the tubes has the same concentration of salt as the tank as a whole. (This is not completely realistic, but as in real physics, we are going to work with the approximate, rather than exact description. The 'real' equations of physics are often too complicated to even write down precisely, much less solve.)

How does the water in each tank change over time?

Let \( p(t) \) and \( q(t) \) be the amount of salt in g at time t in tanks 1 and 2 respectively. Write differential equations for \( p \) and \( q \).

As usual, use the symbols \( p \) and \( q \) rather than \( p(t) \) and \( q(t) \).

\[ \begin{align*}
p' &= \\
q' &=
\end{align*} \]

Give the initial values:

\[ \begin{pmatrix} p(0) \\ q(0) \end{pmatrix} = \begin{pmatrix} 245 \\ 255 \end{pmatrix} \]

Show the equation that needs to be solved to find a constant solution to the differential equation:

\[ \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} \]

A constant solution is obtained if \( p(t) = \) _____ for all time \( t \) and \( q(t) = \) _____ for all time \( t \).

5. (1 pt) setDiffEQ13Systems1stOrder/ur_de_13_5.pg
Match the differential equations and their matrix function solutions:

It's good practice to multiply at least one matrix solution out fully, to make sure that you know how to do it, but you can get the other answers quickly by process of elimination and just multiply out one row or one column.

### 1.
\[ \begin{pmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{pmatrix} \begin{pmatrix} y(t) \end{pmatrix} \]

### 2.
\[ \begin{pmatrix} -97 & 33 & -5 \\ -140 & 84 & 35 \\ -4 & 15 & -8 \end{pmatrix} \begin{pmatrix} y(t) \end{pmatrix} \]

### 3.
\[ \begin{pmatrix} -86 & 218 & -160 \\ 73 & -49 & 80 \\ 111 & -138 & 165 \end{pmatrix} \begin{pmatrix} y(t) \end{pmatrix} \]
Match the differential equations and their vector valued function solutions:

It will be good practice to multiply at least one solution out fully, to make sure that you know how to do it, but you can get the other answers quickly by process of elimination and just multiply out one row element.

1. \( y'(t) = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} y(t) \)

2. \( y'(t) = \begin{pmatrix} 4 & 20 & -15 \\ 15 & 0 & 0 \\ 4 & 30 & -25 \end{pmatrix} y(t) \)

3. \( y'(t) = \begin{pmatrix} 2 & 13 & -8 \\ 2 & -1 & 3 \\ 14 & 0 & 0 \end{pmatrix} y(t) \)

A. \( y(t) = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} e^{13t} \)

B. \( y(t) = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} e^{-2t} \)

C. \( y(t) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{5t} \)

If \( y' = Ay \) is a differential equation, how would the solution curves behave?

- A. All of the solutions curves would converge towards 0. (Stable node)
- B. All of the solutions curves would run away from 0. (Unstable node)
- C. The solution curves converge to different points.
- D. The solution curves would race towards zero and then veer away towards infinity. (Saddle)

Calculate the eigenvalues of this matrix:

[Note– you’ll probably want to use a graphing calculator to estimate the roots of the polynomial which defines the eigenvalues. You can use the web version at xFunctions.

If you select the “integral curves utility” from the main menu, will also be able to plot the integral curves of the associated differential equations.]

A = \begin{pmatrix} -80 & 0 \\ 0 & -80 \end{pmatrix}

smaller eigenvalue = __________ associated eigenvector = ( __________ , __________ )
larger eigenvalue = __________ associated, eigenvector = ( __________ , __________ )

If \( y' = Ay \) is a differential equation, how would the solution curves behave?

- A. All of the solutions curves would converge towards 0. (Stable node)
- B. The solution curves converge to different points.
- C. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
- D. All of the solution curves would run away from 0. (Unstable node)
1. If Tom Bombadil’s house is 16 miles east of Hobbiton and 12 miles south, what is the straight line distance (omit units)?

2. If the distance from the town of Bree to Weathertop is 8 miles on a 45 degree upward slope, what is the elevation gain (omit units)?

3. Frodo and Sam are studying a topographic map of Mordor. Place the letter describing contour lines on a map to the left of the number describing a possible goal.
   1. If Frodo and Sam want to find the River Anduin, they should look for:
   2. If Frodo and Sam want to go directly uphill, they should go:
   3. If Frodo and Sam want to find a level route, they should look at:
   4. If Frodo and Sam want to find Mount Doom, they should look for:
      A. Parallel contour lines
      B. Perpendicular to the contour lines
      C. Concentric contour lines
      D. Single contour lines

4. The nine Ring Wraiths want to fly from Barad-Dur to Rivendell. Rivendell is directly north of Barad-Dur. The Dark Tower reports that the wind is coming from the west at 69 miles per hour. In order to travel in a straight line, the Ring Wraiths decide to head northwest. At what speed should they fly (omit units)?

5. As Aragorn views the Dark Lord in a crystal ball of radius 1, he realizes that:
   The surface area of the ball equals:
   The volume of the ball equals:

6. As Gandalf falls into the depths of Moria, he begins to spin. If he wishes to slow his rate of spinning, he should do which of the following (type the appropriate letter)?
   A. Wiggle His Nose
   B. Spread his arms wide
   C. Think of Sauron
   D. Think of Galadriel
   E. Hug Himself

7. The population of Elves in Lorien is constant. If five Elves per day cross outward over the boundary of Lorien, and none ever return, then we can conclude that Elves in Lorien are being:
   A. Attracted by the promise of a better life
   B. Persecuted
   C. Born
   D. Anti-social

8. In the land of Mordor, where the shadows lie, it is afternoon. In which direction do the shadows point? You may assume that the Earth’s axis of rotation is perpendicular to its plane of revolution about the sun. Type N, S, W, or E as appropriate.
   Type C, D, or I depending on whether the lengths of the shadows are constant, decreasing, or increasing.

9. Two dwarves decide to bore a tunnel through the center of the earth, connecting the mines of Moria with its antipode. They each have identical drills. One dwarf begins drilling from Moria and the other dwarf begins drilling from the antipode. When they meet at the center of the earth, are their two drills turning in the same direction? Type Y if yes, N if no.
What is the distance from the point (2, 8, 8) to the xz-plane?
Distance =

What do the following equations represent in $\mathbb{R}^3$?
Match the two sets of letters:

- a. a vertical plane
- b. a horizontal plane
- c. a plane which is neither vertical nor horizontal

A. $-7x + 5y = 5$
B. $x = -1$
C. $y = -1$
D. $z = -6$

Find an equation of the sphere with center (-5, 1, -1) and radius 2.

Find an equation of the sphere that passes through the origin and whose center is (2, -3, -8).

Find the center and radius of the sphere $x^2 + 20x + y^2 + 18y + z^2 + 8z = -133$
Center: (__, __, __)
Radius:

Find the equation of a sphere if one of its diameters has endpoints: (1, -4, 5) and (5, 0, 9).

Find an equation of the largest sphere with center (5, 7, 10) that is contained completely in the first octant.
1. (1 pt) setVectors2DotProduct/UR_VC_1_8.pg

Let \( \mathbf{a} = (3, 8, 4) \) and \( \mathbf{b} = (-6, 0, 1) \) be vectors. Compute the following vectors.

A. \( \mathbf{a} + \mathbf{b} = (___, ___ , ___) \)
B. \(-4\mathbf{a} = (___, ___ , ___) \)
C. \( \mathbf{a} - \mathbf{b} = (___, ___ , ___) \)
D. \( |\mathbf{a}| = ___ \)

2. (1 pt) setVectors2DotProduct/UR_VC_1_9.pg

A child walks due east on the deck of a ship at 5 miles per hour. The ship is moving north at a speed of 7 miles per hour.
Find the speed and direction of the child relative to the surface of the water.
Speed = ___ mph
The angle of the direction from the north = ___ (radians)

3. (1 pt) setVectors2DotProduct/UR_VC_1_10.pg

A horizontal clothesline is tied between 2 poles, 12 meters apart. When a mass of 5 kilograms is tied to the middle of the clothesline, it sags a distance of 3 meters.
What is the magnitude of the tension on the ends of the clothesline?
Tension = ___ N

4. (1 pt) setVectors2DotProduct/UR_VC_1_11.pg

Find \( \mathbf{a} \cdot \mathbf{b} \) if
\[ |\mathbf{a}| = 10, \]
\[ |\mathbf{b}| = 8, \]
and the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \frac{\pi}{3} \) radians.
\[ \mathbf{a} \cdot \mathbf{b} = ___ \]

5. (1 pt) setVectors2DotProduct/UR_VC_1_12.pg

If \( \mathbf{a} = (1, -6, -1) \) and \( \mathbf{b} = (2, -8, 7) \), find \( \mathbf{a} \cdot \mathbf{b} = ___ \)

6. (1 pt) setVectors2DotProduct/UR_VC_1_13.pg

What is the angle in radians between the vectors \( \mathbf{a} = (-3, -8, -3) \) and \( \mathbf{b} = (8, -3, 8) \)?
Angle: ___ (radians)

7. (1 pt) setVectors2DotProduct/UR_VC_1_14.pg

Find a unit vector in the same direction as \( \mathbf{a} = (4, 9, 7) \).

8. (1 pt) setVectors2DotProduct/UR_VC_1_15.pg

Let \( \mathbf{a} = (2, 8, -3) \) and \( \mathbf{b} = (-7, -3, -9) \) be vectors. Find the scalar, vector, and orthogonal projections of \( \mathbf{b} \) onto \( \mathbf{a} \).
Scalar Projection: ___
Vector Projection: ___
Orthogonal Projection: ___

9. (1 pt) setVectors2DotProduct/UR_VC_1_16.pg

A constant force \( \mathbf{F} = 4\mathbf{i} - 10\mathbf{j} + 6\mathbf{k} \) moves an object along a straight line from point \((8, 8, -1)\) to point \((7, -4, 4)\).
Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.
Work: ___ Nm

10. (1 pt) setVectors2DotProduct/UR_VC_1_17.pg

A woman exerts a horizontal force of 7 pounds on a box as she pushes it up a ramp that is 4 feet long and inclined at an angle of 30 degrees above the horizontal.
Find the work done on the box.
Work: ___ ft-lb

11. (1 pt) setVectors2DotProduct/UR_VC_1_18.pg

Gandalf the Grey started in the Forest of Mirkwood at a point with coordinates \((-3, -3)\) and arrived in the Iron Hills at the point \((-2, 2)\). If he began walking in the direction of the vector \( \mathbf{v} = 5\mathbf{i} + 2\mathbf{j} \) and changes direction only once, when he turns at a right angle, what are the coordinates of the point where he makes the turn.
\((___, ___)\)

12. (1 pt) setVectors2DotProduct/UR_VC_1_F.pg

If Yoda says to Luke Skywalker, “The Force be with you,” then the dot product of the Force and Luke should be:
- A. positive
- B. negative
- C. zero
- D. any real number
1. (1 pt) setVectors3CrossProduct/ur_vc_2_2_1.pg
You are looking down at a map. A vector \( \mathbf{u} \) with \( |\mathbf{u}| = 10 \) points north and a vector \( \mathbf{v} \) with \( |\mathbf{v}| = 3 \) points northeast. The cross-product \( \mathbf{u} \times \mathbf{v} \) points:
   A) south
   B) northwest
   C) up
   D) down
Please enter the letter of the correct answer: ___
The magnitude \( |\mathbf{u} \times \mathbf{v}| = \) ______

2. (1 pt) setVectors3CrossProduct/ur_vc_2_2.pg
Let \( \mathbf{a} = (4, 4, 1) \) and \( \mathbf{b} = (9, 7, 9) \) be vectors.
Compute the cross product \( \mathbf{a} \times \mathbf{b} \). (____ ____ ___)

3. (1 pt) setVectors3CrossProduct/ur_vc_2_3.pg
If \( \mathbf{a} = \mathbf{i} + \mathbf{j} + 1\mathbf{k} \) and \( \mathbf{b} = \mathbf{i} + \mathbf{j} + 3\mathbf{k} \)
Compute the cross product \( \mathbf{a} \times \mathbf{b} \).
   ___ \( \mathbf{i} + ___ \mathbf{j} + ___ \mathbf{k} 

4. (1 pt) setVectors3CrossProduct/ur_vc_2_4.pg
If \( \mathbf{a} = \mathbf{i} + 4\mathbf{j} + \mathbf{k} \) and \( \mathbf{b} = \mathbf{i} + 6\mathbf{j} + \mathbf{k} \), find a unit vector with positive
first coordinate orthogonal to both \( \mathbf{a} \) and \( \mathbf{b} \).
   ______ \( \mathbf{i} + ______ \mathbf{j} + ______ \mathbf{k} 

5. (1 pt) setVectors3CrossProduct/ur_vc_2_5.pg
Find the area of the parallelogram with vertices (5,3), (9, 6), (13, 11), and (17, 14).
   ___
1. Find a vector equation for the line through the point P = (0, -2, 2) and parallel to the vector v = (1, -1, 3).
\[ \mathbf{r}(t) = \mathbf{P} + t\mathbf{v} = (0, -2, 2) + t(1, -1, 3) \]

2. A bicycle pedal is pushed straight downwards by a foot with a 41 Newton force. The shaft of the pedal is 20 cm long. If the shaft is \( \pi/6 \) radians past horizontal, what is the magnitude of the torque about the point where the shaft is attached to the bicycle?

3. Enter T or F depending on whether the statement is true or false. (You must enter T or F – True and False will not work.)
   - Two lines either intersect or are parallel.
   - Two planes either intersect or are parallel.
   - Two planes parallel to a line are parallel.
   - Two lines parallel to a third line are parallel.
   - Two planes perpendicular to a third plane are parallel.
   - Two lines perpendicular to a third line are parallel.
   - Two planes either intersect or are parallel.
   - Two planes parallel to a line are parallel.
   - Two planes perpendicular to a plane are parallel.
   - Two lines parallel to a plane are parallel.

4. Find a vector equation for the line through the point P = (4, 0, -2) and parallel to the vector \( \mathbf{v} = (2, 3, 5) \).
\[ \mathbf{r}(t) = \mathbf{P} + t\mathbf{v} = (4, 0, -2) + t(2, 3, 5) \]

5. Find a vector equation for the line through the point P = (3, 1, 1) and parallel to the vector \( \mathbf{v} = (2, 3, 5) \).
\[ \mathbf{r}(t) = \mathbf{P} + t\mathbf{v} = (3, 1, 1) + t(2, 3, 5) \]

6. Given a the vector equation \( \mathbf{r}(t) = (5 + 2t)i + (0 + t)j + (4 + -2t)k \), rewrite this in terms of the parametric equations for the line.
\[ x(t) = \quad y(t) = \quad z(t) = \]

7. Given the vector equation \( \mathbf{r}(t) = (0 + 4t)i + (1 + -3t)j + (4 + -t)k \), rewrite this in terms of the symmetric equations for the line.
\[ \frac{x}{\quad} = \frac{y}{\quad} = \frac{z}{\quad} \]

8. Consider the planes \( 1x + 2y + 2z = 1 \) and \( 1x + 2z = 0 \).
   - Find the unique point P on the y-axis which is on both planes. (____ ____)
   - Find a unit vector \( \mathbf{u} \) with positive first coordinate that is parallel to both planes. _____ i + _____ j + _____ k
   - Use parts (A) and (B) to find a vector equation for the line of intersection of the two planes. \( \mathbf{r}(t) = \quad \)

9. Find an equation of the plane through the point (1, -4, 2) and parallel to both planes.
\[ (quotient involving x) + (quotient involving y) + (quotient involving z) = \]

10. (A) Find the unique point P on the y-axis which is on both planes. (____ ____)
    (B) At what point Q does this line intersect the yz-plane? \( Q = (____ ____ ) \)

11. Consider the two lines
    \[ L_1 : x = -2t, y = 1 + 2t, z = 3t \]
    \[ L_2 : x = -9 + 5t, y = 2 + 3t, z = 3 + 3t \]
    Find the point of intersection of the two planes.
\[ P = (____ ____ ) \]

12. Find an equation of the plane through the point (-2, 2, -3) and perpendicular to the vector (-4, -2, -3). Do this problem in the standard way or WebWork may not recognize a correct answer.
\[ x + ____ y + ____ z = ____ \]

13. Find an equation of the plane through the point (1, -4, 2) and parallel to the plane \( -4x + 1y + 5z = -5 \). Do this problem in the standard way or WebWork may not recognize a correct answer.
\[ x + ____ y + ____ z = ____ \]

14. Find the point P where the line \( x = 1 + t, y = 2t, z = -3t \) intersects the plane \( x + y - z = -2 \).
\[ P = (____ ____ ) \]
14. (1 pt) setVectors4PlanesLines/ur_vc_2_18.pg
Find the angle in radians between the planes $3x + z = 1$ and $4y + z = 1$.

15. (1 pt) setVectors4PlanesLines/ur_vc_2_19.pg
Find the distance from the point $(1, 3, 5)$ to the line $x = 0, y = 3 + 1t, z = 5 + 2t$.

16. (1 pt) setVectors4PlanesLines/ur_vc_2_20.pg
Find the distance from the point $(1, -2, 5)$ to the plane $-1x + 3y + 4z = 4$.

17. (1 pt) setVectors4PlanesLines/ur_vc_2_21.pg
Match the surfaces with the appropriate descriptions.

   1. $z = x^2$
   2. $z = 2y^2 + 3y^2$
   3. $z = y^2 - 2x^2$
   4. $x^2 + y^2 = 5$

18. (1 pt) setVectors4PlanesLines/ur_vc_2_22.pg
A million years ago, an alien species built a vertical tower on a horizontal plane. When they returned they discovered that the ground had tilted so that measurements of 3 points on the ground gave coordinates of $(0, 0, 0)$, $(1, 3, 0)$, and $(0, 1, 1)$. By what angle does the tower now deviate from the vertical? _______ radians.
1. What are the rectangular coordinates of the point whose cylindrical coordinates are \((r = 2, \theta = 1.6, z = -7)\)?
   \[x = \underline{\phantom{0}}, \quad y = \underline{\phantom{0}}, \quad z = \underline{\phantom{0}}\]

2. What are the rectangular coordinates of the point whose cylindrical coordinates are \((r = 6, \theta = \frac{4\pi}{3}, z = 4)\)?
   \[x = \underline{\phantom{0}}, \quad y = \underline{\phantom{0}}, \quad z = \underline{\phantom{0}}\]

3. What are the cylindrical coordinates of the point whose rectangular coordinates are \((x = 4, y = 2, z = 1)\)?
   \[r = \underline{\phantom{0}}, \quad \theta = \underline{\phantom{0}}, \quad z = \underline{\phantom{0}}\]

4. What are the cylindrical coordinates of the point whose rectangular coordinates are \((x = -1, y = 3, z = 0)\)?
   \[r = \underline{\phantom{0}}, \quad \theta = \underline{\phantom{0}}, \quad z = \underline{\phantom{0}}\]

5. What are the rectangular coordinates of the point whose spherical coordinates are \((2, \frac{4\pi}{6}, \frac{4\pi}{3})\)?
   \[x = \underline{\phantom{0}}, \quad y = \underline{\phantom{0}}, \quad z = \underline{\phantom{0}}\]

6. What are the spherical coordinates of the point whose rectangular coordinates are \((3, 4, -5)\)?
   \[\rho = \underline{\phantom{0}}, \quad \theta = \underline{\phantom{0}}, \quad \phi = \underline{\phantom{0}}\]

7. What are the cylindrical coordinates of the point whose spherical coordinates are \((3, 2, \frac{4\pi}{3})\)?
   \[r = \underline{\phantom{0}}, \quad \theta = \underline{\phantom{0}}, \quad z = \underline{\phantom{0}}\]

8. Match the given equation with the verbal description of the surface:
   
   1. \(z = r^2\)  
   2. \(r = 4\)  
   3. \(\theta = \frac{\pi}{4}\)  
   4. \(r^2 + z^2 = 16\)  
   5. \(\rho = 2\cos(\phi)\)  
   6. \(\rho \cos(\phi) = 4\)  
   7. \(r = 2\cos(\theta)\)  
   8. \(\rho = 4\)  
   9. \(\phi = \frac{\pi}{3}\)

9. If an astronomer is using polar coordinates, then which of the following is the most likely object of study?
   - A. a solar system
   - B. a globular cluster
   - C. the whole universe
   - D. a planet
1. If \( \mathbf{r}(t) = (\sqrt{t+1}) \mathbf{i} + \frac{2-t}{3} \mathbf{j} + \sin(5\pi t) \mathbf{k} \), then
   \[
   \lim_{t \to 1} \mathbf{r}(t) = \mathbf{i} + \mathbf{j} + \mathbf{k}.
   \]

2. The curve \( (x=\cos t, y=\sin t, z=t) \) lies on which of the following surfaces.
   Enter T or F depending on whether the statement is true or false.
   (You must enter T or F – True and False will not work.)
   
   \[\begin{array}{ll}
   1. & \text{a plane} \\
   2. & \text{an elliptic paraboloid} \\
   3. & \text{a circular cylinder} \\
   4. & \text{a sphere}
   \end{array}\]

3. For the given position vectors \( \mathbf{r}(t) \) compute the tangent velocity vector \( \mathbf{r}'(t) \) for the given value of \( t \).
   
   A.)
   If \( \mathbf{r}(t) = (\cos t, \sin t) \)
   Then \( \mathbf{r}'(1) = ( \quad , \quad ) \).

   B.)
   If \( \mathbf{r}(t) = (t^2, t^3) \)
   Then \( \mathbf{r}'(1) = ( \quad , \quad ) \).

   C.)
   If \( \mathbf{r}(t) = e^{t/2} \mathbf{i} + e^{-3t} \mathbf{j} + t \mathbf{k} \).
   Then \( \mathbf{r}'(1) = \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \).

4. For the given position vectors \( \mathbf{r}(t) \) compute the unit tangent vector \( \mathbf{T}(t) \) for the given value of \( t \).
   
   A.)
   If \( \mathbf{r}(t) = (\cos 4t, \sin 4t) \)
   Then \( \mathbf{T}(\frac{\pi}{2}) = ( \quad , \quad ) \).

   B.)
   If \( \mathbf{r}(t) = (t^2, t^3) \)
   Then \( \mathbf{T}(5) = ( \quad , \quad ) \).

   C.)
   If \( \mathbf{r}(t) = e^t \mathbf{i} + e^{-5t} \mathbf{j} + t \mathbf{k} \).
   Then \( \mathbf{T}(-5) = \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \).

5. Find parametric equations for the tangent line at the point \( (\cos(\frac{\pi}{6}), \sin(\frac{\pi}{6}), \frac{\pi}{6}) \) on the curve \( x=\cos t, y=\sin t, z=t \).
   
   \( x(t) = \quad \), \( y(t) = \quad \), \( z(t) = \quad \).

6. Evaluate \( \int_0^9 (t^2 \mathbf{j} + t^3 \mathbf{k}) \) dt = \( \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \).

7. If \( \mathbf{r}(t) = \cos(3t) \mathbf{i} + \sin(3t) \mathbf{j} + 10t \mathbf{k} \)
   compute \( \mathbf{r}'(t) = \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \).

   and \( \int \mathbf{r}(t) \) dt = \( \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \).

8. A particle in space undergoes a constant nonzero acceleration. Depending on the circumstances, the particle’s trajectory can be held by the following curves.
   Enter T or F depending on whether the statement is true or false.
   (You must enter T or F – True and False will not work.)
   
   \[\begin{array}{ll}
   1. & \text{a hyperbola} \\
   2. & \text{a circle} \\
   3. & \text{an ellipse} \\
   4. & \text{a parabola} \\
   5. & \text{a strait line}
   \end{array}\]
1. (1 pt) setVecFunction2Curvature/ur_{vc,4,1}.pg
Find the length of the given curve:
\[ \mathbf{r}(t) = (-1t, 5 \sin t, 5 \cos t) \]
where \(-5 \leq t \leq 2\).

2. (1 pt) setVecFunction2Curvature/ur_{vc,4,2}.pg
Starting from the point \((-5, 3, 0)\) reparametrize the curve
\[ \mathbf{r}(t) = (-5 - 1t)i + (3 + 2t)j + (0 + 0t)k \]
in terms of arclength.
\[ \mathbf{r}(s) = _____ i + _____ j + _____ k \]

3. (1 pt) setVecFunction2Curvature/ur_{vc,4,3}.pg
If \( \mathbf{r}(t) = \cos(-1t)i + \sin(-1t)j - 5t k \), compute:
A. The velocity vector \( \mathbf{v}(t) = _____ i + _____ j + _____ k \)
B. The acceleration vector \( \mathbf{a}(t) = _____ i + _____ j + _____ k \)

4. (1 pt) setVecFunction2Curvature/ur_{vc,4,4}.pg
Consider the helix \( \mathbf{r}(t) = (\cos(-2t), \sin(-2t), 2t) \). Compute, at \( t = \frac{\pi}{4} \):
A. The unit tangent vector \( \mathbf{T} = (_____ _____ _____) \)
B. The unit normal vector \( \mathbf{N} = (_____ _____ _____) \)
C. The unit binormal vector \( \mathbf{B} = (_____ _____ _____) \)
D. The curvature \( \kappa = _____ _____ _____ \)

5. (1 pt) setVecFunction2Curvature/ur_{vc,4,5}.pg
Find the curvature \( \kappa(t) \) of the curve \( \mathbf{r}(t) = (4 \sin t) \mathbf{i} + (4 \sin t) \mathbf{j} + (5 \cos t) \mathbf{k} \)

6. (1 pt) setVecFunction2Curvature/ur_{vc,4,6}.pg
Find the curvature of \( y = \sin(-3x) \) at \( x = \frac{\pi}{4} \).
1. (1 pt) setVecFunction3Motion/ur_vc_4_7.pg
Given that the acceleration vector is \( \mathbf{a}(t) = (-4 \cos(2t)) \mathbf{i} + (-4 \sin(2t)) \mathbf{j} + (-4t) \mathbf{k} \), the initial velocity is \( \mathbf{v}(0) = \mathbf{i} + \mathbf{k} \), and the initial position vector is \( \mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} \), compute:
   A. The velocity vector \( \mathbf{v}(t) = \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \)
   B. The position vector \( \mathbf{r}(t) = \quad \mathbf{i} + \quad \mathbf{j} + \quad \mathbf{k} \)
   Note: the coefficients in your answers must be entered in the form of expressions in the variable \( t \); e.g. “5 cos(2t)”

2. (1 pt) setVecFunction3Motion/ur_vc_4_8.pg
A gun has a muzzle speed of 100 meters per second. What angle of elevation should be used to hit an object 180 meters away? Neglect air resistance and use \( g = 9.8 \text{ m/sec}^2 \) as the acceleration of gravity.

___________________________ radians

3. (1 pt) setVecFunction3Motion/ur_vc_4_9.pg
Match the parametric equations with the verbal descriptions of the surfaces by putting the letter of the verbal description to the left of the letter of the parametric equation.

   1. \( \mathbf{r}(u,v) = u \mathbf{i} + \cos v \mathbf{j} + \sin v \mathbf{k} \)
   2. \( \mathbf{r}(u,v) = u \mathbf{i} + v \mathbf{j} + (2u - 3v) \mathbf{k} \)
   3. \( \mathbf{r}(u,v) = u \mathbf{i} + u \cos v \mathbf{j} + u \sin v \mathbf{k} \)
   4. \( \mathbf{r}(u,v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + u^2 \mathbf{k} \)
   A. plane
   B. cone
   C. circular cylinder
   D. circular paraboloid

4. (1 pt) setVecFunction3Motion/ur_vc_4_10.pg
A factory has a machine which bends wire at a rate of 3 unit(s) of curvature per second. How long does it take to bend a straight wire into a circle of radius 9?

______ seconds
1. (1 pt) setVmultivariable1Functions/UR_VC_5_1.pg

Match the surfaces with the verbal description of the level curves by placing the letter of the verbal description to the left of the number of the surface.

1. \( z = x^2 + y^2 \)  
   A. two straight lines and a collection of hyperbolas
2. \( z = \sqrt{x^2 + y^2} \)  
   B. a collection of unequally spaced concentric circles
3. \( z = xy \)  
   C. a collection of equally spaced parallel lines
4. \( z = \frac{1}{x^2 - y^2} \)  
   D. a collection of concentric ellipses
5. \( z = \sqrt{25 - x^2 - y^2} \)  
   E. a collection of unequally spaced parallel lines
6. \( z = 2x + 3y \)  
   F. a collection of equally spaced concentric circles
7. \( z = 2x^2 + 3y^2 \)

2. (1 pt) setVmultivariable1Functions/UR_VC_5_2.pg

Match the functions with the verbal description of the level surfaces by placing the letter of the verbal description to the left of the number of the function.

1. \( w = x^2 + y^2 + z^2 \)  
   A. two cones and two collections of hyperboloids
2. \( w = x^2 + 2y^2 + 3z^2 \)  
   B. a collection of unequally spaced parallel planes
3. \( w = x + 2y + 3z \)  
   C. a collection of unequally spaced concentric spheres
4. \( w = \sqrt{x^2 + 2y + 3z} \)  
   D. a collection of equally spaced parallel planes
5. \( w = \sqrt{x^2 + y^2 + z^2} \)  
   E. a collection of concentric ellipsoids
6. \( w = \sqrt{(x^2 + 2y^2 + 3z^2)} \)
7. \( w = x^2 - y^2 - z^2 \)

3. (1 pt) setVmultivariable1Functions/UR_VC_5_3.pg

Each of the following functions has a set on which it is continuous and that set has a boundary. Match the verbal description of this boundary with the function by putting the letter of the boundary to the left of the letter of the function.

1. \( f(x, y, z) = \frac{x^2}{x^2 - y^2} \)  
   A. a straight line
2. \( f(x, y) = x\ln y \)  
   B. one point
3. \( f(x, y, z) = \frac{1}{x^2 + y^2 + z^2} \)  
   C. a circular cylinder
4. \( f(x, y) = \frac{1}{4-x^2-y^2} \)  
   D. a circular parabaloid
5. \( f(x, y, z) = \frac{xy}{x^2 + y^2 - z} \)  
   E. a circle

4. (1 pt) setVmultivariable1Functions/UR_VC_5_4.pg

The level curves of a function \( f(x, y) \) consist of a collection of hyperbolas and two lines. If the lines intersect at a point \( P \), what are the possibilities for \( P \)? Type the letters of all possibilities, with no punctuation, in alphabetical order.

A. \( P \) is a local maximum, that is, \( f(P) \geq f(Q) \) for all \( Q \) near \( P \).
B. \( P \) is a local minimum, that is, \( f(P) \leq f(Q) \) for all \( Q \) near \( P \).
C. \( P \) is neither a local maximum nor a local minimum.

5. (1 pt) setVmultivariable1Functions/UR_VC_5_5_F.pg

On a map showing the grave of George Mallory, the contour lines are:

- A. closely spaced
- B. far apart

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Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
1. (1 pt) setVmultivariable2Limits/UR_VC_5_3.pg

Find the limit, if it exists, or type N if it does not exist.

\[ \lim_{(x,y) \to (-1,-1)} e^{\sqrt{x^2+2y^2}} = \]

2. (1 pt) setVmultivariable2Limits/UR_VC_5_4.pg

Find the limit, if it exists, or type N if it does not exist.

\[ \lim_{(x,y) \to (0,0)} \frac{4x^2}{4x^2+4y^2} = \]

3. (1 pt) setVmultivariable2Limits/UR_VC_5_5.pg

Find the limit, if it exists, or type N if it does not exist.

\[ \lim_{(x,y) \to (0,0)} \frac{(x+17y)^2}{x^2+17^2y^2} = \]

4. (1 pt) setVmultivariable2Limits/UR_VC_5_6.pg

Find the limit, if it exists, or type N if it does not exist.

(Hint: use polar coordinates.)

\[ \lim_{(x,y) \to (0,0)} \frac{6x^3+8y^3}{x^2+y^2} = \]

5. (1 pt) setVmultivariable2Limits/UR_VC_5_7.pg

Find the limit, if it exists, or type N if it does not exist.

\[ \lim_{(x,y,z) \to (3,1,1)} \frac{5e^{x^2+y^2}}{3x^2+1y^2+z^2} = \]

6. (1 pt) setVmultivariable2Limits/UR_VC_5_8.pg

Find the limit, if it exists, or type N if it does not exist.

\[ \lim_{(x,y,z) \to (0,0,0)} \frac{1xy + 3yz + 3xz}{x^2 + 9y^2 + 9z^2} = \]
1. (1 pt) setVmultivariable3ParDer/UR_VC_5_10.pg

Find the first partial derivatives of \( f(x, y) = \frac{2x-4y}{2x+4y} \) at the point \((x,y) = (3, 2)\).

\[
\frac{\partial f}{\partial x} (3, 2) =
\]

\[
\frac{\partial f}{\partial y} (3, 2) =
\]

2. (1 pt) setVmultivariable3ParDer/UR_VC_5_11.pg

Find the first partial derivatives of \( f(x, y, z) = z \arctan\left(\frac{x}{y}\right) \) at the point \((1, 1, 1)\).

A. \( \frac{\partial f}{\partial x} (1, 1, 1) =
\]

B. \( \frac{\partial f}{\partial y} (1, 1, 1) =
\]

C. \( \frac{\partial f}{\partial z} (1, 1, 1) =
\]

3. (1 pt) setVmultivariable3ParDer/UR_VC_5_12.pg

Find the first partial derivatives of \( f(x, y) = \sin(x-y) \) at the point \((8, 8)\).

A. \( f_x(8, 8) =
\]

B. \( f_y(8, 8) =
\]

4. (1 pt) setVmultivariable3ParDer/UR_VC_5_13.pg

If \( \sin(-4x - 3y + z) = 0 \), find the first partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) at the point \((0, 0, 0)\).

A. \( \frac{\partial f}{\partial x} (0, 0, 0) =
\]

B. \( \frac{\partial f}{\partial y} (0, 0, 0) =
\]

5. (1 pt) setVmultivariable3ParDer/UR_VC_5_14.pg

Find all the first and second order partial derivatives of \( f(x, y) = 10 \sin(2x + y) - 4 \cos(x - y) \).

A. \( \frac{\partial f}{\partial x} =
\]

B. \( \frac{\partial f}{\partial y} =
\]

C. \( \frac{\partial^2 f}{\partial x^2} =
\]

D. \( \frac{\partial^2 f}{\partial y^2} =
\]

E. \( \frac{\partial^2 f}{\partial y \partial x} =
\]

F. \( \frac{\partial^2 f}{\partial x \partial y} =
\]
1. (1 pt) setVmultivariable4Linearization/ur_vc_6_1.pg
Find the equation of the tangent plane to the surface \( z = 9y^2 - 4x^2 \) at the point \((1, 2, 32)\).
\[ z = \text{expression of } x \text{ and } y \]
Note: Your answer should be an expression of \( x \) and \( y \); e.g. “\(3x - 4y + 6\)”

2. (1 pt) setVmultivariable4Linearization/ur_vc_6_2.pg
Find the equation of the tangent plane to the surface \( z = e^{\frac{3x}{17} \ln (4y)} \) at the point \((-2, 4, 1.948)\).
\[ z = \text{expression of } x \text{ and } y \]
Note: Your answer should be an expression of \( x \) and \( y \); e.g. “\(5x + 2y - 3\)”

3. (1 pt) setVmultivariable4Linearization/ur_vc_6_3.pg
Find the linearization \( L(x, y) \) of the function \( f(x, y) = \sqrt{81 - 4x^2 - 1y^2} \) at \((4, -1)\).
\[ L(x, y) = \text{expression of } x \text{ and } y \]
Note: Your answer should be an expression of \( x \) and \( y \); e.g. “\(3x - 5y + 9\)”

4. (1 pt) setVmultivariable4Linearization/ur_vc_6_4.pg
Find the differential of the function \( w = x \sin (4yz^4) \).
\[ dw = \text{expression of } x, y \text{ and } z \]
Note: Your answers should be expressions of \( x \), \( y \) and \( z \); e.g. “\(3xy + 4z\)”

5. (1 pt) setVmultivariable4Linearization/ur_vc_6_5.pg
The dimensions of a closed rectangular box are measured as 70 centimeters, 80 centimeters, and 90 centimeters, respectively, with the error in each measurement at most .2 centimeters. Use differentials to estimate the maximum error in calculating the surface area of the box.
\( \text{square centimeters} \)

6. (1 pt) setVmultivariable4Linearization/ur_vc_6_6.pg
Find an equation of the tangent plane to the parametric surface \( x = 2r \cos \theta \), \( y = 4r \sin \theta \), \( z = r \) at the point \((-2\sqrt{2}, -4\sqrt{2}, 2)\) when \( r = 2 \), \( \theta = \pi/4 \).
\[ z = \text{expression of } x \text{ and } y \]
Note: Your answer should be an expression of \( x \) and \( y \); e.g. “\(3x - 4y\)”
1. (1 pt) setVmultivariable5ChainRule/ur_vc_6_7.pg
Suppose \( w = \frac{x}{z}, x = e^{5t}, y = 2 + \sin(1t), z = 2 + \cos(5t). \)

A. Use the chain rule to find \( \frac{dw}{dt} \) as a function of \( x, y, z, \) and \( t. \) Do not rewrite \( x, y, \) and \( z \) in terms of \( t, \) and do not rewrite \( e^{5t} \) as \( x. \)

\[ \frac{dw}{dt} = \]

Note: Use \( \text{exp}() \) for the exponential function. Your answer should be an expression in \( x, y, z, \) and \( t; \) e.g. “3x - 4y”

B. Use part A to evaluate \( \frac{dw}{dt} \) when \( t = 0. \)

2. (1 pt) setVmultivariable5ChainRule/ur_vc_6_8.pg
Suppose \( z = x^2 \sin y, x = -3s^2 - 3t^2, y = 0st. \)

A. Use the chain rule to find \( \frac{\partial z}{\partial s} \) and \( \frac{\partial z}{\partial t} \) as functions of \( x, y, s, \) and \( t. \)

\[ \frac{\partial z}{\partial s} = \]

\[ \frac{\partial z}{\partial t} = \]

B. Find the numerical values of \( \frac{\partial z}{\partial s} \) and \( \frac{\partial z}{\partial t} \) when \( (s,t) = (3,0). \)

3. (1 pt) setVmultivariable5ChainRule/ur_vc_6_9.pg
The radius of a right circular cone is increasing at a rate of 2 inches per second and its height is decreasing at a rate of 2 inches per second. At what rate is the volume of the cone changing when the radius is 40 inches and the height is 10 inches?

\[ \] cubic inches per second

4. (1 pt) setVmultivariable5ChainRule/ur_vc_6_10.pg
In a simple electric circuit, Ohm’s law states that \( V = IR, \) where \( V \) is the voltage in volts, \( I \) is the current in amperes, and \( R \) is the resistance in ohms. Assume that, as the battery wears out, the voltage decreases at 0.04 volts per second and, as the resistor heats up, the resistance is increasing at 0.02 ohms per second. When the resistance is 100 ohms and the current is 0.01 amperes, at what rate is the current changing?

\[ \] amperes per second
1. (1 pt) setVmultivariable6Gradient/ur_vc_6_11.pg
If \( f(x, y) = -3x^2 + 3y^2 \), find the value of the directional derivative at the point \((-1, -1)\) in the direction given by the angle \( \theta = \frac{2\pi}{3} \).

2. (1 pt) setVmultivariable6Gradient/ur_vc_6_12.pg
Suppose \( f(x, y) = -2x^2 - 3xy - 3y^2 \), \( P = (0, 3) \), and \( \mathbf{u} = (\frac{12}{19}, \frac{18}{19}) \).
A. Compute the gradient of \( f \).
\[ \nabla f = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} \]
Note: Your answers should be expressions of \( x \) and \( y \); e.g. “\( 3x - 4y \)”
B. Evaluate the gradient at the point \( P \).
\[ (\nabla f) (0, 3) = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} \]
Note: Your answers should be numbers
C. Compute the directional derivative of \( f \) at \( P \) in the direction \( \mathbf{u} \).
\[ (D_u f)(P) = \text{__________} \]
Note: Your answer should be a number
D. Find the maximum rate of change of \( f \) at \( P \).
E. Find the (unit) direction vector in which the maximum rate of change occurs at \( P \).
\[ \mathbf{u} = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} \]
Note: Your answers should be numbers

3. (1 pt) setVmultivariable6Gradient/ur_vc_6_13.pg
Suppose \( f(x, y) = \frac{x}{3} \), \( P = (-1, -3) \) and \( \mathbf{v} = 1\mathbf{i} - 1\mathbf{j} \).
A. Find the gradient of \( f \).
\[ \nabla f = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} \]
Note: Your answers should be expressions of \( x \) and \( y \); e.g. “\( 3x - 4y \)”
B. Find the gradient of \( f \) at the point \( P \).
\[ (\nabla f)(P) = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} \]
Note: Your answers should be numbers
C. Find the directional derivative of \( f \) at \( P \) in the direction of \( \mathbf{v} \).
\[ D_\mathbf{v} f = \text{__________} \]
Note: Your answer should be a number
D. Find the maximum rate of change of \( f \) at \( P \).
E. Find the (unit) direction vector in which the maximum rate of change occurs at \( P \).
\[ \mathbf{u} = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} \]
Note: Your answers should be numbers

Suppose \( f(x, y, z) = \frac{x}{5} + \frac{z}{2} \), \( P = (1, 4, 2) \).
A. Find the gradient of \( f \).
\[ \nabla f = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} + \text{__________} \mathbf{k} \]
Note: Your answers should be expressions of \( x \), \( y \) and \( z \); e.g. “\( 3x - 4y \)”

5. (1 pt) setVmultivariable6Gradient/ur_vc_6_15.pg
Suppose that distances are measured in lightyears and that the temperature \( T \) of a gaseous nebula is inversely proportional to the distance from a fixed point, which we take to be the origin.
Suppose that the temperature 1 lightyear from the origin is 900 degrees celsius. Find the gradient of \( T \) at \((x, y, z)\).
\[ \nabla f = \text{__________} \mathbf{i} + \text{__________} \mathbf{j} + \text{__________} \mathbf{k} \]
Note: Your answers should be expressions of \( x \), \( y \) and \( z \); e.g. “\( 3x - 4y \)”

Consider the surface
\[ 25x^2 + 9y^2 + 25z^2 = 59 \]
and the point \( P = (1, 1, 1) \) on this surface.
A. Starting with the equation \( x = 1 + 5t \), find equations for \( y \) and \( z \) which combine with this equation to give parametric equations for the normal line through \( P \).
\[ y = \text{__________} \]
\[ z = \text{__________} \]
Note: Your answers should be expressions of \( t \); e.g. “\( 3x - 4y \)”
B. Find an equation for the tangent plane through \( P \).
\[ \text{__________} \]
Note: Your answers should be expressions of \( x \) and \( y \); e.g. “\( 3xy + 2y \)”

7. (1 pt) setVmultivariable6Gradient/ur_vc_6_17.pg
The axis of a light in a lighthouse is tilted. When the light points east, it is inclined upward at 10 degree(s). When it points north, it is inclined upward at 7 degree(s). What is its maximum angle of elevation?

8. (1 pt) setVmultivariable6Gradient/ur_vc_6_18.pg
You are hiking the Inca Trail on the way to Machu Picchu. When you arrive at the highest point on the trail, which of the following are possibilities? In alphabetical order without punctuation or spacing, list the letters which indicate possibilities.
(A) The path passes through the center of a set of concentric contour lines.
(B) The path is tangent to a contour line.
(C) The path follows a contour line.
(D) The path crosses a contour line.

Note: Your answer should be a number

B. What is the maximum rate of change of \( f \) at the point \( P \)?

Note: Your answer should be a number
1. (1 pt) setVmultivariable7MaxMin/ur_vc_7.1.png
Suppose \( f(x, y) = x^2 + y^2 - 10x - 10y + 4 \)
(A) How many critical points does \( f \) have in \( \mathbb{R}^2 \)?

(B) If there is a local minimum, what is the value of the discriminant \( D \) at that point? If there is none, type N.

(C) If there is a local maximum, what is the value of the discriminant \( D \) at that point? If there is none, type N.

(D) If there is a saddle point, what is the value of the discriminant \( D \) at that point? If there is none, type N.

(E) What is the maximum value of \( f \) on \( \mathbb{R}^2 \)? If there is none, type N.

(F) What is the minimum value of \( f \) on \( \mathbb{R}^2 \)? If there is none, type N.

2. (1 pt) setVmultivariable7MaxMin/ur_vc_7.2.png
Suppose \( f(x, y) = xy - ax - by \).
(A) How many local minimum points does \( f \) have in \( \mathbb{R}^2 \)?
(The answer is an integer).

(B) How many local maximum points does \( f \) have in \( \mathbb{R}^2 \)?

(C) How many saddle points does \( f \) have in \( \mathbb{R}^2 \)?

3. (1 pt) setVmultivariable7MaxMin/ur_vc_7.3.png
Consider the function \( f(x, y) = x \sin(y) \).
In the following questions, enter an integer value or type INF for infinity.
(A) How many local minima does \( f \) have in \( \mathbb{R}^2 \)?

(B) How many local maxima does \( f \) have in \( \mathbb{R}^2 \)?

(C) How many saddle points does \( f \) have in \( \mathbb{R}^2 \)?

4. (1 pt) setVmultivariable7MaxMin/ur_vc_7.4.png
Suppose \( f(x, y) = xy(1 - 8x - 1y) \).
\( f(x, y) \) has 4 critical points. List them in increasing lexicographic order. By that we mean that \( (x, y) \) comes before \( (z, w) \) if \( x < z \) or if \( x = z \) and \( y < w \). Also, describe the type of critical point by typing MA if it is a local maximum, MI if it is a local minimum, and S if it is a saddle point.

First point (___, ___) of type ___
Second point (___, ___) of type ___
Third point (___, ___) of type ___
Fourth point (___, ___) of type ___

5. (1 pt) setVmultivariable7MaxMin/ur_vc_7.5.png
Each of the following functions has at most one critical point.
Graph a few level curves and a few gradients and, on this basis alone, decide whether the critical point is a local maximum (MA), a local minimum (MI), or a saddle point (S). Enter the appropriate abbreviation for each question, or N if there is no critical point.
(A) \( f(x, y) = e^{-4x^2 - 3y^2} \)
Type of critical point: ______________

(B) \( f(x, y) = e^{4x^2 - 3y^2} \)
Type of critical point: ______________

(C) \( f(x, y) = 4x^2 + 3y^2 + 2 \)
Type of critical point: ______________

(D) \( f(x, y) = 4x + 3y + 2 \)
Type of critical point: ______________

6. (1 pt) setVmultivariable7MaxMin/ur_vc_7.6.png
You are to manufacture a rectangular box with 3 dimensions \( x \), \( y \) and \( z \), and volume \( v = 27 \). Find the dimensions which minimize the surface area of this box.
\( x = \) ______________
\( y = \) ______________
\( z = \) ______________

7. (1 pt) setVmultivariable7MaxMin/ur_vc_7.7.png
Find the coordinates of the point \((x, y, z)\) on the plane \( z = 4x + 2y + 3 \) which is closest to the origin.
\( x = \) ______________
\( y = \) ______________
\( z = \) ______________

8. (1 pt) setVmultivariable7MaxMin/ur_vc_7.8.png
Find the maximum and minimum values of \( f(x, y) = 5x^2 + 6y^2 \) on the disk \( D: x^2 + y^2 \leq 1 \).
maximum value: ______________
minimum value: ______________

9. (1 pt) setVmultivariable7MaxMin/ur_vc_7.9.png
Find the maximum and minimum values of \( f(x, y) = 3x + y \) on the ellipse \( x^2 + 36y^2 = 1 \)
maximum value: ______________
minimum value: ______________

10. (1 pt) setVmultivariable7MaxMin/ur_vc_7.10.png
For each of the following functions, find the maximum and minimum values of the function on the circular disk: \( x^2 + y^2 \leq 1 \). Do this by looking at the level curves and gradients.
(A) \( f(x, y) = x + y + 4 \):
maximum value = ______________
minimum value = ______________

(B) \( f(x, y) = 4x^2 + 5y^2 \):
maximum value = ______________
minimum value = ______________

(C) \( f(x, y) = 4x^2 - 5y^2 \):
maximum value = ______________
For each of the following functions, find the maximum and minimum values of the function on the rectangular region: \(-4 \leq x \leq 4, -5 \leq y \leq 5\).

11. (1 pt) setVmultivariable7MaxMin/ur_vc_7_J11.pg

Do this by looking at level curves and gradients.

(A) \(f(x, y) = x + y + 4\):
maximum value =
minimum value =

(B) \(f(x, y) = 4x^2 + 5y^2\):
maximum value =
minimum value =

(C) \(f(x, y) = (5x^2 - (4)y^2\):
maximum value =
minimum value =

12. (1 pt) setVmultivariable7MaxMin/ur_vc_7_J12.pg

Find the maximum and minimum values of \(f(x, y, z) = 4x + 5y + 1z\) on the sphere \(x^2 + y^2 + z^2 = 1\).

maximum value =
minimum value =

13. (1 pt) setVmultivariable7MaxMin/ur_vc_7_J13.pg

Find the maximum and minimum values of \(f(x, y) = xy\) on the ellipse \(6x^2 + y^2 = 8\).

maximum value =
minimum value =

14. (1 pt) setVmultivariable7MaxMin/ur_vc_7_J14.pg

You are hiking the Inca Trail on the way to Machu Picchu. When you arrive at the highest point on the trail, which of the following are possibilities? In alphabetical order without punctuation or spacing, list the letters which indicate possibilities.

(A) The path passes through the center of a set of concentric contour lines.
(B) The path is tangent to a contour line.
(C) The path follows a contour line.
(D) The path crosses a contour line.

possibilities: _______
1. (1 pt) setVMultIntegrals1Double/ur_vc_5.1.pg
Consider the solid that lies above the square $R = [0, 2] \times [0, 2]$ and below the elliptic paraboloid $z = 36 - x^2 - 2y^2$.
(A) Calculate the volume by dividing $R$ into 4 equal squares and choosing the sample points to lie in the lower left hand corners.

(B) Calculate the volume by dividing $R$ into 4 equal squares and choosing the sample points to lie in the upper right hand corners.

(C) What is the average of the two answers from (A) and (B)?

(D) Using iterated integrals, compute the exact value of the volume.

2. (1 pt) setVMultIntegrals1Double/ur_vc_5.2.pg
Evaluate the iterated integral $\int_0^1 \int_0^2 6x^2y^4 \, dx \, dy$.

3. (1 pt) setVMultIntegrals1Double/ur_vc_5.3.pg
Evaluate the iterated integral $\int_1^2 \int_1^3 (3x + y)^2 \, dy \, dx$.

4. (1 pt) setVMultIntegrals1Double/ur_vc_5.4.pg
Calculate the double integral $\iiint_R (4x + 4y + 16) \, dA$ where $R$ is the region: $0 \leq x \leq 2$, $0 \leq y \leq 2$.

5. (1 pt) setVMultIntegrals1Double/ur_vc_5.5.pg
Calculate the double integral $\iiint_R x \cos(x + y) \, dA$ where $R$ is the region: $0 \leq x \leq \frac{\pi}{6}$, $0 \leq y \leq \frac{\pi}{4}$.

6. (1 pt) setVMultIntegrals1Double/ur_vc_5.6.pg
Calculate the volume under the elliptic paraboloid $z = 4x^2 + 7y^2$ and over the rectangle $R = [-2, 2] \times [-1, 1]$.

7. (1 pt) setVMultIntegrals1Double/ur_vc_5.7.pg
Using geometry, calculate the volume of the solid under $z = \sqrt{16 - x^2 - y^2}$ and over the circular disk $x^2 + y^2 \leq 16$.

8. (1 pt) setVMultIntegrals1Double/ur_vc_5.8.pg
Using the maxima and minima of the function, produce upper and lower estimates of the integral $I = \iint_D e^{x^2+y^2} \, dA$ where $D$ is the circular disk: $x^2 + y^2 \leq 6$.

9. (1 pt) setVMultIntegrals1Double/ur_vc_5.9.pg
Evaluate the iterated integral $I = \int_0^1 \int_{1-x}^{1+x} (3x^2 + 2y) \, dy \, dx$.

10. (1 pt) setVMultIntegrals1Double/ur_vc_5.10.pg
Evaluate the double integral $I = \int_D xy \, dA$ where $D$ is the triangular region with vertices $(0, 0), (2, 0), (0, 4)$.

11. (1 pt) setVMultIntegrals1Double/ur_vc_5.11.pg
Find the volume of the solid bounded by the planes $x = 0, y = 0, z = 0$, and $x + y + z = 4$.

12. (1 pt) setVMultIntegrals1Double/ur_vc_5.12.pg
Evaluate the integral by reversing the order of integration. $\int_0^1 \int_y^2 e^{x^2} \, dx \, dy = \int_0^2 \int_0^x e^{x^2} \, dy \, dx$.

13. (1 pt) setVMultIntegrals1Double/ur_vc_5.13.pg
Match the following integrals with the verbal descriptions of the solids whose volumes they give. Put the letter of the verbal description to the left of the corresponding integral.

1. $\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1 - x^2 - y^2 \, dy \, dx$
A. Solid under a plane and over one half of a circular disk.
B. One half of a cylindrical rod.
C. Solid under an elliptic paraboloid and over a planar region bounded by two parabolas.
D. One eighth of an ellipsoid.
E. Solid bounded by a circular paraboloid and a plane.

2. $\int_0^1 \int_0^{\sqrt{x}} 4x^2 + 3y^2 \, dy \, dx$

3. $\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 4x + 3y \, dy \, dx$

4. $\int_0^1 \int_0^{\sqrt{1-3y^2}} \sqrt{1 - 4x^2 - 3y^2} \, dx \, dy$

5. $\int_0^1 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \sqrt{4-y^2} \, dx \, dy$

If $\int_0^4 f(x) \, dx = 0$ and $\int_{-3}^0 g(x) \, dx = -4$, what is the value of $\int_D f(x)g(y) \, dA$ where $D$ is the square: $2 \leq x \leq 4$, $-3 \leq y \leq 0$?
1. (1 pt) setVMultIntegrals2Polar/UR_VC_9_1.pg

Using polar coordinates, evaluate the integral \( \int \int_R \sin(x^2 + y^2) \, dA \) where \( R \) is the region \( 9 \leq x^2 + y^2 \leq 36 \).

2. (1 pt) setVMultIntegrals2Polar/UR_VC_9_2.pg

Using polar coordinates, evaluate the integral which gives the area which lies in the first quadrant between the circles \( x^2 + y^2 = 16 \) and \( x^2 - 4x + y^2 = 0 \).

3. (1 pt) setVMultIntegrals2Polar/UR_VC_9_3.pg

Use the polar coordinates to find the volume of a sphere of radius 6.

4. (1 pt) setVMultIntegrals2Polar/UR_VC_9_4.pg

A cylindrical drill with radius 4 is used to bore a hole through the center of a sphere of radius 8. Find the volume of the ring shaped solid that remains.

5. (1 pt) setVMultIntegrals2Polar/UR_VC_9_5.pg

A. Using polar coordinates, evaluate the improper integral \( \int \int_{\mathbb{R}^2} e^{-3(x^2+y^2)} \, dx \, dy \).

B. Use part A to evaluate the improper integral \( \int_{-\infty}^{\infty} e^{-3x^2} \, dx \).

6. (1 pt) setVMultIntegrals2Polar/UR_VC_9_6.pg

A sprinkler distributes water in a circular pattern, supplying water to a depth of \( e^{-r} \) feet per hour at a distance of \( r \) feet from the sprinkler.

A. What is the total amount of water supplied per hour inside of a circle of radius 18?

\[ \text{ft}^3/h \]

B. What is the total amount of water that goes through the sprinkler per hour?

\[ \text{ft}^3/h \]
1. (1 pt) setVMultIntegrals3Appl/UR_VC_2_7.pg

Electric charge is distributed over the disk \( x^2 + y^2 \leq 9 \) so that the charge density at \((x,y)\) is \( \sigma(x,y) = 8 + x^2 + y^2 \) coulombs per square meter.
Find the total charge on the disk.

2. (1 pt) setVMultIntegrals3Appl/UR_VC_2_8.pg

A lamina occupies the part of the disk \( x^2 + y^2 \leq 9 \) in the first quadrant and the density at each point is given by the function \( \rho(x,y) = 4(x^2 + y^2) \).
A. What is the total mass? __________
B. What is the moment about the x-axis? __________
C. What is the moment about the y-axis? __________
D. Where is the center of mass? (__________ , __________)
E. What is the moment of inertia about the origin? __________

3. (1 pt) setVMultIntegrals3Appl/UR_VC_2_9.pg

A lamp has two bulbs, each of a type with an average lifetime of 5 hours. The probability density function for the lifetime of a bulb is \( f(t) = \frac{1}{5}e^{-t/5}, t \leq 0 \).
What is the probability that both of the bulbs will fail within 5 hours?

4. (1 pt) setVMultIntegrals3Appl/UR_VC_2_10.pg

You are getting married and your dearest relative has baked you a cake which fills the volume between the two planes, \( z = 0 \) and \( z = 6x + 5y + c \), and inside the cylinder \( x^2 + y^2 = 1 \). You are to cut it in half by making two vertical slices from the center outward. Suppose one of the slices is at \( \theta = 0 \) and the other is at \( \theta = \psi \).
What is the limit, \( \lim_{c \to \infty} \psi ? \)
1. Find the surface area of the part of the plane $5x + 4y + z = 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

2. Find the surface area of the part of the circular paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 25$.

3. The vector equation $\mathbf{r}(u,v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}$, $0 \leq v \leq 10\pi$, $0 \leq u \leq 1$, describes a helicoid (spiral ramp). What is the surface area?

4. Find the surface area of the surface of revolution generated by revolving the graph $y = x^3$, $0 \leq x \leq 9$ around the $x$-axis.
1. Evaluate the triple integral \[ \iiint_E xyz \, dV \]
where \( E \) is the solid: \( 0 \leq z \leq 7, \ 0 \leq y \leq z, \ 0 \leq x \leq y \).

2. Find the volume of the solid enclosed by the paraboloids \( z = 9(x^2 + y^2) \) and \( z = 32 - 9(x^2 + y^2) \).

3. Find the average value of the function \( f(x, y, z) = x^2 + y^2 + z^2 \) over the rectangular prism \( 0 \leq x \leq 4, \ 0 \leq y \leq 2, \ 0 \leq z \leq 5 \).

4. Find the mass of the rectangular prism \( 0 \leq x \leq 3, \ 0 \leq y \leq 3, \ 0 \leq z \leq 3 \), with density function \( \rho(x, y, z) = x \). You might find formula No. 13 on page 1014 of the text helpful.

5. Use cylindrical coordinates to evaluate the triple integral \[ \iiint_E \sqrt{x^2 + y^2} \, dV \]
where \( E \) is the solid bounded by the circular paraboloid \( z = 1 - (x^2 + y^2) \) and the \( xy \)-plane.

6. Use spherical coordinates to evaluate the triple integral \[ \iiint_E x^2 + y^2 + z^2 \, dV \]
where \( E \) is the ball: \( x^2 + y^2 + z^2 \leq 36 \).

7. Match the integrals with the type of coordinates which make them the easiest to do. Put the letter of the coordinate system to the left of the number of the integral.

1. \[ \iiint_E dV \] where \( E \) is: \( x^2 + y^2 + z^2 \leq 4, \ x \geq 0, \ y \geq 0, \ z \geq 0 \)
2. \[ \int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{x} \, dx \, dy \]
3. \[ \iiint_E z \, dV \] where \( E \) is: \( 1 \leq x \leq 2, \ 3 \leq y \leq 4, \ 5 \leq z \leq 6 \)
4. \[ \iiint_E z^2 \, dV \] where \( E \) is: \( -2 \leq z \leq 2, \ 1 \leq x^2 + y^2 \leq 2 \)
5. \[ \int_D \frac{1}{x^2 + y^2} \, dA \] where \( D \) is: \( x^2 + y^2 \leq 4 \)

8. A volcano fills the volume between the graphs \( z = 0 \) and \( z = \frac{1}{(x^2+y^2)^{1/2}} \), and outside the cylinder \( x^2 + y^2 = 1 \). Find the volume of this volcano.
1. (1 pt) setVectorCalculus1/UR_VC_11_1.pg

Consider the transformation \( T : x = \frac{30}{34} u - \frac{16}{34} v, \quad y = \frac{16}{34} u + \frac{30}{34} v \).
A. Compute the Jacobian:
\[ \frac{\partial (x, y)}{\partial (u, v)} = \]

B. The transformation is linear, which implies that it transforms lines into lines. Thus, it transforms the square \( S : -34 \leq u \leq 34, -34 \leq v \leq 34 \) into a square \( T(S) \) with vertices:
\[ T(34, 34) = (\ldots, \ldots) \]
\[ T(-34, 34) = (\ldots, \ldots) \]
\[ T(-34, -34) = (\ldots, \ldots) \]
\[ T(34, -34) = (\ldots, \ldots) \]
C. Use the transformation \( T \) to evaluate the integral \( \iint_{T(S)} x^2 + y^2 \, dA \).

2. (1 pt) setVectorCalculus1/UR_VC_11_2.pg

Compute the gradient vector fields of the following functions:
A. \( f(x, y) = 4x^2 + 8y^2 \)
\[ \nabla f(x, y) = \ldots \hat{i} + \ldots \hat{j} \]
B. \( f(x, y) = x^3 y^7 \)
\[ \nabla f(x, y) = \ldots \hat{i} + \ldots \hat{j} \]
C. \( f(x, y) = 4x + 8y \)
\[ \nabla f(x, y) = \ldots \hat{i} + \ldots \hat{j} \]
D. \( f(x, y, z) = 4x + 8y + 8z \)
\[ \nabla f(x, y) = \ldots \hat{i} + \ldots \hat{j} + \ldots \hat{k} \]
E. \( f(x, y, z) = 4x^2 + 8y^2 + 8z^2 \)
\[ \nabla f(x, y) = \ldots \hat{i} + \ldots \hat{j} + \ldots \hat{k} \]

3. (1 pt) setVectorCalculus1/UR_VC_11_3.pg

Match the following vector fields with the verbal descriptions of the level curves or level surfaces to which they are perpendicular by putting the letter of the verbal description to the left of the number of the vector field.

1. \( F = xi - yj \)
2. \( F = xi + yj \)
3. \( F = xi + yj - zk \)
4. \( F = xi + yj - k \)
5. \( F = xi + yj + zk \)
6. \( F = yi + xj \)
7. \( F = -yi + xj \)
8. \( F = 2xi + yj + zk \)
9. \( F = 2i + j \)
10. \( F = 2i + j + k \)
11. \( F = 2xi + yj \)
A. spheres
B. circles
C. planes

4. (1 pt) setVectorCalculus1/UR_VC_11_4.pg

Compute the total mass of a wire bent in a quarter circle with parametric equations: \( x = 6\cos t, \quad y = 6\sin t, \quad 0 \leq t \leq \frac{\pi}{2} \) and density function \( \rho(x, y) = x^2 + y^2 \).

5. (1 pt) setVectorCalculus1/UR_VC_11_5.pg

Let \( C \) be the curve which is the union of two line segments, the first going from \((0, 0)\) to \((4, -2)\) and the second going from \((4, -2)\) to \((8, 0)\).
Compute the line integral \( \int_C 4dy + 2dx \).

6. (1 pt) setVectorCalculus1/UR_VC_11_6.pg

Let \( \mathbf{F} \) be the radial force field \( \mathbf{F} = xi + yj \). Find the work done by this force along the following two curves, both which go from \((0, 0)\) to \((8, 64)\). (Compare your answers!)
A. If \( C_1 \) is the parabola: \( x = t, \quad y = t^2, \quad 0 \leq t \leq 8 \), then
\[ \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \ldots \]
B. If \( C_2 \) is the straight line segment: \( x = 8t^2, \quad y = 64t^2, \quad 0 \leq t \leq 1 \), then
\[ \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = \ldots \]

7. (1 pt) setVectorCalculus1/UR_VC_11_7.pg

Let \( C \) be the counter-clockwise planar circle with center at the origin and radius \( r > 0 \). Without computing them, determine for the following vector fields \( \mathbf{F} \) whether the line integrals \( \int_C \mathbf{F} \cdot d\mathbf{r} \) are positive, negative, or zero and type P, N, or Z as appropriate.
A. \( \mathbf{F} = \) the radial vector field \( = xi + yj \): \ldots
B. \( \mathbf{F} = \) the circulating vector field \( = -yi + xj \): \ldots
C. \( \mathbf{F} = \) the circulating vector field \( = yi - xj \): \ldots
D. \( \mathbf{F} = \) the constant vector field \( = i + j \): \ldots

8. (1 pt) setVectorCalculus1/UR_VC_11_8.pg

Consider a wire in the shape of a helix \( \mathbf{r}(t) = 4\cos t + 4\sin ti + 7tj + 7tk, \quad 0 \leq t \leq 2\pi \) with constant density function \( \rho(x, y, z) = 1 \).
A. Determine the mass of the wire: \ldots
B. Determine the coordinates of the center of mass: (\ldots, \ldots, \ldots)
C. Determine the moment of inertia about the z-axis:

9. (1 pt) setVectorCalculusI/UR_VC_11_9.pg

Find the work done by the force field \( \mathbf{F}(x, y, z) = 3x \mathbf{i} + 3y \mathbf{j} + 7 \mathbf{k} \) on a particle that moves along the helix \( \mathbf{r}(t) = 2 \cos(t) \mathbf{i} + 2 \sin(t) \mathbf{j} + t \mathbf{k}, 0 \leq t \leq 2\pi \).

10. (1 pt) setVectorCalculusI/UR_VC_11_10.pg

A curve C is given by a vector function \( \mathbf{r}(t), 7 \leq t \leq 10 \), with unit tangent \( \mathbf{T}(t) \), unit normal \( \mathbf{N}(t) \), and unit binormal \( \mathbf{B}(t) \). Indicate whether the following line integrals are positive, negative, or zero by typing P, N, or Z as appropriate:

A. \( \int_C \mathbf{T} \cdot d\mathbf{r} = \)_____

B. \( \int_C \mathbf{N} \cdot d\mathbf{r} = \)_____

11. (1 pt) setVectorCalculusI/UR_VC_11_11.pg

Suppose that \( \iint_D f(x, y) \, dA = 1 \) where D is the disk \( x^2 + y^2 \leq 9 \). Now suppose E is the disk \( x^2 + y^2 \leq 9 \) and \( g(x, y) = 3f(x, y) \). What is the value of \( \iint_E g(x, y) \, dA? \)

12. (1 pt) setVectorCalculusI/UR_VC_11_12.pg

A lattice point in the plane is a point \( (a, b) \) with both coordinates equal to integers. For example, \((-1, 2)\) is a lattice point but \((1/2, 3)\) is not. If \( D(R) \) is the disk of radius R and center the origin, count the lattice points inside \( D(R) \) and call this number \( L(R) \). What is the limit, \( \lim_{R \to \infty} \frac{L(R)}{R^2} \)?
Determine whether the given set is open, connected, and simply connected. For example, if it is open, connected, but not simply connected, type “YYN” standing for “Yes, Yes, No.”
A. \{ (x,y) \mid x > 1, y < 2 \}
to the positively oriented circle \( x^2 + y^2 = 16 \). Compute the flux integral \( \int_C \mathbf{F} \cdot \mathbf{n} \, ds \).

14. (1 pt) setVectorCalculus2/ur_vec12_14.pg

A rock with a mass of 13 kilograms is put aboard an airplane in New York City and flown to Boston. How much work does the gravitational field of the earth do on the rock? Newton-meters

15. (1 pt) setVectorCalculus2/ur_vec12_15.pg

Suppose \( \mathbf{F} = \nabla f \) is a gradient field, \( S \) is a level surface of \( f \), and \( C \) is a curve on \( S \). What is the value of the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \)?

16. (1 pt) setVectorCalculus2/ur_vec12_E.pg

A vector field gives a geographical description of the flow of money in a society. In the neighborhood of a political convention, the divergence of this vector field is:

- A. zero
- B. negative
- C. positive

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1. (1 pt) setVectorCalculus3/ur_ve_13_1.pg
Evaluate \( \int \int_S \sqrt{1 + x^2 + y^2} \, dS \) where \( S \) is the helicoid: \( r(u, v) = u \cos(v)i + u \sin(v)j + v k \), with \( 0 \leq u \leq 5, 0 \leq v \leq 3\pi \).

2. (1 pt) setVectorCalculus3/ur_ve_13_2.pg
Find the surface area of the part of the sphere \( x^2 + y^2 + z^2 = 25 \) that lies above the cone \( z = \sqrt{x^2 + y^2} \).

3. (1 pt) setVectorCalculus3/ur_ve_13_3.pg
A fluid has density 5 and velocity field \( \mathbf{v} = -yi + xj + 5k \). Find the rate of flow outward through the sphere \( x^2 + y^2 + z^2 = 9 \).

4. (1 pt) setVectorCalculus3/ur_ve_13_4.pg
Let \( S \) be the part of the plane \( 2x + 2y + z = 4 \) which lies in the first octant, oriented upward. Find the flux of the vector field \( \mathbf{F} = 2i + j + 4k \) across the surface \( S \).

5. (1 pt) setVectorCalculus3/ur_ve_13_5.pg
Use Gauss’s law to find the charge enclosed by the cube with vertices \((\pm1, \pm1, \pm1)\) if the electric field is \( \mathbf{E}(x, y, z) = 5xi + 3yj + 6k \).

The temperature \( u \) in a star of conductivity 4 is inversely proportional to the distance from the center: \( u = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \). If the star is a sphere of radius 7, find the rate of heat flow outward across the surface of the star.

7. (1 pt) setVectorCalculus3/ur_ve_13_7.pg
Use Stoke’s Theorem to evaluate \( \int \int_S \nabla \times \mathbf{F} \cdot dS \) where \( \mathbf{F}(x, y, z) = -17yi + 17xzj + 18(x^2 + y^2)k \) and \( S \) is the part of the paraboloid \( z = x^2 + y^2 \) that lies inside the cylinder \( x^2 + y^2 = 1 \), oriented upward.

8. (1 pt) setVectorCalculus3/ur_ve_13_8.pg
Use Stoke’s Theorem to evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F}(x, y, z) = xi + yj + 5(x^2 + y^2)k \) and \( C \) is the boundary of the part of the paraboloid \( z = 25 - x^2 - y^2 \) which lies above the xy-plane and \( C \) is oriented counterclockwise when viewed from above.

Use the divergence theorem to find the outward flux of the vector field \( \mathbf{F}(x, y, z) = 4x^2i + 5y^2j + 3z^2k \) across the boundary of the rectangular prism: \( 0 \leq x \leq 3, 0 \leq y \leq 5, 0 \leq z \leq 2 \).

10. (1 pt) setVectorCalculus3/ur_ve_13_10.pg
If a parametric surface given by \( \mathbf{r}_1(u, v) = f(u, v)i + g(u, v)j + h(u, v)k \) and \( -4 \leq u \leq 4, -5 \leq v \leq 5 \), has surface area equal to 2, what is the surface area of the parametric surface given by \( \mathbf{r}_2(u, v) = 3\mathbf{r}_1(u, v) \) with \( -4 \leq u \leq 4, -5 \leq v \leq 5 \)?

11. (1 pt) setVectorCalculus3/ur_ve_13_11.pg
Suppose \( \mathbf{F} \) is a radial force field, \( S_1 \) is a sphere of radius 2 centered at the origin, and the flux integral \( \int \int_{S_1} \mathbf{F} \cdot dS = 9 \). Let \( S_2 \) be a sphere of radius 4 centered at the origin, and consider the flux integral \( \int \int_{S_2} \mathbf{F} \cdot dS \).

(A) If the magnitude of \( \mathbf{F} \) is inversely proportional to the square of the distance from the origin, what is the value of \( \int \int_{S_2} \mathbf{F} \cdot dS \)?

(B) If the magnitude of \( \mathbf{F} \) is inversely proportional to the cube of the distance from the origin, what is the value of \( \int \int_{S_2} \mathbf{F} \cdot dS \)?

12. (1 pt) setVectorCalculus3/ur_ve_13_12.pg
In springtime, the average value over time of the divergence of the vector field which represents air flow is:
- A. positive
- B. negative
- C. zero
1. (1 pt) setProbability1Combinations/ur_pb_j_1.pg
Find the value of the permutation:
\[ P(7, 5) = \]

2. (1 pt) setProbability1Combinations/ur_pb_j_2.pg
Find the value of the combination:
\[ C(12, 6) = \]

3. (1 pt) setProbability1Combinations/ur_pb_j_3.pg
How many 4-digit numbers can be formed using the digits 0 and 1? Repeated digits are allowed.

4. (1 pt) setProbability1Combinations/ur_pb_j_4.pg
How many different 8-letter words (real or imaginary) can be formed from the letters in the word RESEARCH?

5. (1 pt) setProbability1Combinations/ur_pb_j_5.pg
Determine the size of the sample space that corresponds to the experiment of tossing a coin the following number of times:
(a) 4 times
answer:
(b) 6 times
answer:
(c) \( n \) times
answer:

6. (1 pt) setProbability1Combinations/ur_pb_j_6.pg
An experiment consists of choosing objects without regards to order. Determine the size of the sample space when you choose the following:
(a) 7 objects from 15
answer:
(b) 2 objects from 17
answer:
(c) 4 objects from 24
answer:

7. (1 pt) setProbability1Combinations/ur_pb_j_7.pg
Suppose you are managing 18 employees, and you need to form three teams to work on different projects. Assume that all employees will work on a team, and that each employee has the same qualifications/skills so that everyone has the same probability of getting chosen. In how many different ways can the teams be chosen so that the number of employees on each project are as follows:
9, 4, 5
answer:

8. (1 pt) setProbability1Combinations/ur_pb_j_8.pg
A computer retail store has 9 personal computers in stock. A buyer wants to purchase 2 of them. Unknown to either the retail store or the buyer, 2 of the computers in stock have defective hard drives. Assume that the computers are selected at random.
(a) In how many different ways can the 2 computers be chosen?
answer:
(b) What is the probability that exactly one of the computers will be defective?
answer:
(c) What is the probability that at least one of the computers selected is defective?
answer:

9. (1 pt) setProbability1Combinations/ur_pb_j_9.pg
In how many ways can 5 novels, 2 mathematics books, and 1 biology book be arranged on a bookshelf if
(a) the books can be arranged in any order?
answer:
(b) the mathematics books must be together and the novels must be together?
answer:
(c) the novels must be together but the other books can be arranged in any order?
answer:

10. (1 pt) setProbability1Combinations/ur_pb_j_10.pg
From a group of 7 women and 9 men a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
(a) 2 of the men refuse to serve together?
answer:
(b) 2 of the women refuse to serve together?
answer:
(c) 1 man and 1 woman refuse to serve together?
answer:

11. (1 pt) setProbability1Combinations/ur_dis_9_1.pg
(a) A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in 2 sizes for each sex. How many different types of this shirt are made?
(b) How many bit strings of length 8 are there?
(c) How many bit strings of length 8 or less are there? (Count the empty string of length zero also.)
(d) How many strings of 6 lower case English letters are there that have the letter x in them somewhere? Here strings may use the same letter more than once. (Hint: It might be easier to first count the strings that don’t have an x in them.)
12. Find how many positive integers with exactly four decimal digits, that is, positive integers between 1000 and 9999 inclusive, have the following properties:

(a) have distinct digits.
(b) are divisible by 5 but not by 7.
(c) are divisible by 5.
(d) are divisible by 7.

13. How many strings of four decimal digits (Note there are 10 possible digits and a string can be of the form 0014 etc., i.e., can start with zeros.)

(a) do not contain the same digit twice?
(b) begin and end with a 1?

14. How many strings of five uppercase English letters are there

(a) that start with an X, if letters can be repeated?
(b) that start with an X, if no letter can be repeated?
(c) that start with the letters BO (in that order), if letters can be repeated?
(d) if no letter can be repeated?

15. Solve the following two “union” type questions:

(a) How many bit strings of length 7 either begin with 2 0s or end with 3 1s? (inclusive or)
(b) Every student in a discrete math class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 33 computer science majors (including joint majors), 30 math majors (including joint majors) and 9 joint majors?

16. A bowl contains 5 red balls and 5 blue balls. A woman selects balls at random without looking at them.

(a) How many balls must she select (minimum) to be sure of having at least three blue balls?
(b) How many balls must she select (minimum) to be sure of having at least three balls of the same color?

17. This question concerns bit strings of length six. These bit strings can be divided up into four types depending on their initial and terminal bit. Thus the types are: 0XXXX0, 0XXXX1, 1XXXX0, 1XXXX1.

How many bit strings of length six must you select before you are sure to have at least 5 that are of the same type? (Assume that when you select bit strings you always select different ones from ones you have already selected.)

18. Find the value of each of the following quantities:

\[ C(6, 3) = \]
\[ C(6, 1) = \]
\[ C(5, 1) = \]
\[ C(7, 2) = \]
\[ C(6, 4) = \]
\[ C(5, 1) = \]

19. There are 12 different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

20. How many bit strings of length 6 have:

(a) Exactly three 0s? 
(b) The same number of 0s as 1s? 
(d) At least three 1s?

21. 17 players for a softball team show up for a game:

(a) How many ways are there to choose 10 players to take the field?
(b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
(c) Of the 17 people who show up, 7 are women. How many ways are there to choose 10 players to take the field if at least one of these players must be women?

22. Suppose that a department contains 8 men and 18 women. How many ways are there to form a committee with 6 members if it must have strictly more women than men?

23. How many ways are there to select 7 countries in the United Nations to serve on a council if 3 is selected from a block of 55, 3 are selected from a block of 61 and 1 are selected from the remaining 73 countries?

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1. (1 pt) setProbability2BinomialTh/ur_pb_2_1.pg
Evaluate the binomial coefficient: \( \binom{17}{10} \)

2. (1 pt) setProbability2BinomialTh/ur_pb_2_2.pg
Expand the expression using the Binomial Theorem:
\[
(4x + 1)^5 = \ldots x^5 + \ldots x^4 + \ldots x^3 + \ldots x^2 + \ldots x + \ldots
\]

3. (1 pt) setProbability2BinomialTh/ur_pb_2_3.pg
Find the coefficient of \( x^4 \) in the expansion of \( \left( 2x^2 - \frac{5}{x} \right)^8 \)

4. (1 pt) setProbability2BinomialTh/ur_dis_9_14.pg
Find the coefficient of \( x^5 \) in \( (1 + x)^{11} \).

5. (1 pt) setProbability2BinomialTh/ur_dis_9_15.pg
What is the coefficient of \( x^7 y^{12} \) in the expansion of \( (-3x + 3y)^{19} \)?
1. (1 pt) setProbability3Events/ur_pb_3_1.pg

The sample space for an experiment contains five sample points. The probabilities of the sample points are:

- \( P(1) = P(2) = 0.05 \)
- \( P(3) = P(4) = 0.1 \)
- \( P(5) = 0.7 \)

Find the probability of each of the following events:

- \( A \) : { Either 5 or 4 occurs }
- \( B \) : { Either 5, 2, or 1 occurs }
- \( C \) : { 3 does not occur }

\[ P(A) = \quad P(B) = \quad P(C) = \]

2. (1 pt) setProbability3Events/ur_pb_3_2.pg

Two fair dice are tossed, and the up face on each die is recorded. Find the probability of each of the following events:

- \( A \) : { The sum of the numbers is even }
- \( B \) : { A 6 does not appear on either die }
- \( C \) : { The difference of the numbers is 1 }

\[ P(A) = \quad P(B) = \quad P(C) = \]

3. (1 pt) setProbability3Events/ur_pb_3_3.pg

Consider the experiment composed of one roll of a fair die followed by one toss of a fair coin. Determine the probability of each of the following events.

- \( A \) : { An odd number appears on the die; an H appears on the coin. }
- \( B \) : { An H appears on the coin. }
- \( C \) : { A 6 appears on the die; an H appears on the coin. }

\[ P(A) = \quad P(B) = \quad P(C) = \]

4. (1 pt) setProbability3Events/ur_pb_3_5.pg

In the game Roulette, a ball spins on a circular wheel that is divided into 38 arcs of equal length, numbered 00, 0, 1, 2, ..., 35, 36. The number on the arc on which the ball stops is the outcome of one play of the game. The numbers are also colored as follows:

1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36 are red,

2, 4, 6, 8, 10, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 31, 33, 35 are black,

0, 00 are green

Define the following events:

- \( A \) : { Outcome is an even number (0 and 00 are considered neither odd nor even) }
- \( B \) : { Outcome is a red number }
- \( C \) : { Outcome is a green number }
- \( D \) : { Outcome is a low number (1-18) }

Find the following probabilities:

\[ P(A) = \quad P(B) = \quad P(C) = \quad P(D) = \]

5. (1 pt) setProbability3Events/ur_pb_3_7.pg

A couple decided to have 3 children.

(a) What is the probability that they will have at least one girl?  
(b) What is the probability that all the children will be of the same gender?  

6. (1 pt) setProbability3Events/ur_pb_3_8.pg

Consider two people being randomly selected. (For simplicity, ignore leap years.)

(a) What is the probability that two people have a birthday on the 12th of any month?  
(b) What is the probability that two people have a birthday on the same day of the same month?  

7. (1 pt) setProbability3Events/ur_pb_3_10.pg

In a study by the Department of Transportation, there were a total of 98 drivers that were pulled over for speeding. Out of those 98 drivers, 37 were men who were ticketed, 12 were men who were not ticketed, 7 were women who were ticketed, and 42 were women who were not ticketed. Suppose one person was chosen at random.

(a) What is the probability that the selected person is a woman who was not ticketed?  
(b) What is the probability that the selected person is a man who was ticketed?  

8. (1 pt) setProbability3Events/ur_pb_3_12.pg

An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. These classes are open to any of the 97 students in the school. There are 38 in the Spanish class, 30 in the French class, and 23 in the German class. There are 15 students that in both Spanish and French, 8 are in both Spanish and German, and 8 are in both French and German. In addition, there are 4 students taking all 3 classes.

If one student is chosen randomly, what is the probability that he or she is taking exactly one language class?  
If two students are chosen randomly, what is the probability that neither of them is taking a language class?  

9. (1 pt) setProbability3Events/ur_pb_3_13.pg

A group of kids containing 10 boys and 10 girls is lined up in random order - that is, each of the 20! permutations is assumed
to be equally likely. What is the probability that the person in the 6-th position is a boy?

10. (1 pt) setProbability3Events/ur_pb_3_14.png
An instructor gives his class a set of 14 problems with the information that the next quiz will consist of a random selection of 4 of them. If a student has figured out how to do 12 of the problems, what is the probability the he or she will answer correctly
(a) all 4 problems? __________
(b) at least 3 problems? __________

11. (1 pt) setProbability3Events/ur_pb_3_15.png
How many people have to be in a room in order that the probability that at least two of them celebrate their birthday on the same day is at least 0.07? (Ignore leap years, and assume that all outcomes are equally likely.)

12. (1 pt) setProbability3Events/ur_pb_3_14a.png
A fair coin is tossed three times and the events $A$, $B$, and $C$ are defined as follows:
- $A$: { At least one head is observed }
- $B$: { At least two heads are observed }
- $C$: { The number of heads observed is odd }
Find the following probabilities by summing the probabilities of the appropriate sample points:
(a) $P(A) = $ __________
(b) $P(B \cup C) = $ __________
(c) $P(A' \cup B' \cup C) = $ __________

13. (1 pt) setProbability3Events/ur_pb_3_14a.png
A fair coin is tossed three times and the events $A$, $B$, and $C$ are defined as follows:
- $A$: { At least one head is observed }
- $B$: { At least two heads are observed }
- $C$: { The number of heads observed is odd }
Find the following probabilities by summing the probabilities of the appropriate sample points:
(a) $P(C) = $ __________
(b) $P(A \cap B) = $ __________
(c) $P(A' \cup B \cup C) = $ __________

14. (1 pt) setProbability3Events/ur_pb_3_14a.png
A sample space contains 7 sample points and events $A$ and $B$ as seen in the Venn diagram.
Let $P(1) = P(2) = P(3) = P(7) = 0.05$
$P(4) = P(5) = 0.1$
and $P(6) = 0.5$.

Use the Venn diagram and the probabilities of the sample points to find:
(a) $P(A') = $ __________
(b) $P(A \cup B) = $ __________
(c) $P(A \cap A') = $ __________
(d) $P(B') = $ __________

15. (1 pt) setProbability3Events/ur_pb_3_14a.png
A sample space contains 7 sample points and events $A$ and $B$ as seen in the Venn diagram.
Let $P(1) = P(2) = P(3) = P(7) = 0.05$
$P(4) = P(5) = 0.1$
and $P(6) = 0.6$.

Use the Venn diagram and the probabilities of the sample points to find:
(a) $P(A \cup B) = $ __________
(b) $P(B) = $ __________
(c) $P(A \cap B) = $ __________
(d) $P(A \cap B) = $ __________
16. The number 40 is written as a sum of three natural numbers 

\[ 40 = a + b + c \]

(the triple \(a, b, c\) is ordered; e.g., the decompositions \(40 = 1 + 1 + 38\) and \(40 = 1 + 38 + 1\) are different.

Also, assume that all the decompositions have equal probability.)

What is the probability that there exists a triangle with sides \(a, b,\) and \(c\)?

17. A quick quiz consists of 3 multiple choice problems, each of which has 4 answers, only one of which is correct. If you make random guesses on all 3 problems,

(a) What is the probability that all 3 of your answers are incorrect?

answer: 

(b) What is the probability that all 3 of your answers are correct?

answer: 

18. Two six-sided dice are rolled (one red one and one green one). Some possibilities are (Red=1, Green=5) or (Red=2, Green=2) etc.

(a) How many total possibilities are there?

For the rest of the questions, we will assume that the dice are fair and that all of the possibilities in (a) are equally likely.

(b) What is the probability that the sum on the two dice comes out to be 10? (Remember the answer will be a ratio, the denominator of which will be your answer in (a).)

(c) What is the probability that the sum on the two dice comes out to be 7?

(d) What is the probability that the numbers on the two dice are equal?

A card is selected at random from a standard 52-card deck.

(a) What is the probability that it is an ace?

(b) What is the probability that it is a heart?

(c) What is the probability that is an ace or a heart?

20. A five-card poker hand is dealt at random from a standard 52-card deck.

Note the total number of possible hands is \(\binom{52}{5}=2,598,960\).

Find the probabilities of the following scenarios:

(a) What is the probability that the hand contains exactly one ace?

Answer= \(\alpha\) \(\binom{52}{5}\) where \(\alpha = \) 

(b) What is the probability that the hand is a flush? (That is all the cards are of the same suit: hearts, clubs, spades or diamonds.)

Answer= \(\beta\) \(\binom{52}{5}\) where \(\beta = \) 

c) What is the probability that the hand is a straight flush?

Answer= \(\gamma\) \(\binom{52}{5}\) where \(\gamma = \) 

What is the probability that a positive integer \(m\) in the range 

\[ 1 \leq m \leq 100, \text{which is selected randomly, is divisible by 9}\]
1. (1 pt) setProbability4Conditional/ur_pb_A1.pg

If \( P(A) = 0.3 \), \( P(B) = 0.7 \), and \( P(A \cap B) = 0.3 \), then
(a) \( P(A | B) = \) _____ and
(b) \( P(B | A) = \) _____

2. (1 pt) setProbability4Conditional/ur_pb_A2.pg

A sample space contains six sample points and events \( A \), \( B \), and \( C \) as shown in the Venn diagram. The probabilities of the sample points are \( P(1) = 0.05 \), \( P(2) = 0.5 \), \( P(3) = 0.1 \), \( P(4) = 0.15 \), \( P(5) = 0.15 \), \( P(6) = 0.0499999999999999 \).
Use the Venn diagram and the probabilities of the sample points to find:
(a) \( P(\overline{C}) = \) _____
(b) \( P(B | A) = \) _____
(c) \( P(C | \overline{A}) = \) _____

3. (1 pt) setProbability4Conditional/ur_pb_A3a.png

A box contains one yellow, two red, and three green balls. Two balls are randomly chosen without replacement. Define the following events:
\( A : \{ \text{One of the balls is yellow} \} \)
\( B : \{ \text{At least one ball is red} \} \)
\( C : \{ \text{Both balls are green} \} \)
\( D : \{ \text{Both balls are of the same color} \} \)
Find the following conditional probabilities:
(a) \( P(B | A) = \) _____
(b) \( P(B | \overline{D}) = \) _____
(c) \( P(C | \overline{A}) = \) _____

4. (1 pt) setProbability4Conditional/ur_pb_A3a.png

A box contains one yellow, two red, and three green balls. Two balls are randomly chosen without replacement. Define the following events:
\( A : \{ \text{One of the balls is yellow} \} \)
\( B : \{ \text{At least one ball is red} \} \)
\( C : \{ \text{Both balls are green} \} \)
\( D : \{ \text{Both balls are of the same color} \} \)
Find the following conditional probabilities:
(a) \( P(B|A) = \) 
(b) \( P(B|D) = \) 
(c) \( P(C|D) = \) 

6. (1 pt) setProbabilityConditional/ur_pb_4.4.png

“Channel One” is an educational television network for which participating secondary schools are equipped with TV sets in every classroom. It has been found that 50% of secondary schools subscribe to Channel One, where of these subscribers 15% never use Channel One while 25% claim to use it more than 5 times per week.

Find the probability that a randomly selected secondary school subscribes to Channel One and uses it more than 5 times per week.

answer:  

7. (1 pt) setProbabilityConditional/ur_pb_4.5.png

Two fair dice, one blue and one red, are tossed, and the up face on each die is recorded. Define the following events:

- \( E : \{ \) The numbers are equal \( \} \)
- \( F : \{ \) The sum of the numbers is even \( \} \)

Find the following probabilities:

(a) \( P(E) = \) 
(b) \( P(F) = \) 
(c) \( P(E \cap F) = \) 
(d) \( P(E|F) = \) 
(e) \( P(F|E) = \) 

Are events \( E \) and \( F \) independent?

- A. no
- B. yes

8. (1 pt) setProbabilityConditional/ur_pb_4.5a.png

Two fair dice, one blue and one red, are tossed, and the up face on each die is recorded. Define the following events:

- \( E : \{ \) The sum of the numbers is even \( \} \)
- \( F : \{ A 6 \) on the blue die \( \} \)

Find the following probabilities:

(a) \( P(E) = \) 
(b) \( P(F) = \) 
(c) \( P(E \cap F) = \) 

Are events \( E \) and \( F \) independent?

- A. yes
- B. no

A sample space contains six sample points and events \( A, B, \) and \( C \) as shown in the Venn diagram. The probabilities of the sample points are \( P(1) = \frac{1}{12}, \, P(2) = \frac{1}{12}, \, P(3) = \frac{6}{12}, \, P(4) = \frac{1}{12}, \, P(5) = \frac{1}{12}, \, P(6) = \frac{2}{12} \).

Are events \( B \) and \( C \) mutually exclusive?

- A. No
- B. Yes

Use the Venn diagram and the probabilities of the sample points to find:

- \( P(B) = \) 
- \( P(C) = \) 
- \( P(B \cap C) = \) 

Are events \( B \) and \( C \) independent?

- A. Yes
- B. No

10. (1 pt) setProbabilityConditional/ur_pb_4.7.png

Scoring a hole-in-one is the greatest shot a golfer can make. Once 2 professional golfers each made holes-in-one on the 6\(^{th} \) hole at the same golf course at the same tournament. It has been found that the estimated probability of making a hole-in-one is \( \frac{1}{2711} \) for male professionals. Suppose that a sample of 2 professional male golfers is randomly selected.

(a) What is the probability that none of these golfers make a hole-in-one on the 9\(^{th} \) hole at the same tournament?

answer:  

(b) What is the probability that at least one of these golfers makes a hole-in-one on the 9\(^{th} \) hole at the same tournament?

answer:  

11. (1 pt) setProbabilityConditional/ur_pb_4.8.png

For two events \( A \) and \( B, \) \( P(A) = 0.6 \) and \( P(B) = 0.2. \)

(a) If \( A \) and \( B \) are independent, then

\( P(A \cap B) = \) 
\( P(A \cup B) = \) 
\( P(A|B) = \)
(b) If $A$ and $B$ are dependent and $P(A|B) = 0.2$, then
$P(A \cap B) = $ \\
$P(B|A) = $

12. (1 pt) setProbability4Conditional/ur_pb_d_9.pg

If $P(A) = 0.5$, $P(B) = 0.2$, and $P(A \cup B) = 0.66$, then
$P(A \cap B) = $

(a) Are events $A$ and $B$ independent? (Enter YES or NO)

(b) Are $A$ and $B$ mutually exclusive? (Enter YES or NO)

13. (1 pt) setProbability4Conditional/ur_pb_d_10.pg

The number 94 is written as a sum of three natural numbers

$94 = a + b + c$

(the triple $(a, b, c)$ is ordered; e.g., the decompositions $94 = 24 + 25 + 45$ and $94 = 25 + 45 + 24$ are different.

Also, assume that all the decompositions have equal probability.)

Given that there exists a triangle with sides $a$, $b$, and $c$, what is the probability that this triangle is isosceles?

14. (1 pt) setProbability4Conditional/ur_pb_d_11.pg

What is the probability that at least one of a pair of fair dice lands of 3, given that the sum of the dice is 8?

15. (1 pt) setProbability4Conditional/ur_pb_d_12.pg

In a certain community, 20% of the families own a dog, and 20% of the families that own a dog also own a cat. It is also know that 30% of all the families own a cat.

What is the probability that a randomly selected family owns a dog?

16. (1 pt) setProbability4Conditional/ur_pb_d_13.pg

Urn $A$ has 2 white and 11 red balls. Urn $B$ has 14 white and 7 red balls. We flip a fair coin. If the outcome is heads, then a ball from urn $A$ is selected, whereas if the outcome is tails, then a ball from urn $B$ is selected. Suppose that a red ball is selected. What is the probability that the coin landed heads?

17. (1 pt) setProbability4Conditional/ur_pb_d_14/ur_pb_d_14.pg

The probability of the closing of the $i$th relay in the circuits shown is given by $p_i$. Let $p_1 = 0.7$, $p_2 = 0.1$, $p_3 = 0.8$, $p_4 = 0.9$, $p_5 = 0.6$. If all relays function independently, what is the probability that a current flows between $A$ and $B$ for the respective circuits?

(a) $P = $ \\
(b) $P = $
### 1. (1 pt) setProbability5RandomSample/ur_pb_5_1.pg

(a) Count the number of ways to arrange a sample of 5 elements from a population of 9 elements.

answer: 

(b) If random sampling is to be employed, the probability that any particular sample will be selected is 

### 2. (1 pt) setProbability5RandomSample/ur_pb_5_2.pg

A financial firm is performing an assessment test and relies on a random sampling of their accounts. Suppose this firm has 6806 customer accounts numbered from 0001 to 6806.

One account is to be chosen at random. What is the probability that the selected account number is 2372?

answer: 
1. (1 pt) setProbability?RandomVariables/ur_pb_7_1.pg
Determine whether the following are valid probability distributions or not. Type "VALID" if it is valid, or type "INVALID" if it is not a valid probability distributions.

(a) 
\[
\begin{array}{cccc}
  x & 2 & 4 & 5 & 8 \\
P(x) & 0.1 & 0.1 & 0 & 0.8 \\
\end{array}
\]
answer: _____

(b) 
P(x) = \frac{1}{x}, where x = 1, 2, 3, ...
answer: _____

(c) 
\[
\begin{array}{cccc}
  x & 2 & 4 & 5 & 8 \\
P(x) & 0.3 & 0.3 & -0.2 & 0.6 \\
\end{array}
\]
answer: _____

2. (1 pt) setProbability?RandomVariables/ur_pb_7_2.pg
The mean and standard deviation of a random variable \( x \) are 10 and 2 respectively. Find the mean and standard deviation of the given random variables:

(1) \( y = x + 9 \)
\[ \mu = \text{___} \]
\[ \sigma = \text{___} \]

(2) \( v = 4x \)
\[ \mu = \text{___} \]
\[ \sigma = \text{___} \]

(3) \( w = 4x + 9 \)
\[ \mu = \text{___} \]
\[ \sigma = \text{___} \]

3. (1 pt) setProbability?RandomVariables/ur_pb_7_3.pg
Given the discrete probability distribution above, determine the following:

(a) \( P(6 \leq x < 8) = \text{___} \)

(b) \( P(x = 6) = \text{___} \)

(c) \( P(x \leq 9) = \text{___} \)

4. (1 pt) setProbability?RandomVariables/ur_pb_7_4.pg
Three dice are rolled. Let the random variable \( x \) represent the sum of the 3 dice. By assuming that each of the \( 6^3 \) possible outcomes is equally likely, find the probability that \( x \) equals 10.
\[ P(x = 10) = \text{___} \]

5. (1 pt) setProbability?RandomVariables/ur_pb_7_5.pg
Let \( x \) represent the difference between the number of heads and the number of tails when a coin is tossed 39 times. Then
\[ P(x = 3) = \text{___} \]

Four buses carrying 159 high school students arrive to Montreal. The buses carry, respectively, 37, 48, 33, and 41 students. One of the students is randomly selected. Let \( X \) denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let \( Y \) denote the number of students on his bus. Compute the expectations of \( X \) and \( Y \):
\[ E(X) = \text{___} \]
\[ E(Y) = \text{___} \]

7. (1 pt) setProbability?RandomVariables/ur_pb_7_9a.pg
Four buses carrying 148 high school students arrive to Montreal. The buses carry, respectively, 38, 43, 31, and 36 students. One of the students is randomly selected. Let \( X \) denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let \( Y \) denote the number of students on his bus. Compute the expectations and variances of \( X \) and \( Y \):
\[ E(X) = \text{___} \]
\[ Var(X) = \text{___} \]
\[ E(Y) = \text{___} \]
\[ Var(Y) = \text{___} \]

8. (1 pt) setProbability?RandomVariables/ur_pb_7_4a.pg
Two fair dice are rolled 5 times. Let the random variable \( x \) represent the number of times that the sum 7 occurs. The table below describes the probability distribution. Find the value of the missing probability.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.401877572016461</td>
</tr>
<tr>
<td>1</td>
<td>0.401877572016461</td>
</tr>
<tr>
<td>2</td>
<td>0.160751028806584</td>
</tr>
<tr>
<td>3</td>
<td>\text{___}</td>
</tr>
<tr>
<td>4</td>
<td>0.00321502057613169</td>
</tr>
<tr>
<td>5</td>
<td>0.000128600823045267</td>
</tr>
</tbody>
</table>

Would it be unusual to roll a pair of dice 5 times and get no 7s? (enter YES or NO) _____

9. (1 pt) setProbability?RandomVariables/ur_pb_7_5a.pg
A rock concert producer has scheduled an outdoor concert. The producer estimates the attendance will depend on the weather according to the following table.

<table>
<thead>
<tr>
<th>Weather</th>
<th>Attendance</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>wet, cold</td>
<td>50000</td>
<td>0.2</td>
</tr>
<tr>
<td>wet, warm</td>
<td>30000</td>
<td>0.2</td>
</tr>
<tr>
<td>dry, cold</td>
<td>15000</td>
<td>0.1</td>
</tr>
<tr>
<td>dry, warm</td>
<td>60000</td>
<td>0.5</td>
</tr>
</tbody>
</table>
(a) What is the expected attendance?
answer: _____

(b) If tickets cost $15 each, the band will cost $200000, plus $60000 for administration. What is the expected profit?
answer: _____

10. (1 pt) setProbability7RandomVariables/ur.png
Prizes and the chances of winning in a sweepstakes are given in the table below.

<table>
<thead>
<tr>
<th>Prize</th>
<th>Chances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20,000,000</td>
<td>1 chance in 400,000,000</td>
</tr>
<tr>
<td>$250,000</td>
<td>1 chance in 150,000,000</td>
</tr>
<tr>
<td>$25,000</td>
<td>1 chance in 60,000,000</td>
</tr>
<tr>
<td>$5,000</td>
<td>1 chance in 1,000,000</td>
</tr>
<tr>
<td>$900,000</td>
<td>1 chance in 700,000</td>
</tr>
<tr>
<td>A watch valued at $55</td>
<td>1 chance in 3,000</td>
</tr>
</tbody>
</table>

(a) Find the expected value (in dollars) of the amount won by one entry.

(b) Find the expected value (in dollars) if the cost of entering this sweepstakes is the cost of a postage stamp (34 cents)
If $x$ is a binomial random variable, compute $p(x)$ for each of the following cases:

(a) $n = 6, x = 1, p = 0.6$
\[ p(x) = \]

(b) $n = 3, x = 0, p = 0.7$
\[ p(x) = \]

(c) $n = 6, x = 5, p = 0.6$
\[ p(x) = \]

(d) $n = 6, x = 4, p = 0.3$
\[ p(x) = \]

The rates of on-time flights for commercial jets are continuously tracked by the U.S. Department of Transportation. Recently, Southwest Air had the best rate with 80% of its flights arriving on time. A test is conducted by randomly selecting 10 Southwest flights and observing whether they arrive on time.

(a) Find the probability that at least 3 flights arrive on time.
\[ \text{Enter YES or NO} \]

(b) Would it be unusual for Southwest to have 2 flights arrive late?

The Census Bureau reports that 82% of Americans over the age of 25 are high school graduates. A survey of randomly selected residents of certain county included 1150 who were over the age of 25, and 944 of them were high school graduates.

(a) Find the mean and standard deviation for the number of high school graduates in groups of 1150 Americans over the age of 25.
\[ \text{Mean} = \]
\[ \text{Standard deviation} = \]

(b) Is that county result of 944 unusually high, or low, or neither?
\[ \text{Enter HIGH or LOW or NEITHER} \]

To determine whether or not they have a certain disease, 120 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people (we are assuming that the pooled test will be positive if and only if at least one person in the pool has the disease); whereas, if the test is positive each of the 10 people will also be individually tested and, in all, 11 tests will be made on this group. Assume the probability that a person has the disease is 0.07 for all people, independently of each other, and compute the expected number of tests necessary for each group.

answer:

A man claims to have extrasensory perception. As a test, a fair coin is flipped 26 times, and the man is asked to predict the outcome in advance. He gets 23 out of 26 correct. What is the probability that he would have done at least this well if he had no ESP?
1. (1 pt) setProbability9PoissonDist/ur_pb_9_1.pg

Given that \( x \) is a random variable having a Poisson distribution, compute the following:

(a) \( P(x = 7) \) when \( \mu = 2 \)
\[ P(x) = \]

(b) \( P(x \leq 7) \) when \( \mu = 5 \)
\[ P(x) = \]

(c) \( P(x > 9) \) when \( \mu = 1 \)
\[ P(x) = \]

(d) \( P(x < 2) \) when \( \mu = 0.5 \)
\[ P(x) = \]

2. (1 pt) setProbability9PoissonDist/ur_pb_9_2.pg

A statistics professor finds that when he schedules an office hour for student help, an average of 2.3 students arrive. Find the probability that in a randomly selected office hour, the number of student arrivals is 1.

3. (1 pt) setProbability9PoissonDist/ur_pb_9_3.pg

The mean number of patients admitted per day to the emergency room of a small hospital is 1. If, on any given day, there are only 6 beds available for new patients, what is the probability that the hospital will not have enough beds to accommodate its newly admitted patients?

answer:

4. (1 pt) setProbability9PoissonDist/ur_pb_9_4.pg

A certain typing agency employs two typists. The average number of errors per article is 4.7 when typed by the first typist and 1.8 when typed by the second. If your article is equally likely to be typed by either typist, find the probability that it will have no errors.
1. (1 pt) setProbability10NormalDist/ur_pb_10_1.pg
Find the following probabilities for the standard normal random variable $z$:

(a) $P(-0.14 \leq z \leq 1.66) =$

(b) $P(-2.24 \leq z \leq 2.2) =$

(c) $P(z > -1.27) =$

**WARNINGS:**
* This option —8— is not recognized in this subroutine

**WARNING:**

2. (1 pt) setProbability10NormalDist/ur_pb_10_2.pg
Assume that the readings on the thermometers are normally distributed with a mean of 0\(^\circ\)C and a standard deviation of 1.00\(^\circ\)C. A thermometer is randomly selected and tested. Find the probability of each reading in degrees.

(a) Between 0 and 1.59:

(b) Between $-1.18$ and 0:

(c) Between $-1.95$ and 0.12:

(b) Less than $-0.029999999999998$

**WARNINGS:**
* This option —8— is not recognized in this subroutine
Assume that the readings on the thermometers are normally distributed with a mean of 0\degree C and a standard deviation of 1.00\degree C. Find \( P_{70} \), the 70\textsuperscript{th} percentile. This is the temperature reading separating the bottom 70% from the top 30%.

Suppose that the readings on the thermometers are normally distributed with a mean of 0\degree C and a standard deviation of 1.00\degree C. If 8% of the thermometers are rejected because they have readings that are too low, but all other thermometers are acceptable, find the reading that separates the rejected thermometers from the others.
(a) $x = 37$

(b) $x = 33$

(c) $x = 14$

(d) $x = 37$

(e) $x = 11$

(f) $x = 29$

8. (1 pt) setProbability10NormalDist/ur_ph_10.7.pg

Suppose $x$ is a normally distributed random variable with $\mu = 10.9$ and $\sigma = 3.3$. Find each of the following probabilities:

(a) $P(5 \leq x \leq 15.3) = $ 

(b) $P(5.5 \leq x \leq 14.9) = $ 

(c) $P(9.2 \leq x \leq 15) = $ 

(d) $P(x \geq 7.9) = $ 

9. (1 pt) setProbability10NormalDist/ur_ph_10.9.pg

The physical fitness of an athlete is often measured by how much oxygen the athlete takes in (which is recorded in milliliters per kilogram, ml/kg). The mean maximum oxygen uptake for
elite athletes has been found to be 60 with a standard deviation of 7.6. Assume that the distribution is approximately normal.

(a) What is the probability that an elite athlete has a maximum oxygen uptake of at least 50 ml/kg?

answer: 

(b) What is the probability that an elite athlete has a maximum oxygen uptake of 60 ml/kg or lower?

answer: 

(c) Consider someone with a maximum oxygen uptake of 28 ml/kg. Is it likely that this person is an elite athlete? Write "YES" or "NO."

answer: 

**WARNINGS:**

* This option —— is not recognized in this subroutine 

**10.** (1 pt) setProbability10NormalDist/ur_ph_10_10.pg

The combined math and verbal scores for females taking the SAT-I test are normally distributed with a mean of 998 and a standard deviation of 202 (based on data from the College Board). If a college includes a minimum score of 875 among its requirements, what percentage of females do not satisfy that requirement?

**WARNINGS:**

* This option —— is not recognized in this subroutine 

**11.** (1 pt) setProbability10NormalDist/ur_ph_10_11.pg

The extract of a plant native to Taiwan has been tested as a possible treatment for Leukemia. One of the chemical compounds produced from the plant was analyzed for a particular collagen.

The collagen amount was found to be normally distributed with a mean of 77 and standard deviation of 6.8 grams per milliliter.

(a) What is the probability that the amount of collagen is greater than 61 grams per milliliter?

answer: 

(b) What is the probability that the amount of collagen is less than 85 grams per milliliter?

answer: 

(c) What percentage of compounds formed from the extract of this plant fall within 3 standard deviations of the mean?

answer: 

**WARNINGS:**

* This option —— is not recognized in this subroutine 

**12.** (1 pt) setProbability10NormalDist/ur_ph_10_12.pg

IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Mensa is an international society that has one - and only one - qualification for membership: a score in the top 2on an IQ test.

(a) What IQ score should one have in order to be eligible for Mensa?

(b) In a typical region of 155,000 people, how many are eligible for Mensa?

**13.** (1 pt) setProbability10NormalDist/ur_ph_10_13.pg

Using diaries for many weeks, a study on the lifestyles of visually impaired students was conducted. The students kept track
of many lifestyle variables including how many hours of sleep obtained on a typical day. Researchers found that visually impaired students averaged 9.88 hours of sleep, with a standard deviation of 2.57 hours. Assume that the number of hours of sleep for these visually impaired students is normally distributed.

(a) What is the probability that a visually impaired student gets less than 6.1 hours of sleep?

answer: _____

(b) What is the probability that a visually impaired student gets between 6.5 and 7.9 hours of sleep?

answer: _____

(c) Thirty percent of students get less than how many hours of sleep on a typical day?

answer: _____ hours

WARNINGS:

* This option —8— is not recognized in this subroutine

HASH(0x8ccba40) TABLE border = "2" cellpadding = "3" BGCOLOR = "#FFFFFF" www/welcomeAction.pl line 464

--- main::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 464

HASH(0x8ccba40) TABLE border = "2" cellpadding = "3" BGCOLOR = "#FFFFFF" www/welcomeAction.pl line 695

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--- Safe::reval called at /usr/local/lib/perl5/5.6.1/mach/Safe.pm line 222
--- PGtranslator::translate called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 733
--- PGtranslator::translate called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 733
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--- main::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 174
--- main::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 695
--- main::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 174

14.(1 pt) setProbability10NormalDist/ur_pb_10_14.pg
Women’s weights are normally distributed with a mean given by \( \mu = 143 \) lb and a standard deviation given by \( \sigma = 29 \) lb. Find the eighth decile, \( D_8 \), which separates the bottom 80% from the top 20%.

--- main::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 464
--- main::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 174

15.(1 pt) setProbability10NormalDist/ur_pb_10_16.pg
Healthy people have body temperatures that are normally distributed with a mean of 98.20°F and a standard deviation of 0.62°F.

(a) If a healthy person is randomly selected, what is the probability that he or she has a temperature above 99.2°F?

answer: _____

(b) A hospital wants to select a minimum temperature for requiring further medical tests. What should that temperature be, if we want only 1.5% of healthy people to exceed it?

answer: _____

WARNINGS:

* This option —8— is not recognized in this subroutine

HASH(0x8cc9a34) TABLE border = "2" cellpadding = "3" BGCOLOR = "#FFFFFF" www/welcomeAction.pl line 374

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--- main::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 695
--- main::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts/welcomeAction.pl line 174
Assume that women’s weights are normally distributed with a mean given by $\mu = 143$ lb and a standard deviation given by $\sigma = 29$ lb.

(a) If 1 woman is randomly selected, find the probability that her weight is above 176

(b) If 3 women are randomly selected, find the probability that they have a mean weight above 176

(c) If 69 women are randomly selected, find the probability that they have a mean weight above 176

**WARNINGS:**

* This option —8— is not recognized in this subroutine

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1. (1 pt) setProbability11CentralLimitTh/ur_pb11_1J.png

Cans of regular Coke are labeled as containing 12 oz.

---

2. (1 pt) setProbability11CentralLimitTh/ur_pb11_12.png

Cans of regular Coke are labeled as containing 12 oz.
Statistics students weighted the content of 7 randomly chosen cans, and found the mean weight to be 12.14.

Assume that cans of Coke are filled so that the actual amounts are normally distributed with a mean of 12.00 oz and a standard deviation of 0.1 oz. Find the probability that a sample of 7 cans will have a mean amount of at least 12.14 oz.

**WARNINGS:**
* This option —8— is not recognized in this subroutine
HASH@/lib/PG/AnswerCheck.pm line 3184
PGanswer=correct "normal" called at /usr/local/lib/perl5/5.6.1/mach/Safe.pm line 22
— PGerror:deValuations called at (eval 162) line 447
— PGerror:normal called at (eval 57) line 27
— Safe:reveal called at /ww/home/apizer/webwork/system/lib/PGtranslator.pm line 733
— PGtranslator:translate called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174
— main:createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 695
— main:downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174

(1 pt) setProbability11CentralLimitTh/ur_pb_11_3.pg
Scores for men on the verbal portion of the SAT-I test are normally distributed with a mean of 509 and a standard deviation of 112.

(a) If 1 man is randomly selected, find the probability that his score is at least 575.

(b) If 17 men are randomly selected, find the probability that their mean score is at least 575.

17 randomly selected men were given a review course before taking the SAT test. If their mean score is at least 575, is there a strong evidence to support the claim that the course is actually effective?

(Enter YES or NO)

**WARNINGS:**
* This option —8— is not recognized in this subroutine
HASH@/lib/PG/AnswerCheck.pm line 3184
PGanswer=correct "normal" called at /usr/local/lib/perl5/5.6.1/mach/Safe.pm line 22
— PGerror:deValuations called at (eval 162) line 447
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/welcomeAction.pl line 174
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— main:downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174

(1 pt) setProbability11CentralLimitTh/ur_pb_11_4.pg
The Central Limit Theorem says

- A. When $n < 30$, the sampling distribution of $\bar{X}$ will be approximately a normal distribution.
- B. When $n < 30$, the original population will be approximately a normal distribution.
- C. When $n > 30$, the sampling distribution of $\bar{X}$ will be approximately a normal distribution.
- D. When $n > 30$, the original population will be approximately a normal distribution.
- E. None of the above

(1 pt) setProbability11CentralLimitTh/ur_pb_11_5.pg
A soft drink bottler purchases glass bottles from a vendor. The bottles are required to have an internal pressure of at least 150 pounds per square inch (psi). A prospective bottle vendor claims that its production process yields bottles with a mean internal pressure of 157 psi and a standard deviation of 3 psi. The bottler strikes an agreement with the vendor that permits the bottler to sample from the production process to verify the claim. The bottler randomly selects 40 bottles from the list 10000 produced, measures the internal pressure of each, and finds the mean pressure for the sample to be 0.8 psi below the process mean cited by the vendor.

(a) Assuming that the vendor is correct in his claim, what is the probability of obtaining a sample mean this far or further below the process mean?

(b) If the standard deviation were 3 psi as claimed, but the mean was 154 psi, what is the probability of obtaining a sample mean of 156.2 psi or below?

(c) If the process mean were 157 psi as claimed, but the standard deviation was 1.2 psi, what is the probability of obtaining a sample mean of 156.2 psi or below?

**WARNINGS:**
* This option —8— is not recognized in this subroutine
HASH@/lib/PG/AnswerCheck.pm line 3184
PGanswer=correct "normal" called at /usr/local/lib/perl5/5.6.1/mach/Safe.pm line 22
— PGerror:deValuations called at (eval 162) line 447
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— PGtranslator:translate called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174
— main:createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 695
— main:downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174

A soft drink bottler purchases glass bottles from a vendor. The bottles are required to have an internal pressure of at least 150 pounds per square inch (psi). A prospective bottle vendor claims that its production process yields bottles with a mean internal pressure of 157 psi and a standard deviation of 3 psi. The bottler strikes an agreement with the vendor that permits the bottler to sample from the production process to verify the claim. The bottler randomly selects 40 bottles from the list 10000 produced, measures the internal pressure of each, and finds the mean pressure for the sample to be 0.8 psi below the process mean cited by the vendor.
6. (1 pt) setProbability11CentralLimitTh/ur_pb_11_6.pg

Suppose that from the past experience a professor knows that the test score of a student taking his final examination is a random variable with mean 60 and standard deviation 11. How many students would have to take the examination to ensure, with probability at least 0.94, that the class average would be within 2.5 of 60?

7. (1 pt) setProbability11CentralLimitTh/ur_pb_11_7.pg

75 numbers are rounded off to the nearest integer and then summed. If the individual round-off error are uniformly distributed over (−.5,.5) what is the probability that the resultant sum differs from the exact sum by more than 2?

8. (1 pt) setProbability11CentralLimitTh/ur_pb_11_8.pg

A die is continuously rolled until the total sum of all rolls exceeds 225. What is the probability that at least 70 rolls are necessary?
1.(1 pt) setProbability12NormApproxBinom/ur_pb_12_1.pg

Use normal approximation to estimate the probability of getting at most 55 girls in 100 births. Assume that boys and girls are equally likely.

**WARNINGS:**

* This option —S— is not recognized in this subroutine

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--- More details:---
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--- PG:::rcommand called at (eval 163) line 64 ---
--- PG:::nornormal groech called at (eval 57) line 30 ---
--- Safe:::seeval called at /ww/home/apizer/webwork/system/lib/PGtranslator.pm line 733 ---
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/welcomeAction.pl line 695 ---
--- main:::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 464 ---
--- main:::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174 ---

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--- PG:::nornormal groech called at (eval 57) line 30 ---
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--- main:::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 464 ---
--- main:::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174 ---

2.(1 pt) setProbability12NormApproxBinom/ur_pb_12_2.pg

Use normal approximation to estimate the probability of getting a true/false test of 20 questions if the minimum passing grade is 70% and all responses are random guesses.

**WARNINGS:**

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HASH[@S]68be57d4@{TABLE border = "2" cellpadding = "3" BGCOLOR = "#FFFFFF"} to TDS:
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--- More details:---
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--- PG:::rcommand called at (eval 163) line 64 ---
--- PG:::nornormal groech called at (eval 57) line 30 ---
--- Safe:::seeval called at /ww/home/apizer/webwork/system/lib/PGtranslator.pm line 733 ---
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/welcomeAction.pl line 464 ---
--- main:::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174 ---

3.(1 pt) setProbability12NormApproxBinom/ur_pb_12_3.pg

An airline company is considering a new policy of booking as many as 160 persons on an airplane that can seat only 160.5. (Past studies have revealed that only 89% of the booked passengers actually arrive for the flight.) Estimate the probability that if the company books 160 persons, not enough seats will be available.

**WARNINGS:**

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HASH[@S]68be57d4@{TABLE border = "2" cellpadding = "3" BGCOLOR = "#FFFFFF"} to TDS:
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--- More details:---
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--- PG:::nornormal groech called at (eval 57) line 30 ---
--- Safe:::seeval called at /ww/home/apizer/webwork/system/lib/PGtranslator.pm line 733 ---
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--- main:::createTexSource called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 464 ---
--- main:::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174 ---

4.(1 pt) setProbability12NormApproxBinom/ur_pb_12_4.pg

A multiple-choice test consists of 27 questions with possible answers of a, b, c, d, e, f. Estimate the probability that with random guessing, the number of correct answers is at least 12.

**WARNINGS:**

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--- More details:---
--- PG:::print{default options called at (eval 162) line 447 ---
--- PG:::rcommand called at (eval 163) line 64 ---
--- PG:::nornormal groech called at (eval 57) line 30 ---
--- Safe:::seeval called at /ww/home/apizer/webwork/system/lib/PGtranslator.pm line 733 ---
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/welcomeAction.pl line 464 ---
--- main:::downloadAllSets called at /ww/home/apizer/webwork/system/cgi/cgi-scripts-
/welcomeAction.pl line 174 ---
1. (1 pt) setProbability13UniformDist/ur_pb13_1.pg
A manager of an apartment store reports that the time of a customer on the second floor must wait for the elevator has a uniform distribution ranging from 0 to 5 minutes. If it takes the elevator 15 seconds to go from floor to floor, find the probability that a hurried customer can reach the first floor in less than 4 minutes after pushing the elevator button on the second floor.

answer: 

2. (1 pt) setProbability13UniformDist/ur_pb13_2.pg
Suppose the time to process a loan application follows a uniform distribution over the range 5 to 14 days. What is the probability that a randomly selected loan application takes longer than 9 days to process?

answer: 

3. (1 pt) setProbability13UniformDist/ur_pb13_3.pg
Suppose \( x \) is a random variable best described by a uniform probability that ranges from 2 to 5. Compute the following:

(a) the probability density function \( f(x) = \) 
(b) \( \mu = \) 
(c) \( \sigma = \) 
(d) \( P(\mu - \sigma < x < \mu + \sigma) = \) 
(e) \( P(x < 3.83) = \) 

4. (1 pt) setProbability13UniformDist/ur_pb13_4.pg
Suppose a random variable \( x \) is best described by a uniform probability distribution with range 0 to 6. Find the value of \( a \) that makes the following probability statements true.

(a) \( P(x < a) = 0.68 \)
(b) \( P(x < a) = 0.95 \)
(c) \( P(x \geq a) = 0.59 \)
(d) \( P(x > a) = 0.03 \)
(e) \( P(2.99 \leq x \leq a) = 0.5 \)

5. (1 pt) setProbability13UniformDist/ur_pb13_5.pg
The weather in Rochester in December is fairly constant. Records indicate that the low temperature for each day of the month tend to have a uniform distribution over the interval 15\(^\circ\) to 35\(^\circ\)F. A business man arrives on a randomly selected day in December.

(a) What is the probability that the temperature will be above 19\(^\circ\)?

answer: 

(b) What is the probability that the temperature will be between 18\(^\circ\) and 26\(^\circ\)?

answer: 

(c) What is the expected temperature?

answer: 

6. (1 pt) setProbability13UniformDist/ur_pb13_6.pg
If \( a \) is uniformly distributed over \([-19, 16]\), what is the probability that the roots of the equation

\[ x^2 + ax + a + 35 = 0 \]

are both real?
1. (1 pt) setProbability14ExponentialDist/ur_pb_14_1.pg
Suppose that the time (in hours) required to repair a machine is an exponentially distributed random variable with parameter \( \lambda = 0.6 \). What is

(a) the probability that a repair time exceeds 9 hours? __________

(b) the conditional probability that a repair takes at least 6 hours, given that it takes more than 4 hours? __________

2. (1 pt) setProbability14ExponentialDist/ur_pb_14_2.pg
Suppose that the life distribution of an item has hazard rate function \( \lambda(t) = 2.4t^2, t > 0 \). What is the probability that

(a) the item survives to age 3? __________

(b) the item’s lifetime is between 1 and 3? __________

(c) a 0.5-year-old item will survive to age 3.5? __________

3. (1 pt) setProbability14ExponentialDist/ur_pb_14_3.pg
Let \( X \) be an exponential random variable with parameter \( \lambda = 10 \), and let \( Y \) be the random variable defined by \( Y = 10e^X \). Compute the probability density function of \( Y \):

\[
f_Y(t) = \ldots
\]
1. (1 pt) setProbability15OtherContDist/ur_pb_15_1.pg
You’ll need to use the formatted text mode in order to do this problem: click the “formatted text” button on the bottom of the page and then click “submit answers”.

Let $X$ be a random variable with probability density function

$$f(x) = \begin{cases} c(8x - x^2) & \text{if } 0 < x < 8 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of $c$:

$c =$

Find the cumulative distribution function of $X$:

$$F(x) = \begin{cases} \text{if } x \leq 0 \\ \text{if } 0 < x < 8 \\ \text{if } x \geq 8 \end{cases}$$

2. (1 pt) setProbability15OtherContDist/ur_pb_15_2.pg
You’ll need to use the formatted text mode in order to do this problem: click the “formatted text” button on the bottom of the page and then click “submit answers”.

The probability density function of $X$, the lifetime of a certain type of device (measured in months), is given by

$$f(x) = \begin{cases} 0 & \text{if } x < 17 \\ \frac{17}{x^2} & \text{if } x > 17 \end{cases}$$

Find the following:

$P(X > 30) =$

The cumulative distribution function of $X$:

$$F(x) = \begin{cases} \text{if } x < 17 \\ \text{if } x > 17 \end{cases}$$

The probability that at least one out of 5 devices of this type will function for at least 23 months:

3. (1 pt) setProbability15OtherContDist/ur_pb_15_3.pg
The density function of $X$ is given by

$$f(x) = \begin{cases} a + bx^2 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If the expectation of $X$ is $E(X) = 1.75$, find $a$ and $b$.

$a =$

$b =$
1. The joint probability density function of $X$ and $Y$ is given by

$$f(x, y) = c(y^2 - 324x^2)e^{-y}, \quad \frac{y}{18} \leq x \leq \frac{y}{18}, \quad 0 < y < \infty$$

Find $c$ and the expected value of $X$:

$c = \underline{_____}$

$E(X) = \underline{_____}$

2. $x$ and $y$ are uniformly distributed over the interval $[0, 1]$. Find the probability that $|x - y|$, the distance between $x$ and $y$, is less than 0.9.

3. A man and a woman agree to meet at a cafe about noon. If the man arrives at a time uniformly distributed between 11:30 and 12:10 and if the woman independently arrives at a time uniformly distributed between 11:40 and 12:30, what is the probability that the first to arrive waits no longer than 5 minutes?

answer: \underline{_____}

4. Two points are selected randomly on a line of length 26 so as to be on opposite sides of the midpoint of the line. In other words, the two points $X$ and $Y$ are independent random variables such that $X$ is uniformly distributed over $[0, 13]$ and $Y$ is uniformly distributed over $[13, 26]$. Find the probability that the distance between the two points is greater than 9.

answer: \underline{_____}

5. Let

$$f(x) = \begin{cases} cx^3y^2 & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the following:

(a) $c$ such that $f(x, y)$ is a probability density function:

$c = \underline{_____}$

(b) Expected values of $X$ and $Y$:

$E(X) = \underline{_____}$

$E(Y) = \underline{_____}$

(c) Are $X$ and $Y$ independent? (enter YES or NO) \underline{_____}

6. Let $A$, $B$, and $C$ be independent random variables, uniformly distributed over $[0, 2]$, $[0, 7]$, and $[0, 3]$ respectively. What is the probability that both roots of the equation $Ax^2 + Bx + C = 0$ are real? \underline{_____}

7. Assume that the monthly worldwide average number of airplane crashes of commercial airlines is 2.2. What is the probability that there will be

(a) exactly 3 such accidents in the next month? \underline{_____}

(b) less than 6 such accidents in the next 3 months? \underline{_____}

(c) exactly 3 such accidents in the next 5 months? \underline{_____}

8. Sam’s bowling scores are approximately normally distributed with mean 130 and standard deviation 15, while John’s scores are normally distributed with mean 145 and standard deviation 24. If Sam and John each bowl one game, then assuming that their scores are independent random variables, approximate the probability that the total of their scores is above 275.

9. The joint probability mass function of $X$ and $Y$ is given by

$$p(1,1) = 0.3 \quad p(1,2) = 0.15 \quad p(1,3) = 0.1$$

$$p(2,1) = 0.1 \quad p(2,2) = 0.15 \quad p(2,3) = 0.05$$

$$p(3,1) = 0.05 \quad p(3,2) = 0.1 \quad p(3,3) = 0$$

(a) Compute the conditional mass function of $Y$ given $X = 3$: $P(Y = 1|X = 3) = \underline{_____}$

$b) Are X and Y independent? (enter YES or NO) \underline{_____}$

(c) Compute the following probabilities:

$P(X + Y > 4) = \underline{_____}$

$P(XY = 2) = \underline{_____}$

$P(\frac{Y}{X} > 1) = \underline{_____}$

10. The joint probability mass function of $X$ and $Y$ is given by

$$p(1,1) = 0 \quad p(1,2) = 0.05 \quad p(1,3) = 0.1$$

$$p(2,1) = 0.15 \quad p(2,2) = 0.3 \quad p(2,3) = 0.15$$

$$p(3,1) = 0.05 \quad p(3,2) = 0.1 \quad p(3,3) = 0.1$$

Compute the following probabilities:

$P(X + Y > 3) = \underline{_____}$

$P(XY = 3) = \underline{_____}$

$P\left(\frac{Y}{X} > 1\right) = \underline{_____}$

11. The joint probability mass function of $X$ and $Y$ is given by

$$p(1,1) = 0.05 \quad p(1,2) = 0.05 \quad p(1,3) = 0.05$$

$$p(2,1) = 0.1 \quad p(2,2) = 0.2 \quad p(2,3) = 0.05$$

$$p(3,1) = 0.1 \quad p(3,2) = 0.1 \quad p(3,3) = 0.3$$
(a) Compute the conditional mass function of $Y$ given $X = 2$:

$P(Y = 1 | X = 2) =$

$P(Y = 2 | X = 2) =$

$P(Y = 3 | X = 2) =$

(b) Are $X$ and $Y$ independent? (enter YES or NO)

12. (1 pt) setProbability16JointDist/ur_pb_16_10.png

Two points along a straight stick of length 37cm are randomly selected. The stick is then broken at those two points. Find the probability that all of the resulting pieces have length at least 9cm.
1. (1 pt) setProbability17Expectation/ur_pb17_1.pg
A fair die is rolled 16 times. What is the expected sum of the 16 rolls?

2. (1 pt) setProbability17Expectation/ur_pb17_2.pg
24 people arrive separately to a professional dinner. Upon arrival, each person looks to see if he or she has any friends among those present. That person then either sits at the table of a friend or at an unoccupied table if none of those present is a friend. Assuming that each of the $\binom{24}{2}$ pairs of people are, independently, friends with probability 0.4, find the expected number of occupied tables.

3. (1 pt) setProbability17Expectation/ur_pb17_3.pg
Consider $n = 18$ independent flips of a fair coin. Say that a changeover occurs whenever an outcome differs from the one preceding it. For example, if $n = 6$ and the outcome is $T H T T H T$, then there is a total of 4 changeovers. Find the expected number of changeovers for $n = 18$.

4. (1 pt) setProbability17Expectation/ur_pb17_4.pg
If $E[X] = -1$ and $\text{Var}(X) = 3$, then $E[(2 + 4X)^2] =$ ____________
and $\text{Var}(5 + 3X)^2 =$ ____________
1. (1 pt) setStatistics1Data/ur_stat1_1.pg

Determine whether the following examples of data are quantitative or qualitative. Write “QUANTITATIVE” for quantitative and “QUALITATIVE” for qualitative. (without quotations)

(a) Your college GPA.
answer: 

(b) The occupation of your neighbors.
answer: 

(c) The amount of bacteria on a piece of moldy bread.
answer: 

(d) The marital status of your coworkers.
answer: 

2. (1 pt) setStatistics1Data/ur_stat1_2.pg

Determine whether the following examples are discrete or continuous data sets. Write “DISCRETE” for discrete and “CONTINUOUS” for continuous. (without quotations)

(a) The length of time needed for a student to complete a homework assignment.
answer: 

(b) The length of time it takes to fill up your gas tank.
answer: 

(c) The distance traveled by a city bus each day.
answer: 

(d) The number of voters who vote Democratic.
answer: 

3. (1 pt) setStatistics1Data/ur_stat1_3.pg

Determine whether the follow descriptions correspond to an observational study or an experiment. Write "EXPERIMENT" for experiment and "OBSERVATION" for observational study. (without quotations)

(a) The effectiveness of lecture teaching is tested with a sample of students who has completed numerous lecture style courses.
answer: 

(b) A new antibiotic is tested in effectiveness by recording how the drug works on patients that already take the drug.
answer: 

(c) A new study examines how efficiently athletes burn calories by limiting their diets.
answer: 

4. (1 pt) setStatistics1Data/ur_stat1_4.pg

Given the frequency table above, construct the following:
(a) The relative frequency table that corresponds with the above table.

<table>
<thead>
<tr>
<th>Grade on Statistics Exam</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>50 – 59</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>60 – 69</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>70 – 79</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>80 – 89</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>90 – 100</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

(b) The cumulative frequency table that corresponds with the above table.

<table>
<thead>
<tr>
<th>Grade on Statistics Exam</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 50</td>
<td></td>
</tr>
<tr>
<td>50 – 59</td>
<td></td>
</tr>
<tr>
<td>60 – 69</td>
<td></td>
</tr>
<tr>
<td>70 – 79</td>
<td></td>
</tr>
<tr>
<td>80 – 89</td>
<td></td>
</tr>
<tr>
<td>90 – 100</td>
<td></td>
</tr>
</tbody>
</table>

5. (1 pt) setStatistics1Data/ur_stat1_5.pg

Complete the table below.

<table>
<thead>
<tr>
<th>Books read within the past year</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>0 – 4</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>5 – 9</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>10 – 14</td>
<td>0.189655172413793</td>
<td></td>
</tr>
<tr>
<td>15 – 19</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>20 – 25</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>58</td>
<td>1</td>
</tr>
</tbody>
</table>

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, © UR
The length (pgs) of math research projects is given below. Using this information, calculate the range, variance, and standard deviation.

\[
\begin{align*}
47, & \quad 41, \quad 27, \quad 29, \quad 35, \quad 52, \quad 25, \quad 11, \quad 59, \quad 25, \quad 31 \\
\text{range} &= \ldots \\
\text{variance} &= \ldots \\
\text{standard deviation} &= \ldots
\end{align*}
\]

Calculate the mode, mean, and median of the following data:

\[
\begin{align*}
4, & \quad 8, \quad 14, \quad 13, \quad 10, \quad 10, \quad 24 \\
\text{Mode} &= \ldots \\
\text{Mean} &= \ldots \\
\text{Median} &= \ldots
\end{align*}
\]

Given the data set below, calculate the range, variance, and standard deviation.

\[
\begin{align*}
42, & \quad 30, \quad 35, \quad 22, \quad 16, \quad 5, \quad 25, \quad 16, \quad 48 \\
\text{range} &= \ldots \\
\text{variance} &= \ldots \\
\text{standard deviation} &= \ldots
\end{align*}
\]

For each of the given the data sets below, calculate the mean, variance, and standard deviation.

\[
\begin{align*}
(a) & \quad 20, \quad 41, \quad 13, \quad 95, \quad 55, \quad 45, \quad 69, \quad 98, \quad 67 \\
\text{mean} &= \ldots \\
\text{variance} &= \ldots \\
\text{standard deviation} &= \ldots \\
(b) & \quad 49, \quad 49, \quad 42, \quad 60, \quad 45, \quad 53 \\
\text{mean} &= \ldots \\
\text{variance} &= \ldots \\
\text{standard deviation} &= \ldots \\
(c) & \quad 4.2, \quad 4.1, \quad 2.9, \quad 3.3, \quad 3.2 \\
\text{mean} &= \ldots \\
\text{variance} &= \ldots \\
\text{standard deviation} &= \ldots
\end{align*}
\]

Calculate the mean and median of the following grades on a math test:

\[
\begin{align*}
93, & \quad 92, \quad 88, \quad 83, \quad 83, \quad 82, \quad 82, \quad 78, \quad 66, \quad 53, \quad 33 \\
\text{Mean} &= \ldots \\
\text{Median} &= \ldots
\end{align*}
\]

If the average low temperature of a winter month in Rochester, NY is 22° and the standard deviation is 3.3, then according to Chebyshev’s theorem, the percentage of averages low temperatures in Rochester, NY between 15.4° and 28.6° is \( \ldots \)%.

Calculate the mean and median of the following data:

\[
\begin{align*}
-2, & \quad -2, \quad -2, \quad 9, \quad 11 \\
\text{Mean} &= \ldots \\
\text{Median} &= \ldots
\end{align*}
\]

Find the indicated decile of the following data set

\[
\begin{align*}
21, & \quad 28, \quad 56, \quad 12, \quad 25, \quad 41, \quad 48, \quad 31, \quad 62, \quad 37, \quad 24, \quad 21 \\
D_{5} &= \ldots
\end{align*}
\]

IQ scores have a mean of 100 and a standard deviation of 15. Ted has an IQ of 106.

What is the difference between Ted’s IQ and the mean? __________

Convert Ted’s IQ score to a z score: __________

Nick took 4 courses last semester: History, Biology, Spanish, and Physics. The means and standard deviations for the final exams, and Nick’s scores are given in the table below. Convert Nick’s score into z scores.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Mean</th>
<th>Stand. dev.</th>
<th>Nick’s score</th>
<th>Nick’s z score</th>
</tr>
</thead>
<tbody>
<tr>
<td>History</td>
<td>53</td>
<td>16</td>
<td>77</td>
<td>1</td>
</tr>
<tr>
<td>Biology</td>
<td>77</td>
<td>10</td>
<td>94.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Spanish</td>
<td>44</td>
<td>12</td>
<td>59</td>
<td>1.25</td>
</tr>
<tr>
<td>Physics</td>
<td>60</td>
<td>14</td>
<td>67</td>
<td>1.1</td>
</tr>
</tbody>
</table>

On what exam did Nick have the highest relative score? __________

Here is a list of 25 scores on a Math midterm exam:

38.5, 41.5, 52, 52.5, 61, 63, 63.5, 68, 69, 69, 78.5, 79, 80, 83, 87, 88.5, 88.5, 91, 91.5, 92, 92.5, 94, 94, 97, 97

Find \( P_{52} \): __________

Calculate the mean and median of the following data:

\[
\begin{align*}
2, & \quad 6, \quad 53, \quad 44, \quad 77, \quad 5, \quad 6, \quad 10 \\
\text{Mean} &= \ldots \\
\text{Median} &= \ldots
\end{align*}
\]
Here is a list of 27 scores on a Statistics midterm exam:
20, 30, 31, 32, 46, 48, 49, 52, 54,
59, 61, 69, 71, 73, 74, 79, 81, 81,
81, 85, 86, 87, 88, 91, 94, 96, 97

Find $Q_3$: ______
1. (1 pt) setStatistics3Estimates/ur_stt_3.1.pg

Match the confidence level with the confidence interval for \( \mu \).

- 1. \( \bar{x} \pm 2.575 \left( \frac{s}{\sqrt{n}} \right) \)
- 2. \( \bar{x} \pm 1.645 \left( \frac{s}{\sqrt{n}} \right) \)
- 3. \( \bar{x} \pm 1.282 \left( \frac{s}{\sqrt{n}} \right) \)

A. 80%
B. 99%
C. 90%

2. (1 pt) setStatistics3Estimates/ur_stt_3.2.pg

Starting salaries of 130 college graduates who have taken a statistics course have a mean of $43,319 and a standard deviation of $10,480.

Using a 0.99 degree of confidence, find both of the following:

A. The margin of error \( E \)

B. The confidence interval for the mean \( \mu \):

\[ \bar{x} - E < \mu < \bar{x} + E \]

3. (1 pt) setStatistics3Estimates/ur_stt_3.3.pg

A random sample of 100 observations produced a mean of \( \bar{x} = 25.7 \) and a standard deviation \( s = 2.22 \).

- (a) Find a 95% confidence interval for \( \mu \)

- (b) Find a 90% confidence interval for \( \mu \)

- (c) Find a 99% confidence interval for \( \mu \)

4. (1 pt) setStatistics3Estimates/ur_stt_3.4.pg

Listed below are the lengths (in minutes) of randomly selected music CDs. Construct a 91% confidence interval for the mean length of all such CDs.

<table>
<thead>
<tr>
<th>Length (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>68.15</td>
</tr>
<tr>
<td>54.37</td>
</tr>
<tr>
<td>64.4</td>
</tr>
<tr>
<td>29.03</td>
</tr>
<tr>
<td>53.29</td>
</tr>
<tr>
<td>65.6</td>
</tr>
</tbody>
</table>

\[ \bar{x} \pm t \left( \frac{s}{\sqrt{n}} \right) \]

5. (1 pt) setStatistics3Estimates/ur_stt_3.5.pg

A random sample of \( n \) measurements was selected from a population with unknown mean \( \mu \) and standard deviation \( \sigma \). Calculate a 90% confidence interval for \( \mu \) for each of the following situations:

- (a) \( n = 70, \bar{x} = 37.8, s = 3.38 \)

- (b) \( n = 110, \bar{x} = 46.2, s = 4.36 \)

- (c) \( n = 75, \bar{x} = 53.9, s = 4.68 \)

- (d) \( n = 70, \bar{x} = 101, s = 4.63 \)

6. (1 pt) setStatistics3Estimates/ur_stt_3.23.pg

Studies have suggested that twins, in their early years, tend to have lower IQs and pick up language more slowly than non-twins. The slower intellectual growth might be caused by benign parental neglect. Suppose it is desired to estimate the mean attention time given to twin boys by their parents. A sample of 60 sets of 2 year old boys is taken, and after 1 week the attention time received was recorded. The data (in hours) calculated the mean at 24.5 and the standard deviation at 14.8. Use this information to construct a 95% confidence interval for the mean attention time given to all twin boys by their parents.

\[ \bar{x} - E < \mu < \bar{x} + E \]


Use the given data to find the 95% confidence interval estimate of the population mean \( \mu \). Assume that the population has a normal distribution.

IQ scores of professional athletes:

- Sample size \( n = 25 \)
- Mean \( \bar{x} = 104 \)
- Standard deviation \( s = 13 \)

\[ \bar{x} - E < \mu < \bar{x} + E \]

8. (1 pt) setStatistics3Estimates/ur_stt_3.7.pg

Suppose you have selected a random sample of \( n = 5 \) measurements from a normal distribution. Compare the standard normal \( z \) values with the corresponding \( t \) values if you were forming the following confidence intervals:

- (a) 80% confidence interval
  \[ z = \]
  \[ t = \]

- (b) 99% confidence interval
  \[ z = \]
  \[ t = \]

- (c) 90% confidence interval
  \[ z = \]
Weights of 10 red and 36 brown randomly chosen M&M plain candies are listed below.

Red: 0.933 0.898 0.952 0.92 0.898
     0.891 0.877 0.874 0.897 0.909
     0.875 0.872 0.92 0.985 0.857 0.931
     0.912 0.858 0.929 0.889 0.936 0.877
     0.988 0.905 0.92 0.913 0.902 0.867

Brown: 0.918 0.876 0.86 0.931 0.898 0.93
       0.871 0.986 0.904 1.001 0.909 0.928
       0.897 0.909 0.955 0.914 0.923 0.915

1. To construct a 90% confidence interval for the mean weight of red M&M plain candies:
   - A. The normal distribution
   - B. The t distribution with 9 degrees of freedom
   - C. The t distribution with 11 degrees of freedom
   - D. The t distribution with 10 degrees of freedom
   - E. None of the above

2. A 90% confidence interval for the mean weight of red M&M plain candies is _______ < μ < _______.

3. To construct a 90% confidence interval for the mean weight of brown M&M plain candies:
   - A. The normal distribution
   - B. The t distribution with 36 degrees of freedom
   - C. The t distribution with 35 degrees of freedom
   - D. The t distribution with 37 degrees of freedom
   - E. None of the above

4. A 90% confidence interval for the mean weight of brown M&M plain candies is _______ < μ < _______.

Periodically, the county Water Department tests the drinking water of homeowners for contaminants such as lead and copper. The lead and copper levels in water specimens collected in 1998 are shown below.

<table>
<thead>
<tr>
<th>City</th>
<th>Number of papers</th>
<th>City</th>
<th>Number of papers</th>
</tr>
</thead>
<tbody>
<tr>
<td>City 1</td>
<td>7</td>
<td>City 11</td>
<td>23</td>
</tr>
<tr>
<td>City 2</td>
<td>8</td>
<td>City 12</td>
<td>29</td>
</tr>
<tr>
<td>City 3</td>
<td>28</td>
<td>City 13</td>
<td>27</td>
</tr>
<tr>
<td>City 4</td>
<td>19</td>
<td>City 14</td>
<td>19</td>
</tr>
<tr>
<td>City 5</td>
<td>29</td>
<td>City 15</td>
<td>14</td>
</tr>
<tr>
<td>City 6</td>
<td>17</td>
<td>City 16</td>
<td>13</td>
</tr>
<tr>
<td>City 7</td>
<td>4</td>
<td>City 17</td>
<td>12</td>
</tr>
<tr>
<td>City 8</td>
<td>25</td>
<td>City 18</td>
<td>20</td>
</tr>
<tr>
<td>City 9</td>
<td>28</td>
<td>City 19</td>
<td>30</td>
</tr>
<tr>
<td>City 10</td>
<td>22</td>
<td>City 20</td>
<td>22</td>
</tr>
</tbody>
</table>

Construct a 80% confidence interval for the average number of papers published in major world cities.

---

The standard IQ test is designed so that the mean is 100 and the standard deviation is 15 for the population of all adults. We wish to find the sample size necessary to estimate the mean IQ score of statistics students. Suppose we want to be 93% confident that our sample mean is within 1.5 IQ points of the true mean. The mean for this population is clearly greater than 100. The standard deviation for this population is probably less than 15 because it is a group with less variation than a group randomly selected from the general population; therefore, if we use σ = 15, we are being conservative by using a value that will make the sample size at least as large as necessary. Assume then that σ = 15 and determine the required sample size.

Answer:

Periodically, the county Water Department tests the drinking water of homeowners for contaminants such as lead and copper. The lead and copper levels in water specimens collected in 1998 are shown below.

<table>
<thead>
<tr>
<th>lead (µg/L)</th>
<th>copper (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6</td>
<td>0.121</td>
</tr>
<tr>
<td>1.1</td>
<td>0.373</td>
</tr>
<tr>
<td>0.9</td>
<td>0.875</td>
</tr>
<tr>
<td>1.6</td>
<td>0.214</td>
</tr>
<tr>
<td>0.5</td>
<td>0.284</td>
</tr>
<tr>
<td>3.8</td>
<td>0.257</td>
</tr>
<tr>
<td>3.8</td>
<td>0.253</td>
</tr>
<tr>
<td>4.1</td>
<td>0.786</td>
</tr>
<tr>
<td>1.4</td>
<td>0.504</td>
</tr>
<tr>
<td>1.5</td>
<td>0.632</td>
</tr>
</tbody>
</table>

(a) Construct a 99% confidence interval for the mean lead level in water specimens of the subdevelopment.

(b) Construct a 99% confidence interval for the mean copper level in water specimens of the subdevelopment.
14. (1 pt) setStatistics3Estimates/ur_stt_3_12.pg
Suppose that the minimum and maximum ages for typical textbooks currently used in college courses are 0 and 8 years. Use the range rule of thumb to estimate the standard deviation.

Standard deviation = ________

Find the size of the sample required to estimate the mean age of textbooks currently used in college courses. Assume that you want 93% confidence that the sample mean is within 0.4 year of the population mean.

Required sample size = ________

15. (1 pt) setStatistics3Estimates/ur_stt_3_13.pg
A poll is taken in which 397 out of 575 randomly selected voters indicated their preference for a certain candidate. Find a 80% confidence interval for p.

_______ ≤ p ≤ _______

Use the given confidence interval limits to find the point estimate \( \hat{p} \) and the margin of error E.

\[ 0.13 < p < 0.31 \]

\[ \hat{p} = _______ \]

\[ E = _______ \]

17. (1 pt) setStatistics3Estimates/ur_stt_3_15.pg
Astronaunts often report that there are times when they become disoriented as they move around in zero-gravity. Therefore, they usually rely on bright colors and other visual information to help them establish a top-down orientation. A study was conducted to assess the potential of using color as body orienting. 60 college students, reclining on their backs in the dark, found it difficult to establish orientation when positioned on under a rotating disk. This rotating disk was painted half black and half white. Out of the 60 students, 48 believed they were right side up when the white was on top.

Use this information to estimate the true proportion of subjects who use the white color as a cue for right-side-up orientation. That is, construct a 95% confidence interval for the true proportion.

_______ ≤ p ≤ _______

18. (1 pt) setStatistics3Estimates/ur_stt_3_16.pg
Construct the 98% confidence interval estimate of the population proportion \( p \) if the sample size is \( n = 500 \) and the number of successes in the sample is \( x = 177 \).

_______ < p < _______

19. (1 pt) setStatistics3Estimates/ur_stt_3_17.pg
The EPA wants to test a randomly selected sample of \( n \) water specimens and estimate the mean daily rate of pollution produced by a mining operation. If the EPA wants a 98% confidence interval with a bound of error of 1 milligram per liter \((\text{mg/L})\), how many water specimens are required in the sample? Assume prior knowledge indicates that pollution readings in water samples taken during a day have been approximately normally distributed with a standard deviation of 3.9 \((\text{mg/L})\).

\[ n = _______ \]

20. (1 pt) setStatistics3Estimates/ur_stt_3_18.pg
College officials want to estimate the percentage of students who carry a gun, knife, or other such weapon. How many randomly selected students must be surveyed in order to be 98% confident that the sample percentage has a margin of error of 2 percentage points?

(a) Assume that there is no available information that could be used as an estimate of \( p \) is around 0.

Answer: _______

(b) Assume that another study indicated that 6% of college students carry weapons.

Answer: _______

A random sample of elementary school children in New York state is to be selected to estimate the proportion \( p \) who have received a medical examination during the past year. An interval estimate of the proportion \( p \) with a bound of 0.05 and 80% confidence is required.

(a) Assuming no prior information about \( p \) is available, approximately how large of a sample size is needed?

\[ n = _______ \]

(b) If a planning study indicates that \( p \) is around 0.4, approximately how large of a sample size is needed?

\[ n = _______ \]

22. (1 pt) setStatistics3Estimates/ur_stt_3_20.pg
Find the critical values \( \chi^2_1 = \chi_{1-\alpha/2}^2 \) and \( \chi^2_R = \chi_{\alpha/2}^2 \) that correspond to 95% degree of confidence and the sample size \( n = 8 \).

\[ \chi^2_1 = _______ \]

\[ \chi^2_R = _______ \]

23. (1 pt) setStatistics3Estimates/ur_stt_3_21.pg
According to the Food and Drug Administration (FDA), a cup of coffee contains on average 115 milligrams (mg) of caffeine, with the amount per cup ranging from 60 to 180 mg. Suppose you want to repeat the FDA experiment to obtain an estimate of the mean caffeine content in a cup of coffee correct to within 3.2 mg with 95% confidence. How many cups of coffee would have to be included in your sample?

\[ n = _______ \]

24. (1 pt) setStatistics3Estimates/ur_stt_3_22.pg
Find the minimum sample size needed to be 95% confident that the sample variance is within 40% of the population variance.
1. (1 pt) setStatistics4HypothesisTesting/ur_sff4_1.pg

Type I error is:
A. Deciding alternative hypothesis is true when it is false
B. Deciding null hypothesis is false when it is true
C. Deciding alternative hypothesis is true when it is true
D. Deciding null hypothesis is true when it is false
E. All of the above
F. None of the above

Type II error is:
A. Deciding alternative hypothesis is true when it is true
B. Deciding alternative hypothesis is false when it is true
C. Deciding null hypothesis is true when it is false
D. Deciding null hypothesis is false when it is true
E. All of the above
F. None of the above

2. (1 pt) setStatistics4HypothesisTesting/ur_sff4_2.pg

For each statement, express the null hypothesis $H_0$ and alternative hypothesis $H_1$ in symbolic form.
1. The mean salary of statistics professors is less than $70,000$ dollars.
   - $H_0: \mu > 70,000$, $H_1: \mu \leq 70,000$
   - $H_0: \mu < 70,000$, $H_1: \mu > 70,000$
   - $H_0: \mu = 70,000$, $H_1: \mu \neq 70,000$

2. Fewer than one-half of all Internet users make online purchases.
   - $H_0: p < 0.5$, $H_1: p > 0.5$
   - $H_0: p \geq 0.5$, $H_1: p < 0.5$
   - $H_0: p = 0.5$, $H_1: p \neq 0.5$

3. IQ scores of statistics students have a standard deviation less than $15$.
   - $H_0: \sigma \geq 15$, $H_1: \sigma < 15$
   - $H_0: \sigma < 15$, $H_1: \sigma \geq 15$
   - $H_0: \sigma = 15$, $H_1: \sigma \neq 15$
   - $H_0: \sigma = 15$, $H_1: \sigma = 15$

3. (1 pt) setStatistics4HypothesisTesting/ur_sff4_3.pg

Given the significance level $\alpha = 0.085$ find the following:
(a) Lower-tailed $z$ value
   $z = \ldots$
(b) Right-tailed $z$ value
   $z = \ldots$

4. (1 pt) setStatistics4HypothesisTesting/ur_sff4_4.pg

Find the critical $z$ value for a left-tailed test using a significance level of $\alpha = 0.02$.

5. (1 pt) setStatistics4HypothesisTesting/ur_sff4_5.pg

Find the critical $z$ value using a significance level of $\alpha = 0.04$ if the null hypothesis $H_0$ is $\mu \leq 95$.

6. (1 pt) setStatistics4HypothesisTesting/ur_sff4_6.pg

A random sample of 100 observations from a population with standard deviation 20.6677217902406 yielded a sample mean of 93.6.
(a) Given that the null hypothesis is $\mu = 90$ and the alternative hypothesis is $\mu > 90$ using $\alpha = 0.05$, find the following:
   (i) Critical $z$ score
      $z = \ldots$
   (ii) Test statistic
      $z = \ldots$
   (iii) Type I error
      Type I error is:  
      - A. There is insufficient evidence to reject the null hypothesis
      - B. Reject the null hypothesis
      - C. None of the above
   (b) Given that the null hypothesis is $\mu = 90$ and the alternative hypothesis is $\mu \neq 90$ using $\alpha = 0.05$, find the following:
      (i) Positive critical $z$ score
         $z = \ldots$
      (ii) Negative critical $z$ score
         $z = \ldots$
      (iii) Type II error
         Type II error is:
         - A. There is insufficient evidence to reject the null hypothesis
         - B. Reject the null hypothesis
         - C. None of the above

7. (1 pt) setStatistics4HypothesisTesting/ur_sff4_7.pg

It is necessary for an automobile producer to estimate the number of miles per gallon achieved by its cars. Suppose that the sample mean for a random sample of 130 cars is 31 miles and assume the standard deviation is 3.3 miles. Now suppose the
Suppose a random sample of 157 golfers be chosen so that their mean driving distance is 260 yards, with a standard deviation of 0.13. Find the value of the test statistic z for the claim that the population mean is μ = 20.

(a) |positivecriticalz score ______ |
(b) |negativecriticalz score ______ |
(c) |teststatistic ______ |

The final conclusion is
- A. There is not sufficient evidence to reject the null hypothesis that μ = 32.8.
- B. We can reject the null hypothesis that μ = 32.8 and accept that μ ≠ 32.8.

8.(1 pt) setStatistics4HypothesisTesting/ur_stt_428.png
The contents of 32 cans of Coke have a mean of $\bar{x} = 12.15$ and a standard deviation of $s = 0.13$. Find the value of the test statistic z for the claim that the population mean is $\mu = 12$.

9.(1 pt) setStatistics4HypothesisTesting/ur_stt_49.png
Golf-course designers have become concerned that old courses are becoming obsolete since new technology has given golfers the ability to hit the ball so far. Designers, therefore, have proposed that new golf courses need to be built expecting that the average golfer can hit the ball more than 235 yards on average. Suppose a random sample of 187 golfers be chosen so that their mean driving distance is 238.2 yards, with a standard deviation of 44.9.

Conduct a hypothesis test where $H_0 : \mu \leq 235$ and $H_1 : \mu > 235$ by computing the following:
(a) |teststatistic ______ |
(b) |$p-value = ______ |
(c) |Ifthiswasatwo-tailedtest,then $p-value is ______ |

10.(1 pt) setStatistics4HypothesisTesting/ur_stt_42.png
Golf-course designers have become concerned that old courses are becoming obsolete since new technology has given golfers the ability to hit the ball so far. Designers, therefore, have proposed that new golf courses need to be built expecting that the average golfer can hit the ball more than 255 yards on average. Suppose a random sample of 157 golfers be chosen so that their mean driving distance is 260.2 yards, with a standard deviation of 47.

Conduct a hypothesis test where $H_0 : \mu = 255$ and $H_1 : \mu > 255$ by computing the following:

(a) |teststatistic ______ |
(b) |$p-value = ______ |
(c) |If this was a two-tailed test, then $p-value is ______ |

11.(1 pt) setStatistics4HypothesisTesting/ur_stt_410.png
Assume you are using a significance level of $\alpha = 0.05$ to test the claim that $\mu < 8$ and that your sample is a random sample of 34 values. Find $\beta$, the probability of making a type II error (failing to reject a false null hypothesis), given that the population actually has a normal distribution with $\mu = 6$ and $\sigma = 8$.

$\beta = ______$

12.(1 pt) setStatistics4HypothesisTesting/ur_stt_411.png
Physicians at a clinic gave what they thought were drugs to 800 asthma, ulcer, and herpes patients. Although the doctors later learned that the drugs were really placebos, 51% of the patients reported an improved condition. Assume that if the placebo is ineffective, the probability of a patient's condition improving is 0.49. For the hypotheses that the proportion of improving is 0.49 against that it is > 0.49, find the $p$-value.

$p = ______$

13.(1 pt) setStatistics4HypothesisTesting/ur_stt_412.png
Test the claim that the population of sophomore college students has a mean grade point average greater than 2.2. Sample statistics include $n = 110, \bar{x} = 2.4$, and $s = 0.8$. Use a significance level of $\alpha = 0.03$.

The test statistic is ______
The critical value is ______
The $p$-Value is ______
The final conclusion is
- A. There is sufficient evidence to support the claim that the mean grade point average is greater than 2.2.
- B. There is not sufficient evidence to support the claim that the mean grade point average is greater than 2.2.

14.(1 pt) setStatistics4HypothesisTesting/ur_stt_414.png
35 people are randomly selected and the accuracy of their wrist-watches is checked, with positive errors representing watches that are ahead of the correct time and negative errors representing watches that are behind the correct time. The 35 values have a mean of 100sec and a standard deviation of 249sec. Use a 0.02 significance level to test the claim that the population of all watches has a mean of 0sec.

The test statistic is ______
The $p$-Value is ______
The final conclusion is
- A. There is sufficient evidence to warrant rejection of the claim that the mean is equal to 0
- B. There is not sufficient evidence to warrant rejection of the claim that the mean is equal to 0.
A sample of 6 meaurments, randomly selected from a normally distributed population, resulted in a sample mean, $\bar{x} = 9.6$ and sample standard deviation $s = 1.12$. Using $\alpha = 0.01$, test the null hypothesis that the mean of the population is 8.8 against the alternative hypothesis that the mean of the population, $\mu < 8.8$ by giving the following:

(a) $\text{the degree of freedom}$

(b) $\text{the critical value}$

(c) $\text{the test statistic}$

The final conclusion is

- A. There is not sufficient evidence to reject the null hypothesis that $\mu = 8.8$.
- B. We can reject the null hypothesis that $\mu = 8.8$ and accept that $\mu < 8.8$.

The effectiveness of a new bug repellent is tested on 7 subjects for a 10 hour period. Based on the number and location of the bug bites, the percentage of surface area exposed protected from bug bites, the percentage of surface area exposed protected from bug bites was calculated for each of the subjects. The results were as follows:

- $\bar{x} = 89\%$, $s = 15\%$

The new repellent is considered effective if it provides a percent repellency of at least 90. Using $\alpha = 0.05$, construct a hypothesis test with null hypothesis $H_0: \mu = 0.9$ and alternative hypothesis $H_a: \mu > 0.9$ to determine whether the mean repellency of the new bug repellent is greater than 90 by computing the following:

(a) $\text{the degree of freedom}$

(b) $\text{the critical value}$

(c) $\text{the test statistic}$

The final conclusion is

- A. We can reject the null hypothesis that $\mu = 0.9$ and accept that $\mu > 0.9$, that is, the bug repellent is effective.
- B. There is not sufficient evidence to reject the null hypothesis that $\mu = 0.9$.

When a poultry farmer uses his regular feed, the newborn checkens have normally distributed weights with a mean of 62.7 oz. In an experiment with an enriched feed mixture, ten chickens are born with the following weights (in ounces):

64.9, 65.9, 66.2, 68.5, 65.4, 65.2, 65.4, 67, 69.2, 62.9

Use the $\alpha = 0.01$ significance level to test the claim that the mean weight is higher with the enriched feed.

The sample mean is $\bar{x} = $

The sample standard deviation is $s = $

The test statistic is $t = $

The critical value is $t = $

The conclusion is

- A. There is not sufficient evidence to support the claim that with the enriched feed, the mean weight is higher than 62.7.
- B. There is sufficient evidence to support the claim that with the enriched feed, the mean weight is greater than 62.7.

One of the most feared predators in the ocean is the great white shark. It is known that the white shark grows to a mean length of 19 feet; however, one marine biologist believes that great white sharks off the Bermuda coast grow much longer. To test this claim, full-grown white sharks were captured, measured, and then set free. However, this was a difficult, costly and very dangerous task, so only four sharks were actually sampled. Their lengths were 22, 25, 26, and 25 feet. Do the data provide sufficient evidence to support the claim? Use $\alpha = 0.05$ test statistic $t = $

rejection region $t >$

The final conclusion is

- A. We can reject the null hypothesis that the average length of the shark is 19, and accept that the average length of the shark is greater than 19.
- B. There is not sufficient evidence to reject the null hypothesis that the average length of the shark is 19.
20. (1 pt) setStatistics4HypothesisTesting/ur_stt_4_17.png
A random sample of 130 observations is selected from a binomial population with unknown probability of success \( p \). The computed value of \( \hat{p} \) is 0.74.

(1) 
\( Test H_0 : p \leq 0.7 \) against \( H_1 : p > 0.7 \). Use \( \alpha = 0.05 \).

- test statistic \( z = \) __________
- critical \( z \) score

The final conclusion is
- A. There is not sufficient evidence to reject the null hypothesis that \( p \leq 0.7 \).
- B. We can reject the null hypothesis that \( p \leq 0.7 \) and accept that \( p > 0.7 \).

(2) 
\( Test H_0 : p \geq 0.65 \) against \( H_1 : p < 0.65 \). Use \( \alpha = 0.01 \).

- test statistic \( z = \) __________
- critical \( z \) score

The final conclusion is
- A. We can reject the null hypothesis that \( p \geq 0.65 \) and accept that \( p < 0.65 \).
- B. There is not sufficient evidence to reject the null hypothesis that \( p \geq 0.65 \).

(3) 
\( Test H_0 : p = 0.6 \) against \( H_1 : p \neq 0.6 \). Use \( \alpha = 0.01 \).

- test statistic \( z = \) __________
- positive critical \( z \) score
- negative critical \( z \) score

The final conclusion is
- A. There is not sufficient evidence to reject the null hypothesis that \( p = 0.6 \).
- B. We can reject the null hypothesis that \( p = 0.6 \) and accept that \( p \neq 0.6 \).

According to a recent marketing campaign, 100 drinkers of either Diet Coke or Diet Pepsi participated in a blind taste test to see which of the drinks was their favorite. In one Pepsi television commercial, an announcer states that "in recent blind taste tests, more than one half of the surveyed preferred Diet Coke." Suppose that out of those 100, 60 preferred Diet Pepsi. Test the hypothesis, using \( \alpha = 0.05 \) that more than half of all participants will select Diet Pepsi in a blind taste test by giving the following:

(a) 
- the test statistic
- the critical \( z \) score

(b) 
- the test statistic
- the critical \( z \) score

The final conclusion is
- A. We can reject the null hypothesis that \( p \leq 0.5 \) and accept that \( p > 0.5 \).
- B. There is not sufficient evidence to reject the null hypothesis that \( p \leq 0.5 \).

22. (1 pt) setStatistics4HypothesisTesting/ur_stt_4_20.png
A survey of 1395 people who took trips revealed that 141 of them included a visit to a theme park. Based on those survey results, a management consultant claims that less than 11% of trips include a theme park visit. Test this claim using the \( \alpha = 0.01 \) significance level.

- The test statistic is __________
- The critical value is __________
- The conclusion is
  - A. There is not sufficient evidence to reject the claim that less than 11% of trips include a theme park visit.
  - B. There is sufficient evidence to support the claim that less than 11% of trips include a theme park visit.

23. (1 pt) setStatistics4HypothesisTesting/ur_stt_4_23.png
A new cream that advertises that it can reduce wrinkles and improve skin was subject to a recent study. A sample of 68 women over the age of 50 used the new cream for 6 months. Of those 68 women, 63 of them reported skin improvement (as judged by a dermatologist). Is this evidence that the cream can improve the skin of more than 40% of women over the age of 50? Test using \( \alpha = 0.05 \).

- test statistics \( z = \) __________
- rejection region \( z > \) __________

The final conclusion is
- A. We can reject the null hypothesis that \( p = 0.4 \) and accept that \( p > 0.4 \). That is, the cream can improve the skin of more than 40% of women over 50.
- B. There is not sufficient evidence to reject the null hypothesis that \( p = 0.4 \). That is, there is not sufficient evidence to reject the claim that the cream can improve the skin of more than 40% of women over 50.

24. (1 pt) setStatistics4HypothesisTesting/ur_stt_4_21.png
A random sample of \( n = 6 \) observations from a normal population produced the following measurements:

\[ 1 \ 9 \ 9 \ 0 \ 3 \ 8 \]

Do the data provide sufficient evidence to indicate that \( \sigma^2 < 1 \)? Use \( \alpha = 0.05 \), and compute the following:

(a) 
- \( \text{sample standard deviations} = \) __________

(b) 
- \( \text{test statistic} \chi^2 = \) __________

(c) 
- \( \text{critical} \chi^2 = \) __________

The final conclusion is
- A. There is not sufficient evidence to reject the null hypothesis that \( \sigma^2 = 1 \).
- B. We can reject the null hypothesis that \( \sigma^2 = 1 \) and accept that \( \sigma^2 < 1 \).
25. (1 pt) setStatistics4HypothesisTesting/ur_stt4_22.pg

Use a $\alpha = 0.01$ significance level to test the claim that $\sigma = 18$ if the sample statistics include $n = 24$, $\bar{x} = 110$, and $s = 21$.

The test statistic is _______.

The smaller critical number is _______.

The bigger critical number is _______.

What is your conclusion?

- A. There is not sufficient evidence to warrant the rejection of the claim that the population standard deviation is equal to 18

- B. There is sufficient evidence to warrant the rejection of the claim that the population standard deviation is equal to 18
1. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_1.pg
In order to compare the means of two populations, independent random samples of 487 observations are selected from each population, with the following results:

<table>
<thead>
<tr>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x}_1 = 5105 )</td>
<td>( \bar{x}_2 = 5171 )</td>
</tr>
<tr>
<td>( s_1 = 140 )</td>
<td>( s_2 = 145 )</td>
</tr>
</tbody>
</table>

(a) Use a 95% confidence interval to estimate the difference between the population means \( (\mu_1 - \mu_2) \).
\[
(\bar{x}_1 - \bar{x}_2) \pm z_{0.05} \frac{s_p}{\sqrt{n}}
\]

(b) Test the null hypothesis: \( H_0 : (\mu_1 - \mu_2) = 0 \) versus the alternative hypothesis: \( H_a : (\mu_1 - \mu_2) \neq 0 \). Using \( \alpha = 0.05 \), give the following:

(i) the test statistic is \( z = \frac{\bar{x}_1 - \bar{x}_2}{s_p} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \)

(ii) the positive critical \( z \) score

(iii) the negative critical \( z \) score

The final conclusion is

- A. There is sufficient evidence to reject the null hypothesis that \( (\mu_1 - \mu_2) = 0 \).
- B. There is not sufficient evidence to warrant rejection of the claim that the two populations have the same mean.

(c) Test the null hypothesis: \( H_0 : (\mu_1 - \mu_2) = 30 \) versus the alternative hypothesis: \( H_a : (\mu_1 - \mu_2) \neq 30 \). Using \( \alpha = 0.05 \), give the following:

(i) the test statistic is \( z = \frac{\bar{x}_1 - \bar{x}_2}{s_p} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \)

(ii) the positive critical \( z \) score

(iii) the negative critical \( z \) score

The final conclusion is

- A. There is not sufficient evidence to reject the null hypothesis that \( (\mu_1 - \mu_2) = 30 \).
- B. We can reject the null hypothesis that \( (\mu_1 - \mu_2) = 30 \) and accept that \( (\mu_1 - \mu_2) \neq 30 \).

2. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_2.pg
Test the claim that the two samples described below come from populations with the same mean. Assume that the samples are independent simple random samples. Use a significance level of 0.02.
Sample 1: \( n_1 = 92, \bar{x}_1 = 17, s_1 = 3 \).
Sample 2: \( n_2 = 50, \bar{x}_2 = 14, s_2 = 3 \).

The test statistic is \( z = \frac{\bar{x}_1 - \bar{x}_2}{s_p} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \)

The P-Value is

The conclusion is

- A. There is sufficient evidence to warrant rejection of the claim that the two populations have the same mean.
- B. There is not sufficient evidence to warrant rejection of the claim that the two populations have the same mean.

3. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_3.pg
The purpose of this question is to compare the variability of \( \bar{x}_1 \) and \( \bar{x}_2 \) with the variability of \( (\bar{x}_1 - \bar{x}_2) \).

(a) Suppose the first sample of 100 observations is selected from a population with mean \( \mu_1 = 170 \) and variance \( \sigma_1^2 = 1040 \). Construct an interval extending 2 standard deviations of \( \bar{x}_1 \) on each side of \( \mu_1 \).
\[
\mu_1 \pm 2 \frac{\sigma_1}{\sqrt{n}}
\]

(b) Suppose the second sample of 100 observations is selected from a population with mean \( \mu_2 = 170 \) and variance \( \sigma_2^2 = 1460 \). Construct an interval extending 2 standard deviations of \( \bar{x}_2 \) on each side of \( \mu_2 \).
\[
\mu_2 \pm 2 \frac{\sigma_2}{\sqrt{n}}
\]

(c) Consider the difference between the two sample means \( (\bar{x}_1 - \bar{x}_2) \). Compute the mean and the standard deviation of the sampling distribution of \( (\bar{x}_1 - \bar{x}_2) \).

- mean = \( \mu_1 - \mu_2 \)
- standard deviation = \( \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \)

(d) Based on 100 observations, construct an interval extending 2 standard deviations of \( (\bar{x}_1 - \bar{x}_2) \) on each side of \( (\mu_1 - \mu_2) \).
\[
(\mu_1 - \mu_2) \pm 2 \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}
\]

4. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_4.pg
Randomly selected 60 student cars have ages with a mean of 7.5 years and a standard deviation of 3.4 years, while randomly selected 105 faculty cars have ages with a mean of 5.9 years and a standard deviation of 3.3 years.

1. Use 0.04 significance level to test the claim that student cars are older than faculty cars.

- The test statistic is

- The critical value is

- Is there sufficient evidence to support the claim that student cars are older than faculty cars?

- A. Yes

- B. No

- C. Cannot determine
7. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_16.png
Test the given claim using the $\alpha = 0.05$ significance level and assuming that the populations are normally distributed.

Claim: The treatment population and the placebo population have the same mean.

Treatment group: $n = 11, \bar{x} = 115, s = 5.8$.
Placebo group: $n = 7, \bar{x} = 120, s = 5.6$.

The test statistic is ____________
The positive critical value is ____________
The negative critical value is ____________

Is there sufficient evidence to warrant the rejection of the claim that the treatment and placebo populations have the same mean?

- A. No
- B. Yes

8. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_18.png
Randomly selected students were given five seconds to estimate the value of a product of numbers with the results shown below.

Estimates from students given $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$:
750, 10000, 25, 2000, 1252, 1000, 5000, 42200, 4000, 6000

Estimates from students given $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$:
2000, 25000, 1200, 5000, 200, 52836, 50000, 49000, 4000, 23410

Use a 0.05 significance level to test the following claims:

1. \( \text{Claim: the two populations have equal variances.} \)
The test statistic is ____________
The larger critical value is ____________
The conclusion is

- A. There is sufficient evidence to warrant the rejection of the claim that the two populations have equal variances
- B. There is not sufficient evidence to warrant the rejection of the claim that the two populations have equal variances

2. \( \text{Claim: the two populations have the same mean.} \)
The test statistic is ____________
The positive critical value is ____________
The negative critical value is ____________
The conclusion is

- A. There is not sufficient evidence to warrant the rejection of the claim that the two populations have the same mean
- B. There is sufficient evidence to warrant the rejection of the claim that the two populations have the same mean

9. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_6.png
Suppose you want to test the claim that the paired sample data given below come from a population for which the mean difference is $\mu_d = 0$.
A paired difference experiment yielded \( n_D \) pairs of observations. 

In each case described below, what is the rejection region for testing \( H_0 : \mu = 4 \) against \( H_a : \mu > 4 \)? Use \( s_D = 7.5 \).

(a) \( n_D = 39, \alpha = 0.05 \) \( z > \) 
(b) \( n_D = 8, \alpha = 0.1 \) \( t > \) 
(c) \( n_D = 23, \alpha = 0.02 \) \( t > \) 

12. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_11.pg 
A paired difference experiment produced the following results: 
\[ n_D = 46, X_1 = 104, X_2 = 102, \bar{D} = 2, s_D = 62, \]

(a) \( \text{Determine the rejection region for the hypothesis } H_0 : \mu_D = 0 \text{ if } H_a : \mu_D > 0. \text{ Use } \alpha = 0.05. \) \( z > \) 
(b) \( \text{Conduct a paired difference test described above.} \) 
The test statistic is \( \ldots \) 
The final conclusion is 
\[ \begin{array}{ll}
\text{A.} & \text{There is not sufficient evidence to reject the null hypothesis that } \mu_D = 0. \\
\text{B.} & \text{We can reject the null hypothesis that } \mu_D = 0 \text{ and accept that } \mu_D > 0. 
\end{array} \]

13. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_10.pg 
In a study of red/green color blindness, 800 men and 2400 women are randomly selected and tested. Among the men, 68 have red/green color blindness. Among the women, 6 have red/green color blindness. Test the claim that men have a higher rate of red/green color blindness. 
The test statistic is \( \ldots \) 

Is there sufficient evidence to support the claim that men have a higher rate of red/green color blindness than women?  
\[ \begin{array}{ll}
\text{A.} & \text{No} \\
\text{B.} & \text{Yes} 
\end{array} \] 

Construct the 99% confidence interval for the difference between the color blindness rates of men and women.  
\[ \ldots < (p_1 - p_2) < \ldots \]

14. (1 pt) setStatistics5Inferences2Samples/ur_stt_5_13.pg 
Independent random samples, each containing 600 observations, were selected from two binomial populations. The samples from populations 1 and 2 produced 113 and 284 successes, respectively. 
(a) \( \text{Test } H_0 : (p_1 - p_2) = 0 \text{ against } H_a : (p_1 - p_2) \neq 0. \text{ Use } \alpha = 0.05 \) 

\begin{align*}
\text{test statistic} & = \ldots \\
\text{rejection region } |z| & > \ldots \\
\text{The final conclusion is} & \\
\text{A.} & \text{We can reject the null hypothesis that } (p_1 - p_2) = 0 \text{ and accept that } (p_1 - p_2) \neq 0. 
\end{align*}
• B. There is not sufficient evidence to reject the null hypothesis that \((p_1 - p_2) = 0\).

15. (1 pt) supptestStatistics5Inferences2Samples/ur_slt_5_15.pg

Suppose a group of 1000 smokers (who all wanted to give up smoking) were randomly assigned to receive an antidepressant drug or a placebo for six weeks. Of the 585 patients who received the antidepressant drug, 19 were not smoking one year later. Of the 415 patients who received the placebo, 10 were not smoking one year later. Given the null hypothesis \(H_0 : (p_1 - p_2) = 0\) and the alternative hypothesis \(H_a : (p_1 - p_2) \neq 0\), conduct a test to see if taking an antidepressant drug can help smokers stop smoking. Use \(\alpha = 0.05\)

(a)

**TEST**

**Hypothesis region** \(|z| > \phantom{1234567890} \)

(b)

**Test statistic** \(z = \phantom{1234567890} \)

The final conclusion is

- A. There is not sufficient evidence to reject the null hypothesis that \((p_1 - p_2) = 0\).
- B. We can reject the null hypothesis that \((p_1 - p_2) = 0\) and accept that \((p_1 - p_2) > 0\).

16. (1 pt) supptestStatistics5Inferences2Samples/ur_slt_5_14.png

Test the given claim using the \(\alpha = 0.05\) significance level and assuming that the populations are normally distributed.

Claim: The treatment population and the placebo population have different variances.

Treatment group: \(n = 5, \bar{x} = 122.2, s = 19.1\).

Placebo group: \(n = 11, \bar{x} = 115.1, s = 11.3\).

The test statistic is \(z = \phantom{1234567890} \)

The larger critical value is \(\phantom{1234567890} \)

What is your conclusion?

17. (1 pt) supptestStatistics5Inferences2Samples/ur_slt_5_17.png

Suppose you wanted to estimate the difference between two population means correct to within 3 with probability 0.99. If prior information suggests that the population variances are approximately equal to \(\sigma_1^2 = \sigma_2^2 = 19\) and you want to select independent random samples of equal size from the populations, how large should the sample sizes, \(n_1\) and \(n_2\) be?

Answer: \(n_1 = n_2 = \phantom{1234567890} \)

18. (1 pt) supptestStatistics5Inferences2Samples/ur_slt_5_12.png

Find the size of each sample needed to estimate the difference between the proportions of boys and girls under 10 years old who are afraid of spiders. Assume that we want 97% confidence that the error is smaller than 0.02.

\(n = \phantom{1234567890} \)

19. (1 pt) supptestStatistics5Inferences2Samples/ur_slt_5_12a.png

The sample size needed to estimate the difference between two population proportions to within a margin of error \(E\) with a significance level of \(\alpha\) can be found as follows. In the expression

\[ E = z_{\alpha/2} \sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}} \]

we replace both \(n_1\) and \(n_2\) by \(n\) (assuming that both samples have the same size) and replace each of \(p_1, p_2, q_1,\) and \(q_2\) by 0.5 (because their values are not known). Then we solve for \(n\), and get

\[ n = \frac{(z_{\alpha/2})^2}{2E^2}. \]

Finally, increase the value of \(n\) to the next larger integer number.

Use the above formula to find the size of each sample needed to estimate the difference between the proportions of boys and girls under 10 years old who are afraid of spiders. Assume that we want 99% confidence that the error is smaller than 0.04.

\(n = \phantom{1234567890} \)
1. (1 pt) setStatistics6CorrelationRegression/ur_stt_6_2.pg

Use a scatterplot and the linear correlation coefficient \( r \) to determine whether there is a correlation between the two variables.

\[
\begin{array}{c|cccccccccccc}
 x & 0.7 & 1.6 & 2.2 & 3.8 & 4.7 & 5 & 6.1 & 7.2 & 8.9 & 9.3 & 10.8 & 11.9 & 12.3 & 13.5 & 14.3 \\
 y & 12.3 & 12.3 & 12.8 & 10.1 & 10.8 & 11 & 7.9 & 8.8 & 4.3 & 6.9 & 2.8 & 4.5 & 1.1 & 1.9 & 0.9 \\
\end{array}
\]

\[ r = \] 

There is

- A. no correlation between \( x \) and \( y \)
- B. a positive correlation between \( x \) and \( y \)
- C. a negative correlation between \( x \) and \( y \)
- D. a perfect positive correlation between \( x \) and \( y \)
- E. a nonlinear correlation between \( x \) and \( y \)
- F. a perfect negative correlation between \( x \) and \( y \)

2. (1 pt) setStatistics6CorrelationRegression/ur_stt_6_1.pg

Match the following sample correlation coefficients with the explanation of what that correlation coefficient means.

- 1. \( r = 0 \)
- 2. \( r = .1 \)
- 3. \( r = -.97 \)
- 4. \( r = -1 \)

A. a perfect negative relationship between \( x \) and \( y \)
B. a strong negative relationship between \( x \) and \( y \)
C. a weak positive relationship between \( x \) and \( y \)
D. no relationship between \( x \) and \( y \)

3. (1 pt) setStatistics6CorrelationRegression/ur_stt_6_3.pg

Given the following data set,

\[
\begin{array}{c|ccccc}
 x & 4 & -1 & 2 & 0 & 2 \\
 y & 3 & 5 & 5 & 1 & 6 \\
\end{array}
\]

Compute the coefficient of correlation \( r \)

\[ r = \] 

4. (1 pt) setStatistics6CorrelationRegression/ur_stt_6_4.pg

Heights (in centimeters) and weights (in kilograms) of 7 supermodels are given below. Find the regression equation, letting the first variable be the independent \( (x) \) variable, and predict the weight of a supermodel who is 167 cm tall.

<table>
<thead>
<tr>
<th>Height</th>
<th>174</th>
<th>176</th>
<th>168</th>
<th>176</th>
<th>178</th>
<th>172</th>
<th>178</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>55</td>
<td>55</td>
<td>50</td>
<td>56</td>
<td>57</td>
<td>52</td>
<td>58</td>
</tr>
</tbody>
</table>

The regression equation is \( y = 14.52 + 0.47x \).

The best predicted weight of a supermodel who is 167 cm tall is \( 72.75 \) kg.

5. (1 pt) setStatistics6CorrelationRegression/ur_stt_6_5.pg

Is the number of games won by a major league baseball team in a season related to the team batting average? The table below shows the number of games won and the batting average of 8 teams.

<table>
<thead>
<tr>
<th>Team</th>
<th>Games Won</th>
<th>Batting Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94</td>
<td>0.281</td>
</tr>
<tr>
<td>2</td>
<td>79</td>
<td>0.289</td>
</tr>
<tr>
<td>3</td>
<td>82</td>
<td>0.29</td>
</tr>
<tr>
<td>4</td>
<td>87</td>
<td>0.272</td>
</tr>
<tr>
<td>5</td>
<td>63</td>
<td>0.28</td>
</tr>
<tr>
<td>6</td>
<td>66</td>
<td>0.271</td>
</tr>
<tr>
<td>7</td>
<td>99</td>
<td>0.26</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
<td>0.273</td>
</tr>
</tbody>
</table>

Using games won as the independent variable \( x \), do the following:

a. The correlation coefficient is \( r = \) 

(b) The equation of the least squares line is \( y = \) 

6. (1 pt) setStatistics6CorrelationRegression/ur_stt_6_6.pg

The amounts of 6 restaurant bills and the corresponding amounts of the tips are given in the below.

<table>
<thead>
<tr>
<th>Bill</th>
<th>88.01</th>
<th>106.27</th>
<th>49.72</th>
<th>52.44</th>
<th>64.30</th>
<th>43.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip</td>
<td>10.00</td>
<td>16.00</td>
<td>5.28</td>
<td>7.00</td>
<td>7.70</td>
<td>5.50</td>
</tr>
</tbody>
</table>

Use a 0.05 confidence level to find the following:

The test statistic \( r = \) 

Is there a significant correlation?

- A. Yes
- B. No

The regression equation is \( \hat{y} = \) 

If the amount of the bill is $75, the best prediction for the amount of the tip is \( \)
The amounts of 6 restaurant bills and the corresponding amounts of the tips are given in the below.

<table>
<thead>
<tr>
<th>Bill</th>
<th>32.98</th>
<th>52.44</th>
<th>43.58</th>
<th>88.01</th>
<th>97.34</th>
<th>49.72</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip</td>
<td>4.50</td>
<td>7.00</td>
<td>5.50</td>
<td>10.00</td>
<td>16.00</td>
<td>5.28</td>
</tr>
</tbody>
</table>

Use a 0.05 confidence level to find the following:

The test statistic \( r = \) ________

The test statistic \( t = \) ________

The critical value \( t = \) ________

Is there a significant correlation?

- A. Yes
- B. No

The regression equation is \( \hat{y} = \) ________ + ________ \( x \).

If the amount of the bill is $75, the best prediction for the amount of the tip is ________ and a prediction interval estimate of the amount amount of the tip is ________ < tip < ________.

Construct both a 95% and a 98% confidence interval for \( \beta_1 \).

95%: \( \beta_1 \leq \) ________

98%: \( \beta_1 \leq \) ________

Find the following:

(a) The correlation coefficient \( r = \) ________

(b) The least squares line \( y = \) ________ + ________ \( x \)

Find the multiple regression equation for the data given below.

\[
\begin{array}{cccccc}
\text{Student} & \text{Verbal Score} x & \text{Final Grade} y \\
1 & 41 & 66 \\
2 & 50 & 60 \\
3 & 59 & 79 \\
4 & 67 & 84 \\
5 & 39 & 63 \\
6 & 56 & 68 \\
7 & 41 & 61 \\
8 & 28 & 61 \\
9 & 62 & 68 \\
10 & 73 & 79 \\
\end{array}
\]

Find the following:

(a) The test statistic \( t = \) ________

(b) The degree of freedom \( df = \) ________

(c) The rejection region \( |t| > \) ________

The final conclusion is

- A. There is not sufficient evidence to reject the null hypothesis that \( \beta_1 = 0 \).
- B. We can reject the null hypothesis that \( \beta_1 = 0 \) and accept that \( \beta_1 \neq 0 \).
1. (1 pt) setStatistics7MultinomialContingency/ur_stt_7_1.pg
A multinomial experiment with \( k = 3 \) cells and \( n = 360 \) produced the data shown below.

\[
\begin{array}{c|c|c|c}
\text{Number of Nights} & \text{Pre-retirement} & \text{Post-retirement} & \text{Total} \\
\hline
4 - 7 & 249 & 166 & 415 \\
8 - 13 & 80 & 63 & 143 \\
14 - 21 & 37 & 57 & 94 \\
22 or more & 21 & 31 & 52 \\
\hline
\text{Total} & 387 & 317 & 704
\end{array}
\]

With this information, construct a table of estimated expected values.

\[
\begin{array}{c|c|c|c}
\text{Number of Nights} & \text{Pre-retirement} & \text{Post-retirement} \\
\hline
4 - 7 & 249 & 166 \\
8 - 13 & 80 & 63 \\
14 - 21 & 37 & 57 \\
22 or more & 21 & 31 \\
\hline
\text{Total} & 387 & 317
\end{array}
\]

Now, with that information, determine whether the length of stay is independent of retirement using \( \alpha = 0.01 \)

\[\chi^2 = \ldots\]

The rejection region is \( \chi^2 > \ldots \)

The final conclusion is

- A. We can reject the null hypothesis that the length of stay is independent of retirement and accept the alternative hypothesis that the two are dependent.
- B. There is not sufficient evidence to reject the null hypothesis that the length of stay is independent of retirement.

2. (1 pt) setStatistics7MultinomialContingency/ur_stt_7_2.pg
A computer random number generator was used to generate 750 random digits \((0,1,\ldots,9)\). The observed frequencies of the digits are given in the table below.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
\text{Digit} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
\text{Frequency} & 67 & 65 & 63 & 70 & 78 & 60 & 61 & 77 & 82 & 127 \\
\end{array}
\]

Test the claim that all the outcomes are equally likely using the significance level \( \alpha = 0.05 \).

The expected frequency of each outcome is \( E = \ldots \)

The test statistic is \( \chi^2 = \ldots \)

The critical value is \( \chi^2 = \ldots \)

Is there sufficient evidence to warrant the rejection of the claim that all the outcomes are equally likely?

- A. No
- B. Yes

3. (1 pt) setStatistics7MultinomialContingency/ur_stt_7_3.pg
It has been suggested that the highest priority of retirees is travel. Thus, a study was conducted to investigate the differences in the length of stay of a trip for pre and postretirees. A sample of 704 travelers were asked how long they stayed on a typical trip. The observed results of the study are found below.

\[
\begin{array}{c|c|c|c}
\text{Age} & \text{Under 25} & 25-44 & 45-64 \\
\hline
\text{Drivers} & 78 & 48 & 28 \\
\hline
\text{Over 64} & 46 \\
\end{array}
\]

If all ages have the same crash rate, we would expect (because of the age distribution of licensed drivers) the given categories to have 16%, 44%, 27%, 13% of the subjects, respectively. At the 0.025 significance level, test the claim that the distribution of crashes conforms to the distribution of ages.

The test statistic is \( \chi^2 = \ldots \)

The critical value is \( \chi^2 = \ldots \)

The conclusion is

- A. There is sufficient evidence to warrant the rejection of the claim that the distribution of crashes conforms to the distribution of ages.
- B. There is not sufficient evidence to warrant the rejection of the claim that the distribution of crashes conforms to the distribution of ages.

4. (1 pt) setStatistics7MultinomialContingency/ur_stt_7_4.pg
Among drivers who have had a car crash in the last year, 200 were randomly selected and categorized by age, with the results listed in the table below.

\[
\begin{array}{c|c|c|c|c}
\text{Number of Nights} & \text{Pre-retirement} & \text{Post-retirement} \\
\hline
4 - 7 & 249 & 166 \\
8 - 13 & 80 & 63 \\
14 - 21 & 37 & 57 \\
22 or more & 21 & 31 \\
\hline
\text{Total} & 387 & 317
\end{array}
\]

Now, with that information, determine whether the length of stay is independent of retirement using \( \alpha = 0.01 \)

\[\chi^2 = \ldots\]

The rejection region is \( \chi^2 > \ldots \)

The final conclusion is

- A. We can reject the null hypothesis that the length of stay is independent of retirement and accept the alternative hypothesis that the two are dependent.
- B. There is not sufficient evidence to reject the null hypothesis that the length of stay is independent of retirement.

5. (1 pt) setStatistics7MultinomialContingency/ur_stt_7_5.pg
Test the null hypothesis of independence of the two classifications, A and B, of the 3 \times 3 contingency table shown below. Test using \( \alpha = 0.01 \)
\[
\chi^2 = \ldots
\]

The rejection region is \( \chi^2 > \ldots \)

The final conclusion is

- A. There is not sufficient evidence to reject the null hypothesis that A and B are independent.
- B. We can reject the null hypothesis that A and B are independent and accept that A and B are dependent.

Test the claim that the gender of a professor is independent of the department. Use the significance level \( \alpha = 0.01 \)

The test statistic is \( \chi^2 = \ldots \)

The critical value is \( \chi^2 = \ldots \)

Is there sufficient evidence to warrant the rejection of the claim that the gender of a professor is independent of the department?

- A. Yes
- B. No
1. **(1 pt) setStatistics8ANOVA/ur_sst8_1.pg**

Complete the ANOVA table for a completely randomized design below.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>17</td>
<td>16.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>47.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**2. (1 pt) setStatistics8ANOVA/ur_sst8_2.pg**

The table below lists the body temperatures of six randomly selected subjects from each of three different age groups. Use the $\alpha = 0.05$ significance level to test the claim that the three age-group populations have different mean body temperatures.

<table>
<thead>
<tr>
<th>Subject</th>
<th>16-20</th>
<th>21-29</th>
<th>30 and older</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 1</td>
<td>97.3</td>
<td>98.6</td>
<td>97.2</td>
</tr>
<tr>
<td>Subject 2</td>
<td>98.6</td>
<td>98.2</td>
<td>97.2</td>
</tr>
<tr>
<td>Subject 3</td>
<td>98.1</td>
<td>99.4</td>
<td>97.7</td>
</tr>
<tr>
<td>Subject 4</td>
<td>97.8</td>
<td>97.8</td>
<td>97.2</td>
</tr>
<tr>
<td>Subject 5</td>
<td>97.4</td>
<td>99.2</td>
<td>98.3</td>
</tr>
<tr>
<td>Subject 6</td>
<td>97.6</td>
<td>99.1</td>
<td>97.6</td>
</tr>
<tr>
<td>Mean</td>
<td>97.8</td>
<td>98.717</td>
<td>97.533</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.486</td>
<td>0.627</td>
<td>0.437</td>
</tr>
</tbody>
</table>

The variance between samples is $s^2_s = \frac{(97.2 - 98.717)^2 + (97.7 - 98.717)^2 + \ldots + (97.6 - 98.717)^2}{5}$
The variance within samples is $s^2_p = \frac{0.486^2 + 0.627^2 + 0.437^2}{5}$
The test statistic is $F = \frac{s^2_s}{s^2_p}$
The critical value is $F = \text{critical value}$

Is there sufficient evidence to warrant the rejection of the claim that the three age-group populations have the same mean body temperature?

- A. No
- B. Yes

**3. (1 pt) setStatistics8ANOVA/ur_sst8_3.pg**

An experiment is conducted to determine whether there is a difference among the mean increases in growth produced by five inoculins (A, B, C, D and E) of growth hormones for plants. The experimental material consists of 20 cuttings of a shrub (all of equal weight), with four cuttings randomly assigned to each of the five different inoculins. The increase in weight and the standard deviation of the experiment are given in the table below.

<table>
<thead>
<tr>
<th>Plant</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1</td>
<td>9</td>
<td>30</td>
<td>25</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Plant 2</td>
<td>8</td>
<td>27</td>
<td>22</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Plant 3</td>
<td>9</td>
<td>30</td>
<td>21</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>Plant 4</td>
<td>8</td>
<td>29</td>
<td>21</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Mean</td>
<td>8.5</td>
<td>29</td>
<td>22.25</td>
<td>13</td>
<td>10.75</td>
</tr>
<tr>
<td>Standard Dev.</td>
<td>0.5000</td>
<td>1.2247</td>
<td>1.6394</td>
<td>1.0000</td>
<td>4.2647</td>
</tr>
</tbody>
</table>

Compute the following:
(a) $\text{SST} = \ldots$
(b) $\text{SSE} = \ldots$
(c) $\text{MST} = \ldots$
(d) $\text{MSE} = \ldots$
(e) $F = \ldots$

**4. (1 pt) setStatistics8ANOVA/ur_sst8_4.pg**

Which of the following changes the analysis of variance results?

- A. each of the sample values is converted to a different scale
- B. the same constant is added to each value in one of the samples
- C. the same constant is added to every one of the sample values
- D. each of the sample values is multiplied by the same constant
- E. the order of the samples is changed
- F. each value in one of the samples is multiplied by the same constant

**5. (1 pt) setStatistics8ANOVA/ur_sst8_5.pg**

423 47 Suppose the Total Sum of Squares for a completely randomized design with $p = 6$ treatments and $n = 30$ total measurements (SS(Total)) is equal to 470. In each of the following cases, conduct an $F$ -test of the null hypothesis that the mean responses for the 6 treatments are the same. Use $\alpha = 0.1$.

(a) Sum of Squares for Treatment (SST) is 90% of SS(Total)

$F = \ldots$

Rejection region $F > \ldots$

The final conclusion is

- A. There is not sufficient evidence to reject the null hypothesis that the mean responses for the treatments are the same.
- B. We can reject the null hypothesis that the mean responses for the treatments are the same and accept the alternative hypothesis that at least two treatment means differ.

(b) Sum of Squares for Treatment (SST) is 40% of SS(Total)

$F = \ldots$

Rejection region $F > \ldots$
A study was conducted to see how people reacted to certain facial expressions. A sample group of \( n = 36 \) was randomly divided into six groups. Each group was assigned to view one picture of a person making a facial expression. Each group saw a different picture, and the different expressions were (1) Surprised (2) Nervous (3) Scared (4) Sad (5) Excited (6) Angry. After viewing the pictures, the subjects were asked to rank the degree of dominance they inferred from the facial expression they saw. (The scale ranged from -10 to 10) The data collected is summarized in the table below.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Surprised</th>
<th>Nervous</th>
<th>Scared</th>
<th>Sad</th>
<th>Excited</th>
<th>Angry</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.6</td>
<td>-0.3</td>
<td>1.9</td>
<td>-0.5</td>
<td>-1.6</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>-0.5</td>
<td>1</td>
<td>-0.5</td>
<td>-0.7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1.7</td>
<td>1.2</td>
<td>-0.09999999999999999</td>
<td>-0.5</td>
<td>-0.2</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.3</td>
<td>2</td>
<td>1.9</td>
<td>-0.2</td>
<td></td>
</tr>
<tr>
<td>1.9</td>
<td>1.7</td>
<td>0.6</td>
<td>-0.4</td>
<td>0.4</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>-0.8</td>
<td>-1.5</td>
<td>0.6</td>
<td>-0.8</td>
<td>-0.2</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>

Complete the following ANOVA table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td></td>
<td>28</td>
<td>1</td>
<td></td>
<td>0.057</td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>51</td>
<td>4.61</td>
<td></td>
<td>0.031</td>
</tr>
</tbody>
</table>

The P-value is 0.05. The sample data are the numbers of support beams manufactured by 3 different operators using 4 different machines. Assume that there is no interaction effect from operator and machine.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator</td>
<td>2</td>
<td>59.28</td>
<td>28.10</td>
<td>2.51</td>
<td>0.161</td>
</tr>
<tr>
<td>Machine</td>
<td>3</td>
<td>93.11</td>
<td>46.22</td>
<td>5.76</td>
<td>0.04</td>
</tr>
<tr>
<td>Error</td>
<td>6</td>
<td>47.87</td>
<td>8.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>200.26</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test the claim that the four operators have the same mean production output.

The F—test statistic is (1 pt) setStatistics8ANOVA/ur_stt_8_8.png

The P-value is (1 pt) setStatistics8ANOVA/ur_stt_8_7.png

Is there sufficient evidence to warrant the rejection of the claim that the four machine operators have the same mean production output?

• A. No
• B. Yes
Test the claim that the choice of machine has no effect on the production output.

The $F$-test statistic is _____.

The P-value is _____.

Is there sufficient evidence to warrant the rejection of the claim that the choice of machine has no effect on the production output?

- A. Yes
- B. No
1. (1 pt) setLinearAlgebra1Systems/ur_la_1_3.pg
Solve the system using substitution
\[
\begin{cases}
    x - 5y = 1 \\
    3x - 17y = -1
\end{cases}
\]
x = ___
y = ___

2. (1 pt) setLinearAlgebra1Systems/ur_la_1_4.pg
Solve the system using substitution
\[
\begin{cases}
    -2x + 7y = 82 \\
    -7x + 4y = 82
\end{cases}
\]
x = ___
y = ___

3. (1 pt) setLinearAlgebra1Systems/ur_la_1_4a.pg
Solve the system using elimination
\[
\begin{cases}
    -8x + 9y = -81 \\
    7x - 7y = 70
\end{cases}
\]
x = ___
y = ___

4. (1 pt) setLinearAlgebra1Systems/ur_la_1_5.pg
Solve the system using elimination
\[
\begin{cases}
    4x + 5y + 5z = 25 \\
    -3x - 4y + 4z = -11
\end{cases}
\]
x = ___
y = ___
z = ___

5. (1 pt) setLinearAlgebra1Systems/ur_la_1_23.pg
Write the system
\[
\begin{cases}
    -5y - 2z = -3 \\
    11x - 6y = 2 \\
    3x - 4y + 4z = 10
\end{cases}
\]
in matrix form.
\[
\begin{pmatrix}
    -5 & 0 & -2 \\
    11 & -6 & 0 \\
    3 & -4 & 4
\end{pmatrix}
\begin{pmatrix}
    x \\
    y \\
    z
\end{pmatrix}
= \begin{pmatrix}
    -3 \\
    2 \\
    10
\end{pmatrix}
\]

6. (1 pt) setLinearAlgebra1Systems/ur_la_1_7.pg
Write the augmented matrix of the system
\[
\begin{cases}
    -45y - 2z = 5 \\
    -91x + 32z = -8 \\
    -7x - 9y - z = 3
\end{cases}
\]
\[
\begin{pmatrix}
    -45 & 0 & -2 \\
    -91 & 32 & 0 \\
    -7 & -9 & -1
\end{pmatrix}
\]

7. (1 pt) setLinearAlgebra1Systems/ur_la_1_1.pg
Perform one step of row reduction, in order to calculate the values for x and y by back substitution. Then calculate the values for x and for y. Also calculate the determinant of the original matrix.

You can let webwork do much of the calculation for you if you want (e.g. enter 45-(-56)/76/(-3) instead of calculating the value out). You can also use the preview feature in order to make sure that you have used the correct syntax in entering the answer.

[Note- since the determinant is unchanged by row reduction it will be easier to calculate the determinant of the row reduced matrix.]
\[
\begin{pmatrix}
    8 & 14 \\
    14 & -13 \\
    0 & 14
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
= \begin{pmatrix}
    -2 \\
    4 \\
    3
\end{pmatrix}
\]
x = ___
y = ___
det = ___

8. (1 pt) setLinearAlgebra1Systems/ur_la_1_2.pg
Perform one step of row reduction, in order to calculate the values for x and y by back substitution. Then calculate the values for x and for y. Also calculate the determinant of the original matrix.

You can let webwork do much of the calculation for you if you want (e.g. enter 45-(-56)/76/(-3) instead of calculating the value out). You can also use the preview feature in order to make sure that you have used the correct syntax in entering the answer.

This problem has rather difficult complex calculations.
[Note- since the determinant is unchanged by row reduction it will be easier to calculate the determinant of the row reduced matrix.]
\[
\begin{pmatrix}
    3 - 5i & -2 - 4i \\
    -5 + 3i & 5 + 4i \\
    3 - 5i & -2 - 4i
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
= \begin{pmatrix}
    -14 - 22i \\
    9 + 46i \\
    -14 - 22i
\end{pmatrix}
\]
x = ___
y = ___
det = ___

9. (1 pt) setLinearAlgebra1Systems/ur_la_1_4b.pg
Solve the system using matrices (row operations)
\[
\begin{cases}
    8x + 9y = 1 \\
    3x + 4y = 1
\end{cases}
\]
x = ___
y = ___

10. (1 pt) setLinearAlgebra1Systems/ur_la_1_5a.pg
Solve the system using matrices (row operations)
\[
\begin{cases}
    4x - 3y - 5z = -3 \\
    5x + 4y + 2z = -13 \\
    4x + 3y - 2z = 0
\end{cases}
\]
x = ___
Solve the system
\[
\begin{align*}
  x + y &= -13 \\
  4x - 2y &= 8 \\
  14x - 4y &= -2
\end{align*}
\]
\[x = \]  
\[y = \]

Solve the system
\[
\begin{align*}
  x_1 + 3x_3 + 3x_4 &= -24 \\
  x_2 - 4x_3 - 2x_4 &= 23 \\
  3x_1 - 2x_2 + 20x_3 + 13x_4 &= -127 \\
  -x_2 + 4x_3 + 6x_4 &= -43
\end{align*}
\]
\[x_1 = \]  
\[x_2 = \]  
\[x_3 = \]  
\[x_4 = \]

For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1. \[
\begin{align*}
  3x - 10y &= 22 \\
  4x - 7y &= 23
\end{align*}
\]
   - A. Unique solution: \(x = -1, y = 4\)
   - B. Infinitely many solutions
   - C. No solutions
   - D. Unique solution: \(x = 0, y = 0\)
   - E. Unique solution: \(x = 4, y = -1\)
   - F. None of the above

2. \[
\begin{align*}
  5x + 2y &= 18 \\
  -5x - 2y &= -17
\end{align*}
\]
   - A. No solutions
   - B. Unique solution: \(x = -17, y = 18\)
   - C. Unique solution: \(x = 18, y = -17\)
   - D. Unique solution: \(x = 0, y = 0\)
   - E. Infinitely many solutions
   - F. None of the above

3. \[
\begin{align*}
  -2x + 4y &= 0 \\
  3x - 9y &= 0
\end{align*}
\]
   - A. Infinitely many solutions
   - B. Unique solution: \(x = 0, y = 0\)
   - C. Unique solution: \(x = 2, y = -6\)
   - D. No solutions
   - E. Unique solution: \(x = 9, y = -2\)
   - F. None of the above

The reduced row-echelon forms of the augmented matrices of four systems are given below. How many solutions does each system have?

1. \[
\begin{pmatrix}
  1 & 0 & -2 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 \\
  0 & 0 & 0 & 0
\end{pmatrix}
\]
   - A. Unique solution
   - B. No solutions
   - C. Infinitely many solutions
   - D. None of the above

2. \[
\begin{pmatrix}
  0 & 1 & 0 & -13 \\
  0 & 0 & 1 & 14
\end{pmatrix}
\]
   - A. Infinitely many solutions
   - B. Unique solution
   - C. No solutions
   - D. None of the above

3. \[
\begin{pmatrix}
  1 & 0 & 0 & 19 \\
  0 & 0 & 1 & -9
\end{pmatrix}
\]
   - A. No solutions
   - B. Infinitely many solutions
   - C. Unique solution
   - D. None of the above

4. \[
\begin{pmatrix}
  1 & 0 & 17 \\
  0 & 0 & 16 \\
  0 & 0 & 0
\end{pmatrix}
\]
   - A. Unique solution
   - B. Infinitely many solutions
   - C. No solutions
   - D. None of the above

Solve the equation
\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix}
= \begin{pmatrix}
  -\infty \\
  -\infty \\
  -\infty
\end{pmatrix} + \begin{pmatrix}
  \infty \\
  \infty \\
  \infty
\end{pmatrix} t.
\]

Solve the system
\[
\begin{align*}
  -2x + x_2 &= -3 \\
  -8x_1 + 4x_2 &= -12
\end{align*}
\]
17. (1 pt) setLinearAlgebra1Systems/ur_la_1_14.png
Solve the system
\[
\begin{align*}
    x_1 + x_2 - 2x_3 &= -6 \\
    4x_1 + 5x_2 + 4x_3 &= -2
\end{align*}
\]
\[
\begin{pmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
\end{pmatrix}
= \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
+ \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
s.
\]

18. (1 pt) setLinearAlgebra1Systems/ur_la_1_15.png
Solve the system
\[
\begin{align*}
    x_1 - x_2 + 4x_3 &= -1 \\
    6x_1 - 7x_2 + 9x_3 &= -4 \\
    2x_1 + 38x_3 &= -6
\end{align*}
\]
\[
\begin{pmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
\end{pmatrix}
= \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
+ \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
s.
\]

19. (1 pt) setLinearAlgebra1Systems/ur_la_1_18.png
Solve the system
\[
\begin{align*}
    x_1 + x_2 &= -2 \\
    x_2 + x_3 &= -4 \\
    x_3 + x_4 &= 4 \\
    x_1 + x_4 &= 6
\end{align*}
\]
\[
\begin{pmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
\end{pmatrix}
= \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
+ \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
s.
\]

20. (1 pt) setLinearAlgebra1Systems/ur_la_1_19.png
Solve the system
\[
\begin{align*}
    5x_1 - 4x_2 + 4x_3 + 4x_4 &= 4 \\
    -x_1 + x_2 + 3x_3 + 2x_4 &= 5 \\
    4x_1 - 3x_2 + 7x_3 + 6x_4 &= 9 \\
    3x_1 - 3x_2 - 9x_3 - 6x_4 &= -15
\end{align*}
\]
\[
\begin{pmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
\end{pmatrix}
= \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
s + \begin{pmatrix}
    \quad \\
    \quad \\
    \quad \\
    \quad \\
\end{pmatrix}
t.
\]

21. (1 pt) setLinearAlgebra1Systems/ur_la_1_20.png
Solve the system
\[
\begin{align*}
    x_1 - 5x_2 - 2x_3 + 3x_4 - 4x_6 &= 5 \\
    -x_4 + 2x_5 + 5x_6 &= -6 \\
    x_1 - 5x_2 - 7x_3 - 8x_6 &= 3
\end{align*}
\]

22. (1 pt) setLinearAlgebra1Systems/ur_la_1_4c.png
Solve the system by using Cramer’s Rule.
\[
\begin{align*}
    6x + 3y &= -30 \\
    9x + 2y &= -40
\end{align*}
\]
x = 6

23. (1 pt) setLinearAlgebra1Systems/ur_la_1_18.png
Determine the value of \(h\) such that the matrix is the augmented matrix of a consistent linear system.
\[
\begin{pmatrix}
    4 & -4 & h \\
    -12 & 12 & 4
\end{pmatrix}
\]
h =

24. (1 pt) setLinearAlgebra1Systems/ur_la_1_19.png
Determine the value of \(h\) such that the matrix is the augmented matrix of a linear system with infinitely many solutions.
\[
\begin{pmatrix}
    4 & -5 & 2 \\
    8 & h & 4
\end{pmatrix}
\]
h =

25. (1 pt) setLinearAlgebra1Systems/ur_la_1_11.png
Solve the system
\[
\begin{align*}
    4x - 5y &= a \\
    5x - 6y &= b
\end{align*}
\]
x =
y =

26. (1 pt) setLinearAlgebra1Systems/ur_la_1_12.png
Determine the value of \(k\) for which the system
\[
\begin{align*}
    x + y + 3z &= 0 \\
    x + 2y - 3z &= -2 \\
    4x + 10y + kz &= -11
\end{align*}
\]
has no solutions.
k =
The dot product of two vectors $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$ in $\mathbb{R}^n$ is defined by $\mathbf{x} \cdot \mathbf{y} = x_1y_1 + x_2y_2 + \ldots + x_ny_n$.

The vectors $\mathbf{x}$ and $\mathbf{y}$ are called perpendicular if $\mathbf{x} \cdot \mathbf{y} = 0$.

Then any vector in $\mathbb{R}^3$ perpendicular to $\begin{pmatrix} 1 \\ 8 \\ 1 \end{pmatrix}$ can be written in the form $s \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.
1. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_1.pg
Fred and Sasha are brother and sister. Fred has twice as many brothers as sisters, and Sasha has five times as many brothers as sisters. How many boys and girls are there in this family?
Answer: _____ boys and _____ girls.

2. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_2.pg
Find the quadratic polynomial whose graph goes through the points \((-1,4), (0,2),\) and \((2,22)\).
\[ f(x) = \_\_\_\_ x^2 + \_\_\_\_ x + \_\_\_\_ \]

3. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_3.pg
Find the polynomial of degree 4 whose graph goes through the points \((-2, -51), (-1, -5), (0, 1), (1, 9),\) and \((2, 13)\).
\[ f(x) = \_\_\_\_ x^4 + \_\_\_\_ x^3 + \_\_\_\_ x^2 + \_\_\_\_ x + \_\_\_\_ \]

4. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_4.pg
Find the cubic polynomial \(f(x)\) such that \(f(-2) = 9, f'(-2) = 0, f''(-2) = -8,\) and \(f'''(-2) = 6.\)
\[ f(x) = \_\_\_\_ x^3 + \_\_\_\_ x^2 + \_\_\_\_ x + \_\_\_\_ \]

5. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_5.pg
Consider the chemical reaction
\[ aC_2H_6 + bCO_2 + cH_2O \rightarrow dC_2H_5OH, \]
where \(a, b, c,\) and \(d\) are unknown positive integers. The reaction must be balanced; that is, the number of atoms of each element must be the same before and after the reaction. For example, because the number of oxygen atoms must remain the same,
\[ 2b + c = d. \]
While there are many possible choices for \(a, b, c,\) and \(d\) that balance the reaction, it is customary to use the smallest possible integers. Balance this reaction.
\[ a = \_\_\_\_ \]
\[ b = \_\_\_\_ \]
\[ c = \_\_\_\_ \]
\[ d = \_\_\_\_ \]

6. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_6.pg
In a grid of wires, the temperature at exterior mesh points is maintained at constant values as shown in the figure. When the grid is in thermal equilibrium, the temperature at each interior mesh point is the average of the temperatures at the four adjacent points. For instance,
\[ T_1 = \frac{T_2 + T_3 + 20 - 90}{4}. \]
Find the temperatures \(T_1, T_2, T_3, T_4\), when the grid is in thermal equilibrium.

7. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_7.pg
Consider a two-commodity market. When the unit prices of the products are \(P_1\) and \(P_2\), the quantities demanded, \(D_1\) and \(D_2\), and the quantities supplied, \(S_1\) and \(S_2\), are given by
\[ D_1 = 119 - 3P_1 + P_2 \]
\[ D_2 = 79 + P_1 - 2P_2 \]
\[ S_1 = -59 + 3P_1 \]
\[ S_2 = -14 + 2P_2 \]
(a) What is the relationship between the two commodities? Do they compete, as do Volvos and BMWs, or do they complement one another, as do shirts and ties? (type in “compete” or “complement”)

(b) Find the equilibrium prices (i.e. the prices for which supply equals demand), for both products.
\[ P_1 = \_\_\_\_\_, P_2 = \_\_\_\_\_ \]

8. (1 pt) setLinearAlgebra2SystemsApplications/ur_ja_2_8.pg
A dietician is planning a meal that supplies certain quantities of vitamin C, calcium, and magnesium. Three foods will be used, their quantities measured in milligrams. The nutrients supplied by one unit of each food and the dietary requirements are given in the table below.

<table>
<thead>
<tr>
<th>Nutrient</th>
<th>Food 1</th>
<th>Food 2</th>
<th>Food 3</th>
<th>Total Required (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vitamin C</td>
<td>30</td>
<td>45</td>
<td>30</td>
<td>397.5</td>
</tr>
<tr>
<td>Calcium</td>
<td>40</td>
<td>70</td>
<td>50</td>
<td>610</td>
</tr>
<tr>
<td>Magnesium</td>
<td>10</td>
<td>25</td>
<td>20</td>
<td>212.5</td>
</tr>
</tbody>
</table>

Find the total required (mg) for each nutrient.
Write the augmented matrix for this problem.
\[
\begin{pmatrix}
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_ \\
\end{pmatrix}
\]

What quantity (in units) of Food 1 is necessary to meet the dietary requirements?

What quantity (in units) of Food 2 is necessary to meet the dietary requirements?

What quantity (in units) of Food 3 is necessary to meet the dietary requirements?
1. (1 pt) setLinearAlgebra3Matrices/ur_la_3_21.pg

If \( A = \begin{pmatrix} 6 & 1 & 17 & 2 \\ 5 & -7 & -15 & 5 \\ 12 & 13 & 17 & -18 \end{pmatrix} \), determine the following entries:

\[ a_{14} = \quad , \quad a_{24} = \quad , \quad a_{32} = \quad . \]

2. (1 pt) setLinearAlgebra3Matrices/ur_la_3_22.pg

Write a 2 \times 2 matrix with \( A \) with entries

\[ a_{11} = -18, \quad a_{12} = 11, \quad a_{13} = -6, \quad a_{21} = 17, \quad a_{22} = 20, \quad a_{23} = -7. \]

\[ A = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \]

3. (1 pt) setLinearAlgebra3Matrices/ur_la_3_23.pg

If \( A = \begin{pmatrix} -7 & 8 \\ 6 & 6 \end{pmatrix} \) then \( \text{tr}(A) = \quad . \)

4. (1 pt) setLinearAlgebra3Matrices/ur_la_3_24.pg

If \( A = \begin{pmatrix} 4 & 8 & -6 \\ 2 & 2 & -7 \\ -5 & -6 & -8 \end{pmatrix} \) then \( \text{tr}(A) = \quad . \)

5. (1 pt) setLinearAlgebra3Matrices/ur_la_3_29.pg

Let \( A = \begin{pmatrix} -3 & 1 & 2 \\ 3 & 9 & -9 \end{pmatrix} \).

Perform the following row operations.

(a) \( P_{12} \), permute the 1st and 2nd rows:

\[ \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \]

(b) \( M_1 \left( \frac{1}{3} \right) \), multiply every element of the 1st row by \( \frac{1}{3} \):

\[ \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \]

(c) \( A_{12}(3) \), add to the elements of the 2nd row, 3 times the corresponding elements of the 1st row:

\[ \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \]

6. (1 pt) setLinearAlgebra3Matrices/ur_la_3_35.pg

Given the augmented matrix \( A \), perform each row operation in order, (a) followed by (b) followed by (c).

\( A = \begin{pmatrix} 1 & 3 & 2 & 6 \\ 3 & 10 & -6 & -5 \\ -4 & -14 & 1 & -3 \end{pmatrix} \)

(a) \( R_2 = -3r_1 + r_2 \)

(b) \( R_3 = 4r_1 + r_3 \)

(c) \( R_3 = 2r_2 + r_3 \)

\[ \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \]

7. (1 pt) setLinearAlgebra3Matrices/ur_la_3_30.pg

Reduce the matrix \( \begin{pmatrix} 4 & -1 & 16 \\ -1 & -3 & 9 \end{pmatrix} \) to reduced row-echelon form.

\[ \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \]

8. (1 pt) setLinearAlgebra3Matrices/ur_la_3_31.pg

Reduce the matrix \( \begin{pmatrix} 3 & -3 & -12 \\ -2 & 1 & -7 \end{pmatrix} \) to reduced row-echelon form.

\[ \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \]

9. (1 pt) setLinearAlgebra3Matrices/ur_la_3_32.pg

Reduce the matrix \( \begin{pmatrix} 1 & 2 & 3 & 14 \\ 2 & 3 & 3 & 17 \end{pmatrix} \) to reduced row-echelon form.

\[ \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \]

10. (1 pt) setLinearAlgebra3Matrices/ur_la_3_36.pg

If \( A \) and \( B \) are 3 \times 5 matrices, and \( C \) is a 9 \times 3 matrix, which of the following are defined?

- A. \( B^T C^T \)
- B. \( C - A \)
- C. \( A^T \)
- D. \( BA \)
- E. \( B + A \)
- F. \( CB \)

11. (1 pt) setLinearAlgebra3Matrices/ur_la_3_13.pg

If \( A, B, \) and \( C \) are 3 \times 3, 3 \times 2, and 2 \times 8 matrices respectively, determine which of the following products are defined. For those defined, enter the size of the resulting matrix (e.g. "3 x 4", with spaces between numbers and "x"). For those undefined, enter "undefined".

- \( AB: \)
- \( AC: \)
- \( BA: \)
- \( CB: \)

12. (1 pt) setLinearAlgebra3Matrices/ur_la_3_14.pg

Find the ranks of the following matrices.

\[
\begin{pmatrix} 1 & 5 \\ 8 & -5 \end{pmatrix} = \quad
\begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}
\]

\[
\begin{pmatrix} 0 & -2 & 0 \\ -5 & 0 & 5 \end{pmatrix} = \quad
\begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}
\]
13. Compute the following product.

\[
\begin{pmatrix}
  2 & 5 & -5 \\
  -2 & 8 & 0 \\
  2 & 0 & 0
\end{pmatrix}
\]

Then \(2A - B = \)

14. Compute the following product.

\[
\begin{pmatrix}
  -3 & -1 & 3 \\
  -4 & -2 & 0 \\
  -3 & 2 & -2
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  0 & -4 & 3 \\
  4 & 3 & 0 \\
  -1 & -2 & -1
\end{pmatrix}
\]

Then \(2A - B = \)

15. Compute the following product.

\[
\begin{pmatrix}
  0 & -2 & -4 \\
  -4 & -3 & 1 \\
  -3 & -2 & -1
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  -2 & -1 & -3 \\
  0 & 2 & -4 \\
  3 & 0 & 1
\end{pmatrix}
\]

Then \(4A + B = \)

16. Compute the following product.

\[
\begin{pmatrix}
  3 & 2 & -2 \\
  2 & 2 & 0 \\
  -1 & 4 & 3
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  2 & 1 & 4 \\
  0 & 1 & -1 \\
  1 & 3 & 0
\end{pmatrix}
\]

Then \(4A - 4B = \)

17. Compute the following product.

\[
\begin{pmatrix}
  3 & 2 \\
  5 & 3
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  -2 & 8 \\
  -5 & 6
\end{pmatrix}
\]

Then \(AB = \)

18. Compute the following product.

\[
\begin{pmatrix}
  1 & 2 & 4 \\
  2 & 3 & -3
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  3 & 5 \\
  3 & 5
\end{pmatrix}
\]

Then \(\begin{pmatrix} 1 \end{pmatrix} A = \)

19. Compute the following product.

\[
\begin{pmatrix}
  4 & -3 & -4 & -3 \\
  -3 & -2 & -4
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  5 \\
  1 \\
  -2 \\
  -4
\end{pmatrix}
\]

Then \(AB = \)

20. Compute the following product.

\[
\begin{pmatrix}
  -4 & 7 \\
  -3 & 0 \\
  0 & 4
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  6 & \ \\
  -2 & \ \\
  0 & \ \\
\end{pmatrix}
\]

Then \(2A - B = \)

21. Compute the following product.

\[
\begin{pmatrix}
  5 & 5 & 1 \\
  -4 & 3 & -4 \\
  3 & -5 & 5
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  -5 & \ \\
  -1 & \ \\
  2 & \ \\
\end{pmatrix}
\]

Then \(B = \)

22. Compute the following product.

\[
\begin{pmatrix}
  -1 & 2 \\
  -4 & 4 \\
  1 & 1
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  -2 & 1 \\
  -3 & 2 \\
  1 & 1
\end{pmatrix}
\]

Then \(AB = \)

23. Compute the following product.

\[
\begin{pmatrix}
  -4 & -2 & 3 \\
  3 & 1 & 2 \\
  3 & 2
\end{pmatrix}
\]

Then \(A^T = \)

24. Compute the following products.

\[
\begin{pmatrix}
  3 & 1 & 2 \\
  -3 & 1 & 1 \\
  -3 & 1 & 1
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  -2 & 1 \\
  1 & 1 \\
  1 & 1
\end{pmatrix}
\]

Then \(\begin{pmatrix} -3 \ 2 \
\end{pmatrix} A = \)

25. Find a \(3 \times 3\) matrix \(A\) such that

\[
\begin{pmatrix}
  1 \\
  0 \\
  0
\end{pmatrix} A = \)

26. Compute the following product.

\[
\begin{pmatrix}
  -3 & -4 & 0 \\
  4 & 3 & 4 \\
  -4 & -2 & 0
\end{pmatrix}
\] and

\[
\begin{pmatrix}
  2 & 3 & -4 \\
  2 & 0 & -3 \\
  1 & 3 & -2
\end{pmatrix}
\]

Then \(AB = \)
If \( A = \begin{pmatrix} -1 + 4i & -3 + 2i \\ -2i & -1 + 2i \end{pmatrix} \) and \( B = \begin{pmatrix} -2 - 2i & 2 - 3i \\ -4 - 4i & 3 - 4i \end{pmatrix} \)

Then \( AB = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \) and \( BA = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \)

If \( A = \begin{pmatrix} 1 + 4i & 2 + 3i \\ 2i & 1 + 2i \end{pmatrix} \) and \( B = \begin{pmatrix} 2 - 2i & 2 - 3i \\ 3 - 4i & 3 - 4i \end{pmatrix} \)

Then \( AB = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \) and \( BA = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \)

If \( A = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \)

Then \( \text{rank } A = \_ \), and \( A^2 = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \)

If \( A = \begin{pmatrix} x & -9 \\ y & 3 \end{pmatrix} \), determine the values of \( x \) and \( y \) for which \( A^2 = A \).

\( x = \_ \), \( y = \_ \).

Find the value of \( k \) for which the matrix
\[
A = \begin{pmatrix} -2 & -8 & 14 \\ -6 & 7 & -20 \\ 4 & -2 & k \end{pmatrix}
\]
has rank 2.

\( k = \_ \).

Let \( A = \begin{pmatrix} 1 & 4 \\ -4 & -1 \end{pmatrix} \) and \( B = \begin{pmatrix} -4 & -5 \\ 1 & 2 \end{pmatrix} \).

Find the commutator of \( A \) and \( B \).

\([A, B] = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \).\]
1. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_1.pg
If \( A = \begin{pmatrix} -3 & 8 \\ 9 & 3 \end{pmatrix} \)
Then \( A^{-1} = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \)

2. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_2.pg
The matrix \( \begin{pmatrix} 2 & 3 \\ 5 & k \end{pmatrix} \) is invertible if and only if \( k \neq \_ \)

3. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_3.pg
If \( A = \begin{pmatrix} 1 & -5 & -6 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix} \)
Then \( A^{-1} = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \)

4. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_4.pg
If \( A = \begin{pmatrix} 1 & 1 & 0 \\ -5 & -6 & 4 \\ -1 & -1 & 1 \end{pmatrix} \)
Then \( A^{-1} = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \)

5. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_5.pg
A square matrix is called a permutation matrix if it contains the entry 1 exactly once in each row and in each column, with all other entries being 0. All permutation matrices are invertible. Find the inverse of the following permutation matrix
\[
A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}
\]
\[
A^{-1} = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix}
\]

6. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_6.pg
If \( A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -4 & 4 & 1 & 0 \\ -4 & -7 & -3 & 1 \end{pmatrix} \)
Then \( A^{-1} = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \)

Then \( A^{-1} = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \)

7. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_7.pg
If \( A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -2 & 7 \end{pmatrix} \)
Then \( A^{-1} = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \)

8. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_8.pg
Determine which of the formulas hold for all invertible \( n \times n \) matrices \( A \) and \( B \)
- A. \( A^6B^5 \) is invertible
- B. \( ABA^{-1} = B \)
- C. \((A+A^{-1})^3 = A^3 + A^{-3}\)
- D. \( A+I_n \) is invertible
- E. \((A(AA^{-1})^7 = AB^3A^{-1}\)
- F. \((A + B)^2 = A^2 + B^2 + 2AB\)

9. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_9.pg
If \( A = \begin{pmatrix} i & i \\ 1 & 1+i \end{pmatrix} \)
then \( A^{-1} = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \).

10. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_10.pg
If \( A = \begin{pmatrix} 1 & 0 & -2 & -2i \\ 2 & i & 1 & -7 - i \\ -1 + 3i & 0 & 8 & 5i \end{pmatrix} \)
then \( A^{-1} = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \).

11. (1 pt) setLinearAlgebra4InverseMatrix/ur_la_4_11.pg
If \( A = \begin{pmatrix} 4e^{5i} \sin(2t) & 4e^{2i} \cos(2t) \\ 3e^{5i} \cos(2t) & -3e^{2i} \sin(2t) \end{pmatrix} \)
then \( A^{-1} = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \).
Find the LU factorization of $A = \begin{pmatrix} 5 & 1 \\ -15 & 2 \end{pmatrix}$, i.e. write $A = LU$ where $L$ is a lower triangular matrix with 1’s on the diagonal, and $U$ is an upper triangular matrix.

$A = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$.

Find the LU factorization of $A = \begin{pmatrix} 1 & 4 & -1 \\ -1 & -1 & -2 \end{pmatrix}$.

$A = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}$.

Find the LU factorization of $A = \begin{pmatrix} -2 & 1 \\ 6 & -5 \end{pmatrix}$.

$A = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}$.

Find the LU factorization of $A = \begin{pmatrix} 4 & -3 & -2 \\ -12 & 7 & 5 \\ -20 & 11 & 3 \end{pmatrix}$.

$A = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}$.

Find the LU factorization of $A = \begin{pmatrix} -5 & 3 & -3 \\ 15 & -6 & 7 \end{pmatrix}$.

$A = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}$.

Find the LU factorization of $A = \begin{pmatrix} 1 & -2 & -1 & -3 \\ -2 & 2 & 1 & 8 \\ -1 & 6 & 4 & -6 \\ -3 & 10 & 3 & 17 \end{pmatrix}$.

$A = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix}$.

Find the LU factorization of $A = \begin{pmatrix} 1 & -2 \\ 4 & -12 \end{pmatrix}$, and use it to solve the system $\begin{pmatrix} 1 & -2 \\ 4 & -12 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 12 \\ 64 \end{pmatrix}$.

$x_1 = \_ \\
x_2 = \_ \\

Find the LU factorization of $A = \begin{pmatrix} -1 & -2 \\ 3 & 9 \end{pmatrix}$, and use it to solve the system $\begin{pmatrix} -1 & -2 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -5 \\ 28 \end{pmatrix}$.

$x_1 = \_ \\
x_2 = \_ \\

Find the LU factorization of $A = \begin{pmatrix} -1 & -1 & 4 & -1 \\ 1 & 3 & -1 & -1 \\ -3 & -9 & 5 & 1 \\ -4 & 0 & 26 & -14 \end{pmatrix}$, and use it to solve the system $\begin{pmatrix} -1 & -1 & 4 & -1 \\ 1 & 3 & -1 & -1 \\ -3 & -9 & 5 & 1 \\ -4 & 0 & 26 & -14 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -13 \\ 15 \\ -45 \\ -40 \end{pmatrix}$.

$x_1 = \_ \\
x_2 = \_ \\
x_3 = \_ \\
x_4 = \_
10. Find the LDU factorization of \( A = \begin{pmatrix} 4 & 12 \\ -16 & -50 \end{pmatrix} \), i.e. write \( A = LDU \) where \( L \) is a lower triangular matrix with 1’s on the diagonal, \( D \) is a diagonal matrix, and \( U \) is an upper triangular matrix with 1’s on the diagonal. 
\[
A = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}.
\]

11. Find the LDU factorization of \( A = \begin{pmatrix} 1 & 2 & 2 \\ -3 & -2 & 14 \\ -2 & -4 & -2 \end{pmatrix} \). 
\[
A = LDU \text{ where } \\
L = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}, \\
D = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}, \\
U = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}.
\]

12. Find the LDU factorization of 
\[
A = \begin{pmatrix} 2 & -4 & -4 & 2 \\ -4 & 7 & 10 & -2 \\ -2 & 2 & 12 & -14 \\ 4 & -7 & -10 & 5 \end{pmatrix}.
\]
\[
A = LDU \text{ where } \\
L = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix}, \\
D = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix}, \\
U = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix}.
\]
Find the determinant of the matrix
\[ A = \begin{pmatrix} 9 & 11 \\ -4 & -1 \end{pmatrix}. \]
det(A) = _____

Find the determinant of the matrix
\[ M = \begin{pmatrix} -3 & -6 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & -5 \end{pmatrix}. \]
det(M) = _____

A square matrix is called a permutation matrix if each row and each column contains exactly one entry 1, with all other entries being 0. An example is
\[ P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]
Find the determinant of this matrix.
det(P) = _____

Find the determinant of the matrix
\[ B = \begin{pmatrix} 4 & -4 & 1 \\ -3 & 5 & -4 \\ -2 & 0 & 3 \end{pmatrix}. \]
det(B) = _____

Find the determinant of the matrix
\[ A = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 9 & -3 & 0 & 0 \\ -1 & -8 & -3 & 0 \\ 8 & 7 & 5 & 1 \end{pmatrix}. \]
det(A) = _____

Find the determinant of the matrix
\[ M = \begin{pmatrix} -3 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 1 & -3 & 0 \end{pmatrix}. \]
det(M) = _____

Find the determinant of the matrix
\[ C = \begin{pmatrix} 2 & -1 & -2 & 1 \\ 0 & -3 & -2 & 0 \\ 2 & -2 & 0 & -1 \\ -1 & 0 & -1 & -1 \end{pmatrix}. \]
det(C) = _____

Find the determinant of the matrix
\[ A = \begin{pmatrix} 1 & 7 \\ -6 & 3 \end{pmatrix}. \]
then \( \det(A) = \) _____ and \( A^{-1} = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \)

Find \( k \) such that the matrix
\[ M = \begin{pmatrix} -3 & -14 & -14 \\ -18 + k & -43 & -40 \end{pmatrix} \]
is singular.
\( k = \) _____

If the determinant of a 3 \( \times \) 3 matrix \( A \) is \( \det(A) = 9 \), and the matrix \( B \) is obtained from \( A \) by multiplying the third column by 5, then \( \det(B) = \) _____

If the determinant of a 5 \( \times \) 5 matrix \( A \) is \( \det(A) = 3 \), and the matrix \( C \) is obtained from \( A \) by swapping the third and fourth rows, then \( \det(C) = \) _____

If the determinant of a 4 \( \times \) 4 matrix \( A \) is \( \det(A) = 2 \), and the matrix \( D \) is obtained from \( A \) by adding 4 times the fourth row to the second, then \( \det(D) = \) _____

If \( A \) and \( B \) are 4 \( \times \) 4 matrices, \( \det(A) = -5 \), \( \det(B) = 9 \), then \( \det(AB) = \) _____
\( \det(3A) = \) _____
\( \det(A^T) = \) _____
\( \det(B^{-1}) = \) _____
\( \det(B^3) = \) _____

If \( \det \begin{pmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{pmatrix} = 3 \)
\( \text{and} \ \det \begin{pmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{pmatrix} = 5 \),
By definition, the determinant of \( M \) is given by

\[
\det(M) = m_{11}m_{22} - m_{12}m_{21}
\]
23. (1 pt) setLinearAlgebra6Determinants/ur_la_6_24.pg
Let \( A = \begin{pmatrix} 3 & 4 \\ 6 & -5 \end{pmatrix} \).
Find the following:
(a) \( \det(A) = \) 
(b) the matrix of cofactors \( C = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} \),
(c) \( \text{adj}(A) = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} \),
(d) \( A^{-1} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} \).

24. (1 pt) setLinearAlgebra6Determinants/ur_la_6_25.pg
Let \( A = \begin{pmatrix} 0 & 2 & -3 \\ -1 & -1 & 3 \\ 3 & -2 & 2 \end{pmatrix} \).
Find the following:
(a) \( \det(A) = \) 
(b) the matrix of cofactors \( C = \begin{pmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix} \),
(c) \( \text{adj}(A) = \begin{pmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix} \),
(d) \( A^{-1} = \begin{pmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix} \).

25. (1 pt) setLinearAlgebra6Determinants/ur_la_6_26.pg
Let \( A = \begin{pmatrix} 3e^{2t} & -2e^{3t} \\ 6e^{2t} & -3e^{3t} \end{pmatrix} \).
Find the following:
(a) \( \det(A) = \) 
(b) the matrix of cofactors \( C = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} \),
(c) \( \text{adj}(A) = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} \),
(d) \( A^{-1} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix} \).

26. (1 pt) setLinearAlgebra6Determinants/ur_la_6_27.pg
Find the determinant of the matrix
\[
M = \begin{pmatrix} -2 - 2x^3 & 6 + 2x^2 + 4x^3 & 0 \\ -x^2 & 1 + x^2 + 2x^3 & 0 \\ 2 + 4x^2 & -4 - 8x^2 & -3 - 2x^2 \end{pmatrix},
\]
and use the adjoint method to find \( M^{-1} \).
\( \det(M) = \) 
\( M^{-1} = \begin{pmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix} \).
1. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_2.pg
Find the area of the parallelogram defined by the vectors 
\[
\begin{pmatrix} 4 \\ 2 \\ 3 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -5 \\ -3 \\ 1 \\ 3 \end{pmatrix}.
\]
Area = ________

2. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_1.pg
Find the area of the parallelogram defined by the vectors 
\[
\begin{pmatrix} 3 \\ 2 \\ 3 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -2 \\ -3 \\ 1 \\ 3 \end{pmatrix}.
\]
Area = ________

3. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_3.pg
Find the area of the parallelogram with vertices at (2, -5), (-10, -3), (1, -6), and (-11, -4).
Area = ________

4. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_4.pg
Find the area of the triangle with vertices (3, 3), (8, 5), and (2, 11).
Area = ________

5. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_6.pg
Find the area of the quadrangle with vertices (4, 1), (-6, 4), (-1, -4), and (3, -3).

6. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_10.pg
Find the area of the ellipse given by \[
\frac{x^2}{49} + \frac{y^2}{36} = 1.
\]
Area = ________

7. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_8.pg
Find the volume of the parallelepiped defined by the vectors 
\[
\begin{pmatrix} -5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 3 \\ 5 \end{pmatrix}, \text{ and } \begin{pmatrix} 2 \\ -2 \end{pmatrix}.
\]
Volume = ________

8. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_5.pg
Find the volume of the parallelepiped defined by the vectors 
\[
\begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}, \text{ and } \begin{pmatrix} -1 \\ 1 \end{pmatrix}.
\]
Volume = ________

9. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_6.pg
Find the volume of the parallelepiped with one vertex at (3, -4, -4), and adjacent vertices at (7, -9, -3), (3, -2, -5), and (9, -7, -10).
Volume = ________

10. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_9.pg
Find the volume of the tetrahedron with vertices (-4, -3, -2), (-5, 2, -4), (-7, 1, -2), and (-5, -7, -2).
Volume = ________

11. (1 pt) setLinearAlgebra7AreaVolume/ur_la_7_11.pg
Find the volume of the ellipsoid given by \[
\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{9} = 1.
\]
Volume = ________
1. Let $x = (-2, 1)$ and $y = (-6, 4)$. Find the vectors $v = 6x$, $u = 3y$, and $w = 6x + 3y$.

$v = (\ldots, \ldots)$,
$u = (\ldots, \ldots)$,
$w = (\ldots, \ldots)$.

2. Let $x = \begin{pmatrix} -4 \\ -1 \\ -3 \end{pmatrix}$ and $y = \begin{pmatrix} 0 \\ -5 \\ -1 \end{pmatrix}$. Find the vectors $v = 4x$, $u = x + y$, and $w = 4x + y$.

$v = (\ldots, \ldots, \ldots)$,
$u = (\ldots, \ldots, \ldots)$,
$w = (\ldots, \ldots, \ldots)$.

3. Let $u = (3, -5, -1)$ and $v = (0, -1, -5)$. Find the vector $w = 3u - 2v$ and its additive inverse.

$w = (\ldots, \ldots, \ldots)$,
$-w = (\ldots, \ldots, \ldots)$.

4. Let $x = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ and $y = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$. Find the vector $v = 2x - 6y$ and its additive inverse.

$v = (\ldots, \ldots, \ldots)$,
$-v = (\ldots, \ldots, \ldots)$.

5. Which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$?

- A. $\{ (x, y, z) \mid x < y < z \}$
- B. $\{ (x, x - 7, x + 2) \mid x \text{ arbitrary number} \}$
- C. $\{ (x, 0, 0) \mid x \text{ arbitrary number} \}$
- D. $\{ (x, y, z) \mid -4x + 8y + 3z = 5 \}$
- E. $\{ (-5x, 4x, -8x) \mid x \text{ arbitrary number} \}$
- F. $\{ (x, y, z) \mid x + y + z = 0 \}$

6. Which of the following subsets of $P_2$ are subspaces of $P_2$?

- A. $\{ p(t) \mid p(2) = 0 \}$
- B. $\{ p(t) \mid p'(t) \text{ is constant} \}$
- C. $\{ p(t) \mid \int_0^6 p(t) \, dt = 0 \}$
- D. $\{ p(t) \mid p(4) = 7 \}$
- E. $\{ p(t) \mid p(t) + 8p(t) + 9 = 0 \}$
- F. $\{ p(t) \mid p'(3) = p(5) \}$

7. Which of the following subsets of $\mathbb{R}^{3 \times 3}$ are subspaces of $\mathbb{R}^{3 \times 3}$?

- A. The $3 \times 3$ matrices in reduced row-echelon form
- B. The $3 \times 3$ matrices with all zeros in the second row
- C. The $3 \times 3$ matrices with trace 0 (the trace of a matrix is the sum of its diagonal entries)
- D. The diagonal $3 \times 3$ matrices
- E. The $3 \times 3$ matrices of rank 1
- F. The $3 \times 3$ matrices whose entries are all integers

8. Determine whether the given set $S$ is a subspace of the vector space $V$.

- A. $V = P_3$, and $S$ is the subset of $P_3$ consisting of those polynomials satisfying $p(0) = 0$.
- B. $V = C^1(I)$, and $S$ is the subset of $V$ consisting of those functions satisfying the differential equation $y'' + 4y = x^2$.
- C. $V = C^2(I)$, and $S$ is the subset of $V$ consisting of those functions satisfying the differential equation $y'' - 4y' + 3y = 0$.
- D. $V$ is the vector space of all real-valued functions defined on the interval $[a, b]$, and $S$ is the subset of $V$ consisting of those functions satisfying $\int_a^b f(t) \, dt = 0$.
- E. $V = \mathbb{R}^2$, and $S$ consists of all vectors $(x_1, x_2)$ satisfying $x_1^2 - x_2^2 = 0$.
- F. $V$ is the vector space of all real-valued functions defined on the interval $(-\infty, \infty)$, and $S$ is the subset of $V$ consisting of those functions satisfying $f(0) = 0$.
- G. $V = \mathbb{R}^n$, and $S$ is the set of solutions to the homogeneous linear system $Ax = 0$ where $A$ is a fixed $m \times n$ matrix.

9. (a) If $S$ is the subspace of $M_6(\mathbb{R})$ consisting of all upper triangular matrices, then dim $S =$ ___.
(b) If $S$ is the subspace of $M_7(\mathbb{R})$ consisting of all skew-symmetric matrices, then dim $S =$ ___.

10. Find the dimensions of the following linear spaces.

(a) The space of all lower triangular $2 \times 2$ matrices ___.
(b) $\mathbb{R}^{5 \times 3}$ ___.
(c) The space of all diagonal $4 \times 4$ matrices ___.
1. Determine whether or not the three vectors listed above are linearly independent or linearly dependent. Enter 0’s for the coefficients, since that relationship always holds.

\[ A+\quad B+\quad C = 0. \]

You can use this row reduction tool to help with the calculations.

2. Which of the following sets of vectors are linearly independent?

- A. \[
\begin{pmatrix} 0 \\ 0 \\ -6 \\ -7 \end{pmatrix} \]
- B. \[
\begin{pmatrix} 1 \\ -9 \\ -2 \\ 1 \end{pmatrix} \]
- C. \[
\begin{pmatrix} -8 \\ 4 \\ 8 \\ -4 \end{pmatrix} \]
- D. \[
\begin{pmatrix} -6 \\ 2 \\ 2 \\ -4 \end{pmatrix} \]
- E. \[
\begin{pmatrix} 9 \\ 6 \\ 7 \\ -8 \end{pmatrix} \]
- F. \[
\begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix} \]

3. Let \( A = \begin{pmatrix} 10 \\ -1 \\ -3 \end{pmatrix} \), \( B = \begin{pmatrix} -5 \\ 1 \\ 3 \end{pmatrix} \), and \( C = \begin{pmatrix} -35 \\ 2 \\ 6 \end{pmatrix} \).

4. Let \( A = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \), \( B = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} \), and \( C = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \).

5. Let \( A = \begin{pmatrix} -3 \\ -3 \\ 3 \end{pmatrix} \), \( B = \begin{pmatrix} -9 \\ -11 \\ 5 \end{pmatrix} \), and \( C = \begin{pmatrix} 12 \\ 14 \\ -8 \end{pmatrix} \), and \( D = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix} \).

6. The vectors \( v = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix} \), \( u = \begin{pmatrix} -2 \\ -2 \\ 20+k \end{pmatrix} \), and \( w = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \) are linearly independent if and only if \( k \neq \cdots \)

7. Let \( v_1 = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} \), \( v_2 = \begin{pmatrix} -3 \\ -14 \\ h \end{pmatrix} \), and \( y = \begin{pmatrix} 7 \\ 32 \end{pmatrix} \).

8. Find a linearly independent set of vectors that spans the same subspace of \( \mathbb{R}^3 \) as that spanned by the vectors \( \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \).

Linearly independent set: \( \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} \)
9. Find a linearly independent set of vectors that spans the same subspace of \( \mathbb{R}^4 \) as that spanned by the vectors
\[
\begin{pmatrix}
-4 \\
-7 \\
5 \\
-4
\end{pmatrix}, \quad \begin{pmatrix}
2 \\
3 \\
-2 \\
1
\end{pmatrix}, \quad \begin{pmatrix}
0 \\
-1 \\
1 \\
-2
\end{pmatrix}, \quad \begin{pmatrix}
2 \\
0 \\
1 \\
-5
\end{pmatrix}.
\]
Linearly independent set: \( \text{___, ___} \).

10. Find a linearly independent set of vectors that spans the same subspace of \( \mathbb{R}^4 \) as that spanned by the vectors
\[
\begin{pmatrix}
1 \\
1 \\
-3 \\
-2
\end{pmatrix}, \quad \begin{pmatrix}
0 \\
-2 \\
9 \\
4
\end{pmatrix}, \quad \begin{pmatrix}
-1 \\
0 \\
-1 \\
-2
\end{pmatrix}, \quad \begin{pmatrix}
-2 \\
-1 \\
2 \\
0
\end{pmatrix}.
\]
Linearly independent set: \( \text{___, ___} \).

11. Let \( f = \exp(t) \), \( g = t \), and \( h = 2 + 3 \cdot t \). Give the answer 1 if \( f \), \( g \), and \( h \) are linearly dependent and 0 if they are linearly independent.

12. Determine which of the following pairs of functions are linearly independent.

13. Determine whether the following pairs of functions are linearly independent or not.
Let $v_1 = \left( \begin{array}{c} 2 \\ -5 \\ 0 \end{array} \right)$, $v_2 = \left( \begin{array}{c} -5 \\ 2 \\ 7 \end{array} \right)$, and $v_3 = \left( \begin{array}{c} -16 \\ 2 \\ k \end{array} \right)$ form a basis for $\mathbb{R}^3$ if and only if $k \neq \text{____}$.  

Let $A = \left( \begin{array}{ccc} 4 & -4 & 2 \\ -2 & 2 & \text{____} \end{array} \right)$.  
Find bases of the kernel and image of $A$ (or the linear transformation $T(x) = Ax$).  
Kernel: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  
Image: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} -8 & -4 & -2 \\ -12 & -6 & -3 \end{array} \right)$.  
Find bases of the kernel and image of $A$ (or the linear transformation $T(x) = Ax$).  
Kernel: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  
Image: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} -2 & -8 & -6 \\ 8 & 6 & 11 \end{array} \right)$.  
Find bases of the kernel and image of $A$ (or the linear transformation $T(x) = Ax$).  
Kernel: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  
Image: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} 12 & -12 & -12 \\ -12 & 12 & -12 \end{array} \right)$.  
Find bases of the kernel and image of $A$ (or the linear transformation $T(x) = Ax$).  
Kernel: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  
Image: \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} -3 & -2 & -4 \\ -6 & -4 & -8 \end{array} \right)$.  
Find a basis of the kernel of $A$ (or the linear transformation $T(x) = Ax$).  
\left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} 3 & -1 & -4 \\ 12 & -4 & -16 \end{array} \right)$.  
Find a basis of nullspace(A).  
\left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} 6 & 8 & -1 \\ -2 & 6 & -4 \end{array} \right)$.  
Find a basis of the image of $A$ (or the linear transformation $T(x) = Ax$).  
\left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 2 & -1 \end{array} \right)$.  
Find a basis of the kernel of $A$ (or the linear transformation $T(x) = Ax$).  
\left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} 6 & 6 & -3 & -9 \\ 6 & 6 & -3 & -9 \end{array} \right)$.  
Find a basis of nullspace(A).  
\left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).  

Let $A = \left( \begin{array}{ccc} 3 & 4 & 6 \\ -6 & 6 & 2 \end{array} \right)$.  
Find a basis of nullspace(A).  
\left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \), \left( \begin{array}{c} \text{____} \\ \text{____} \\ \text{____} \end{array} \right) \).
Find bases of the orthogonal projection
onto the plane \(-2x + 5y + z = 0\) in \(\mathbb{R}^3\).

\[
\begin{pmatrix}
2 & -2 \\
3 & 4 \\
0 & 0
\end{pmatrix}
\]
Find a basis of the subspace of \( \mathbb{R}^4 \) spanned by the following vectors:
\[
\begin{pmatrix}
0 & 2 & -1 & 0 \\
1 & 1 & 0 & 0 \\
0 & -2 & 1 & 1 \\
2 & 5 & -1 & 0 \\
1 & 3 & -1 & 0 \\
\end{pmatrix}
\]

Find a basis of the subspace of \( \mathbb{R}^4 \) consisting of all vectors of the form
\[
\begin{pmatrix}
x_1 \\
-5x_1 + x_2 \\
9x_1 - 5x_2 \\
9x_1 - 5x_2 \\
\end{pmatrix}
\]

Find a basis of the subspace of \( \mathbb{R}^3 \) defined by the equation

\[6x_1 + 3x_2 + 6x_3 = 0.
\]

Find a basis of the subspace of \( \mathbb{R}^4 \) defined by the equation

\[4x_1 + 9x_2 - 4x_3 + 8x_4 = 0.
\]

Find a basis of the subspace of \( \mathbb{R}^4 \) that consists of all vectors perpendicular to both
\[
\begin{pmatrix}
1 \\
0 \\
-2 \\
-6 \\
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
0 \\
1 \\
9 \\
-1 \\
\end{pmatrix}
\]

Find a basis of the row space of the matrix
\[
A = \begin{pmatrix}
0 & -2 & -2 & -2 \\
3 & 0 & 1 & 0 \\
3 & -2 & -1 & -2 \\
\end{pmatrix}
\]

Find a basis of the row space of the matrix
\[
A = \begin{pmatrix}
0 & 4 & -1 & -4 & -2 \\
-4 & 6 & 1 & 8 & 3 \\
-4 & -2 & 1 & 4 & 1 \\
-4 & 2 & 1 & 0 & -1 \\
\end{pmatrix}
\]

Find a basis of the column space of the matrix
\[
A = \begin{pmatrix}
-3 & -1 & 1 & 0 \\
0 & -1 & 4 & -4 \\
3 & 2 & -5 & 4 \\
\end{pmatrix}
\]

Find a basis of the column space of the matrix
\[
A = \begin{pmatrix}
0 & -1 & 4 & 2 & 2 \\
1 & 0 & -2 & -3 & -1 \\
-2 & 2 & -4 & 2 & -1 \\
0 & 1 & -4 & -2 & -2 \\
\end{pmatrix}
\]

Consider the basis \( B \) of \( \mathbb{R}^2 \) consisting of vectors
\[
\begin{pmatrix}
-1 \\
1 \\
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
1 \\
2 \\
\end{pmatrix}
\]

Find \( x \) in \( \mathbb{R}^2 \) whose coordinate vector relative to the basis \( B \) is
\[
[x]_B = \begin{pmatrix}
2 \\
6 \\
\end{pmatrix}
\]

The set \( B = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \end{pmatrix} \right\} \) is a basis for \( \mathbb{R}^2 \).

Find the coordinates of the vector \( x = \begin{pmatrix} 5 \\ 13 \end{pmatrix} \) relative to the basis \( B \):
\[
[x]_B = \begin{pmatrix}
\end{pmatrix}
\]
33. Find the coordinate vector of \( x = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \) with respect to the basis \( B = \{ \begin{pmatrix} 1 \\ 8 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -4 \\ 0 \\ 0 \\ 1 \end{pmatrix} \} \) or \( \mathbb{R}^3 \). 

\[ [x]_B = \begin{pmatrix} \_ \\ \_ \\ \_ \end{pmatrix} \]

34. Let \( B \) be the basis of \( \mathbb{R}^2 \) consisting of the vectors \( \begin{pmatrix} 5 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} -1 \\ 5 \end{pmatrix} \), and let \( R \) be the basis consisting of \( \begin{pmatrix} 2 \\ 3 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 2 \end{pmatrix} \). Find a matrix \( P \) such that \([x]_R = P[x]_B\) for all \( x \) in \( \mathbb{R}^2 \). 

\[ P = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \]

35. The set \( B = \{ 1 + 4x^2, 5 + 4x + 20x^2, -12 - 12x - 52x^2 \} \) is a basis for \( P_2 \). Find the coordinates of \( p(x) = -23 - 24x - 100x^2 \) relative to this basis: 

\[ [p(x)]_B = \begin{pmatrix} \_ \\ \_ \end{pmatrix} \]

36. The set \( B = \{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \} \) is called the standard basis of the space of \( 2 \times 2 \) matrices. Find the coordinates of \( M = \begin{pmatrix} -5 & 8 \\ -8 & 1 \end{pmatrix} \) with respect to this basis. 

\[ [M]_B = \begin{pmatrix} \_ \\ \_ \end{pmatrix} \]

37. The set \( B = \{ \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} \} \) is a basis of the space of upper-triangular \( 2 \times 2 \) matrices. Find the coordinates of \( M = \begin{pmatrix} 8 & -1 \\ 0 & -4 \end{pmatrix} \) with respect to this basis. 

\[ [M]_B = \begin{pmatrix} \_ \\ \_ \end{pmatrix} \]

38. Find a basis for the space of \( 2 \times 2 \) diagonal matrices: 

\[ \{ \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}, \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \} \]

39. Find a basis for the space of \( 2 \times 2 \) lower triangular matrices: 

\[ \{ \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}, \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \} \]
Find the characteristic polynomial of the matrix
\[ A = \begin{pmatrix} -2 & 9 \\ 6 & -2 \end{pmatrix} \]
\[ p(x) = \] 

Find the characteristic polynomial of the matrix
\[ A = \begin{pmatrix} 3 & 1 \\ 0 & 5 \end{pmatrix} \]
\[ p(x) = \] 

Find the eigenvalues of the matrix \[ A = \begin{pmatrix} 86 & 270 \\ -27 & -85 \end{pmatrix} \].
The smaller eigenvalue is \( \lambda_1 = \) 
The bigger eigenvalue is \( \lambda_2 = \) 

The matrix \( B = \begin{pmatrix} -7 & -5 & 3 \\ 0 & 5 & -9 \\ 0 & 0 & 6 \end{pmatrix} \) has three distinct eigenvalues, \( \lambda_1 < \lambda_2 < \lambda_3 \), where \( \lambda_1 = \), \( \lambda_2 = \), and \( \lambda_3 = \) 

The matrix \( C = \begin{pmatrix} 43 & 20 & -96 \\ 6 & 0 & -12 \\ 20 & 10 & -45 \end{pmatrix} \) has three distinct eigenvalues, \( \lambda_1 < \lambda_2 < \lambda_3 \), where \( \lambda_1 = \), \( \lambda_2 = \), and \( \lambda_3 = \) 

The matrix \( C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{pmatrix} \) has two distinct eigenvalues, \( \lambda_1 < \lambda_2 ; \lambda_1 = \) has multiplicity \( \), and \( \lambda_2 = \) has multiplicity \( \) 

Find the eigenvalues of the matrix
\[ A = \begin{pmatrix} -7 & -6 & 10 & -7 \\ 8 & 7 & -10 & 2 \\ -3 & -3 & 4 & -5 \\ 0 & 0 & 0 & -4 \end{pmatrix} \].
Place all your answers in the following blank, separated by commas.

The matrix \( A = \begin{pmatrix} 1 & k \\ 9 & -7 \end{pmatrix} \) has two distinct real eigenvalues if and only if \( k > \) 

The matrix \( A = \begin{pmatrix} -2 & 1 & 0 \\ -12 & 6 & 1 \\ k & 0 & 0 \end{pmatrix} \) has three distinct real eigenvalues if and only if \( k < \) 

Suppose that the trace of a \( 2 \times 2 \) matrix \( A \) is \( \text{tr}(A) = 1 \), and the determinant is \( \text{det}(A) = -6 \). Find the eigenvalues of \( A \): smaller eigenvalue = \( \), larger eigenvalue = \( \) 

Suppose a \( 3 \times 3 \) matrix \( A \) has only two distinct eigenvalues. Suppose that \( \text{tr}(A) = 0 \) and \( \text{det}(A) = 54 \). Find the eigenvalues of \( A \) with their algebraic multiplicities: smaller eigenvalue = \( \) has multiplicity \( \) and larger eigenvalue = \( \) has multiplicity \( \) 

For which value of \( k \) does the matrix
\[ A = \begin{pmatrix} -1 & k \\ -4 & -8 \end{pmatrix} \] have one real eigenvalue of multiplicity 2? 
\[ k = \] 

Given that \( v_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \) and \( v_2 = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \) are eigenvectors of the matrix \( A = \begin{pmatrix} -42 & -60 \\ 30 & 43 \end{pmatrix} \), determine the corresponding eigenvalues. 
\( \lambda_1 = \) 
\( \lambda_2 = \) 

If \( v_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) and \( v_2 = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \) are eigenvectors of a matrix \( A \) corresponding to the eigenvalues \( \lambda_1 = 2 \) and \( \lambda_2 = -4 \), respectively, then \( A(v_1 + v_2) = \) and \( A(3v_1) = \) 

Let \( v_1 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \), \( v_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \), and \( v_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \) be eigenvectors of the matrix \( A \) which correspond to the eigenvalues \( \lambda_1 = -3 \), \( \lambda_2 = 0 \), and \( \lambda_3 = 2 \), respectively, and let \( \lambda = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \). Express \( \lambda \) as a linear combination of \( v_1, v_2, \) and \( v_3 \), and find \( Av \). 
\( v = \) \( \lambda \).
16. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_6.png
Find the eigenvalues of the matrix
\[
A = \begin{pmatrix}
3 & 2 \\
-12 & -7
\end{pmatrix}
\]
smaller eigenvalue =
associated eigenvector =
larger eigenvalue =
associated eigenvector =

17. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_8.png
Find the eigenvalues of the following matrix.
[Note— you may want to use a graphing calculator to estimate the roots of the polynomial which defines the eigenvalues. You can use the web version at xFunctions. Also, You can use this row reduction tool to help with the calculations.]
\[
A = \begin{pmatrix}
5 & 0 & 0 \\
10 & -5 & 0 \\
15 & -5 & 0
\end{pmatrix}
\]
The eigenvalues are \(\lambda_1 < \lambda_2 < \lambda_3\), where \(\lambda_1 =
\lambda_2 =
\lambda_3 =
associated eigenvector =

18. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_9.png
Suppose \(A\) is an invertible \(n \times n\) matrix and \(v\) is an eigenvector of \(A\) with associated eigenvalue 8. Convince yourself that \(v\) is an eigenvalue of the following matrices, and find the associated eigenvalues:
1. \(A^5\)
\(\lambda_1 = \) eigenvalue =

2. \(A^{-1}\)
\(\lambda_1 = \) eigenvalue =

3. \(A + 9I_n\)
\(\lambda_1 = \) eigenvalue =

4. \(2A\)
\(\lambda_1 = \) eigenvalue =

19. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_17.png
The matrix \(A = \begin{pmatrix} 11 & -1 \\ 4 & 7 \end{pmatrix}\) has one eigenvalue of multiplicity 2. Find this eigenvalue and the dimension of the eigenspace.
eigenvalue =
dimension of the eigenspace =

20. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_17a.png
The matrix \(A = \begin{pmatrix} -3 & 2 \\ -8 & -11 \end{pmatrix}\) has one eigenvalue of multiplicity 2. Find this eigenvalue and the dimension of the eigenspace.
eigenvalue =
dimension of the eigenspace =

21. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_20.png
The matrix \(A = \begin{pmatrix} 12 & 4 & 20 \\ 6 & 6 & 14 \\ -4 & -2 & -6 \end{pmatrix}\) has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.
eigenvalue =
multiplicity =
dimension of the eigenspace =

22. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_20a.png
The matrix \(A = \begin{pmatrix} -7 & -2 & 10 \\ -3 & -4 & 7 \\ -2 & -1 & 2 \end{pmatrix}\) has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.
eigenvalue =
multiplicity =
dimension of the eigenspace =

23. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_7.png
The matrix \(C = \begin{pmatrix} 12 & 2 & 24 \\ 1 & 2 & 3 \\ -4 & -1 & -8 \end{pmatrix}\) has two distinct eigenvalues, \(\lambda_1 < \lambda_2\):
\(\lambda_1 = \) has multiplicity 
\(\lambda_2 = \) has multiplicity 
Is the matrix \(C\) diagonalizable? (enter YES or NO)

24. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_22.png
The matrix \(A = \begin{pmatrix} -6 & 4 & -8 & -4 \\ -4 & 2 & -6 & -2 \\ 3 & -2 & 4 & 2 \\ -3 & 2 & -4 & -2 \end{pmatrix}\) has two real eigenvalues \(\lambda_1 < \lambda_2\). Find these eigenvalues, their multiplicities, and dimensions of the corresponding eigenspaces.
\(\lambda_1 = \) has multiplicity 
\(\lambda_2 = \) has multiplicity 
Is the matrix \(A\) defective? (Type ”yes” or ”no”)

25. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_22a.png
The matrix \(A = \begin{pmatrix} 1 & 2 & -5 & 0 \\ 0 & 2 & -4 & 1 \\ 1 & 2 & -5 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\) has two real eigenvalues \(\lambda_1 < \lambda_2\). Find these eigenvalues, their multiplicities, and dimensions of the corresponding eigenspaces.
\[ \lambda_1 = \_ \text{ has multiplicity } \_ \text{. The dimension of the corresponding eigenspace is } \_. \]
\[ \lambda_2 = \_ \text{ has multiplicity } \_ \text{. The dimension of the corresponding eigenspace is } \_. \]
Is the matrix \( A \) defective? (Type "yes" or "no") 

\[ \lambda_1 = \_ \]
\[ \lambda_2 = \_ \]

26. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_18.pg

The matrix \( A = \begin{pmatrix} 1 & 0 & -1 \\ -2 & 2 & -2 \\ 1 & 0 & 3 \end{pmatrix} \) has one real eigenvalue. Find this eigenvalue and a basis of the eigenspace.

Eigenvector = 
Basis: ( ), ( ).

27. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_19.pg

The matrix \( A = \begin{pmatrix} 4 & 4 & -4 \\ -4 & -4 & 4 \\ 4 & 4 & -4 \end{pmatrix} \) has two real eigenvalues, one of multiplicity 1 and one of multiplicity 2. Find the eigenvalues and a basis of each eigenspace.

\( \lambda_1 = \_ \) has multiplicity 1,

Basis: ( ) ,
\[ \lambda_2 = \_ \] has multiplicity 2,

Basis: ( ) , ( ) .

28. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_23.pg

Find a basis of the eigenspace associated with the eigenvalue 2 of the matrix \( A = \begin{pmatrix} -2 & 0 & 8 & -2 \\ -4 & 2 & 8 & 4 \\ -2 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \).

Basis: ( ), ( ), ( ), ( ) .

29. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_24.pg

The matrix \( A = \begin{pmatrix} -2 & -2 & 0 & 4 \\ 2 & 2 & 0 & -3 \\ -3 & -2 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \) has two distinct eigenvalues \( \lambda_1 < \lambda_2 \). Find the eigenvalues and a basis of each eigenspace.

\( \lambda_1 = \_ \)
Basis: ( ) ,
\[ \lambda_2 = \_ \]
Basis: ( ) , ( ) .

30. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_11.pg

Find a \( 2 \times 2 \) matrix \( A \) such that 
\[ \begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} -2 \\ -2 \end{pmatrix} \]
are eigenvectors of \( A \), with eigenvalues 8 and \(-2\) respectively.

\( A = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \).

31. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_10.pg

Find a \( 2 \times 2 \) matrix \( A \) for which 
\[ E_4 = \text{span} \left( \begin{pmatrix} -1 \\ -5 \end{pmatrix} \right) \text{ and } E_{-4} = \text{span} \left( \begin{pmatrix} -1 \\ -6 \end{pmatrix} \right) \]
where \( E_\lambda \) is the eigenspace associated with the eigenvalue \( \lambda \).

\[ A = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \).

32. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_29.pg

The matrix \( A = \begin{pmatrix} 5 & 0 & -1 \\ 1 & 4 & -1 \\ 0 & 0 & 4 \end{pmatrix} \) has two real eigenvalues, \( \lambda_1 = 4 \) of multiplicity 2, and \( \lambda_2 = 5 \) of multiplicity 1. Find an orthonormal basis for the eigenspace \( E_1 \).

\[ \begin{pmatrix} \_ \\ \_ \end{pmatrix} , \begin{pmatrix} \_ \\ \_ \end{pmatrix} . \)

33. (1 pt) setLinearAlgebra11Eigenvalues/ur_la_11_30.pg

Give an example of a 2x2 matrix without any real eigenvalues:

\[ \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \).

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
1. (1 pt) setLinearAlgebra12Diagonalization/ur_la_12_1.pg
Let $M = \begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$.
Find formulas for the entries of $M^n$, where $n$ is a positive integer.
$M^n = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$.

2. (1 pt) setLinearAlgebra12Diagonalization/ur_la_12_2.pg
Let $M = \begin{pmatrix} 8 & 4 \\ -8 & -4 \end{pmatrix}$.
Find formulas for the entries of $M^n$, where $n$ is a positive integer.
$M^n = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$.

3. (1 pt) setLinearAlgebra12Diagonalization/ur_la_12_3.pg
Let $M = \begin{pmatrix} 11 & 1 \\ -49 & -3 \end{pmatrix}$.
Find formulas for the entries of $M^n$, where $n$ is a positive integer.
$M^n = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$.

4. (1 pt) setLinearAlgebra12Diagonalization/ur_la_12_4.pg
Let $A = \begin{pmatrix} 3 & -0.25 \\ 63 & -5 \end{pmatrix}$.
Find an invertible $S$ and a diagonal $D$ such that $S^{-1}AS = D$.
$S = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$ $D = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$.
1. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_1.pg
The matrix
\[ A = \begin{pmatrix} -7 & 3 \\ -5 & -7 \end{pmatrix} \]
has complex eigenvalues, \( \lambda_{1,2} = a \pm bi \), where \( a = \) _____ and \( b = \) _____.

2. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_2.pg
Find all the eigenvalues (real and complex) of the matrix
\[ M = \begin{pmatrix} -6 & -10 & -16 \\ 5 & 6 & 11 \\ 0 & 0 & -2 \end{pmatrix} \]
Enter your answers in the following blank, separated by commas:

3. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_3.pg
Find all the eigenvalues (real and complex) of the matrix
\[ M = \begin{pmatrix} 11 & -2 & 6 & -6 \\ -22 & 6 & -11 & 11 \\ -10 & 1 & -2 & 3 \\ 24 & -8 & 16 & -15 \end{pmatrix} \]
Enter your answers in the following blank, separated by commas:

4. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_4.pg
Find all the eigenvalues (real and complex) of the matrix
\[ M = \begin{pmatrix} 2 & 5 & -13 & -8.88178419700125e-16 \\ -9 & -8 & 7 & 6 \\ -2 & -2 & 1 & 2 \\ -6 & -2 & -10 & 5 \end{pmatrix} \]
Enter your answers in the following blank, separated by commas:

5. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_5.pg
Find all the values of \( k \) for which the matrix
\[ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & k-6 & -k+7 \end{pmatrix} \]
is not diagonalizable over \( \mathbb{C} \).
Enter your answers in the following blank, separated by commas:

6. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_6.pg
Let \( M = \begin{pmatrix} -5 & 4 \\ -4 & -5 \end{pmatrix} \).
Find formulas for the entries of \( M^n \) where \( n \) is a positive integer.
(Your formulas should not contain complex numbers.)
\[ M^n = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \]

7. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_7.pg
Let \( M = \begin{pmatrix} 1 & -9 & -8 \\ 9 & 1 & -10 \\ 0 & 0 & 2 \end{pmatrix} \).
Find formulas for the entries of \( M^n \) where \( n \) is a positive integer.
(Your formulas should not contain complex numbers.)
\[ M^n = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \]

8. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_12.pg
Determine for which of the following matrices \( M \) the zero state is a stable equilibrium of the dynamical system \( x(t+1) = Mx(t) \).

- A. \( M = \begin{pmatrix} 9 & 0 \\ 0 & 0.2 \end{pmatrix} \)
- B. \( M = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.7 \end{pmatrix} \)
- C. \( M = \begin{pmatrix} 0 & -3 \\ 2 & 0 \end{pmatrix} \)
- D. \( M = \begin{pmatrix} -0.4 & 0.4 \\ -0.4 & -0.4 \end{pmatrix} \)
- E. \( M = \begin{pmatrix} -0.2 & -0.2 & -0.2 \\ -0.2 & -0.2 & -0.2 \end{pmatrix} \)
- F. \( M = \begin{pmatrix} 6 & 0 & 5 \\ 0 & 1 & 0 \\ -2 & 0 & -4 \end{pmatrix} \)

9. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_8.pg
Find real closed formulas for the trajectory \( x(t+1) = Ax(t) \), where
\[ A = \begin{pmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{pmatrix} \] and \( x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \).
\[ x(t) = \begin{pmatrix} \_ \\ \_ \end{pmatrix} \]

10. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_9.pg
Find real closed formulas for the trajectory \( x(t+1) = Ax(t) \), where
\[ A = \begin{pmatrix} 12 & -9 \\ 9 & 12 \end{pmatrix} \] and \( x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).
\[ x(t) = \begin{pmatrix} \_ \\ \_ \end{pmatrix} \]

11. (1 pt) setLinearAlgebra13ComplexEigenvalues/ur_la_13_10.pg
Find real closed formulas for the trajectory \( x(t+1) = Ax(t) \), where
\[ A = \begin{pmatrix} 39 & -18 \\ 45 & -15 \end{pmatrix} \] and \( x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \).
\[ x(t) = \begin{pmatrix} \_ \\ \_ \end{pmatrix} \]
Find the equilibrium state of the dynamical system
\[ x_1(t + 1) = -0.5x_1(t) - 0.1x_2(t) - 2, \]
\[ x_1(t + 1) = 1.7x_1(t) - 0.7x_2(t) + 1. \]
Consider a linear transformation $T$ from $\mathbb{R}^3$ to $\mathbb{R}^2$ for which

$$
\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
= \begin{pmatrix}
7 \\
5 \\
0
\end{pmatrix},
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
= \begin{pmatrix}
9 \\
8 \\
0
\end{pmatrix},
$$

and

$$
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
= \begin{pmatrix}
4 \\
3 \\
0
\end{pmatrix}.
$$

Find the matrix $A$ of $T$.

$A = \begin{pmatrix} \_ & \_ & \_ \end{pmatrix}$.
The set $B = \{b_1, b_2\}$ is a basis for $\mathbb{R}^2$.
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that
$T(b_1) = 6b_1 + 5b_2$ and $T(b_2) = 8b_1 + 8b_2$.
Then the matrix of $T$ relative to the basis $B$ is
$$[T]_B = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix},$$
and the matrix of $T$ relative to the standard basis $E$ for $\mathbb{R}^2$ is
$$[T]_E = \begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}.$$

12. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_6.png
A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ whose matrix is
$$\begin{pmatrix} 1 & 3 & -2 \\ 3 & 9 & -15.5 + k \end{pmatrix}$$
is onto if and only if $k \neq ___$.

13. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_7.png
The matrix
$$A = \begin{pmatrix} 1 & 4 & 2 \\ 1.5 & 6 & 3 \\ 3.5 & 14 & 7 \end{pmatrix}$$
is a matrix of a linear transformation $T : \mathbb{R}^k \to \mathbb{R}^n$ where
$k = \text{____}$, $n = \text{____}$.
$\dim(Ker(T)) = \text{____}$, $\dim(Range(T)) = \text{____}$.
Is $T$ onto? (enter YES or NO) ____
Is $T$ one-to-one? (enter YES or NO) ____

14. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_8.png
Let $A = \begin{pmatrix} 6 & 12 \\ 6 & 14 \\ 7 & 12 \end{pmatrix}$ and $b = \begin{pmatrix} -78 \\ -88 \\ -81 \end{pmatrix}$.
A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(x) = Ax$.
Find an $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ in $\mathbb{R}^2$ whose image under $T$ is $b$.
$x_1 = \text{____}$, $x_2 = \text{____}$.

15. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_13.png
Match each linear transformation with its matrix.

1. \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}
2. \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}
3. \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}
4. \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}
5. \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
6. \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}

A. Dilation by a factor of 2
B. Rotation through an angle of 90° in the counterclockwise direction
C. Projection onto the $y$-axis
D. Projection onto the $x$-axis
E. Reflection in the line $y = x$
F. Reflection in the $y$-axis

16. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_14.png
Find a $3 \times 3$ matrix $A$ such that $Ax = 6x$ for all $x$ in $\mathbb{R}^3$.
$A = \begin{pmatrix} ____ & ____ & ____ \\ ____ & ____ & ____ \\ ____ & ____ & ____ \end{pmatrix}$.

17. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_15.png
Find the matrix $A$ of the linear transformation $T$ from $\mathbb{R}^2$ to $\mathbb{R}^2$ that rotates any vector through an angle of 135° in the clockwise direction.
$A = \begin{pmatrix} ____ & ____ \\ ____ & ____ \end{pmatrix}$.

18. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_16.png
The dot product of two vectors in $\mathbb{R}^3$ is defined by
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3.$$ Let $v = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$. Find the matrix $A$ of the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}$ given by $T(x) = v \cdot x$.
$A = \begin{pmatrix} ____ & ____ & ____ \\ ____ & ____ & ____ \end{pmatrix}$.

19. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_17.png
The cross product of two vectors in $\mathbb{R}^3$ is defined by
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}.$$ Let $v = \begin{pmatrix} -6 \\ 4 \\ -4 \end{pmatrix}$. Find the matrix $A$ of the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ given by $T(x) = v \times x$.
$A = \begin{pmatrix} ____ & ____ & ____ \\ ____ & ____ & ____ \\ ____ & ____ & ____ \end{pmatrix}$.

20. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_18.png
Let $L$ be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$. Find the orthogonal projection of the vector $v = \begin{pmatrix} 3 \\ 5 \\ 3 \end{pmatrix}$ onto $L$.
$\text{proj}_Lv = \begin{pmatrix} ____ \\ ____ \\ ____ \end{pmatrix}$.

21. (1 pt) setLinearAlgebra14TransfOfRn/ur Ja Ja_14_19.png
Let $L$ be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the
vector \( \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} \). Find the reflection of the vector \( v = \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix} \) in the line \( L \).

\[ \text{proj}_L v = \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \]

22. (1 pt) Find the matrix \( A \) of the orthogonal projection onto the line \( L \) in \( \mathbb{R}^2 \) that consists of all scalar multiples of the vector \( \begin{pmatrix} 1 \\ 5 \end{pmatrix} \).

\( A = \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \).

23. (1 pt) Find the matrix \( A \) of the reflection in the line \( L \) in \( \mathbb{R}^2 \) that consists of all scalar multiples of the vector \( \begin{pmatrix} 6 \\ 5 \end{pmatrix} \).

\( A = \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \).

24. (1 pt) Find the matrices of the following linear transformations from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \).

The orthogonal projection onto the \( y \)-axis:

\( \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \).

The reflection in the \( xy \)-plane:

\( \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \).

25. (1 pt) Find the matrix \( A \) of the rotation about the \( x \)-axis through an angle of \( \frac{\pi}{2} \), counterclockwise as viewed from the positive \( x \)-axis.

\( A = \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \).

26. (1 pt) Let \( A = \begin{pmatrix} 0 & 4 \\ 1 & 6 \end{pmatrix} \) and \( B = \begin{pmatrix} 9 & 8 \\ 5 & 3 \end{pmatrix} \). Find the matrix \( C \) of the linear transformation \( T(x) = B(A(x)) \).

\( C = \begin{pmatrix} \_ \_ \\ \_ \_ \end{pmatrix} \).

27. (1 pt) Which of the following linear transformations from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \) are invertible?

- A. Dilation by a factor of 4
- B. Reflection in the \( xy \)-plane
- C. Identity transformation (i.e. \( T(v) = v \) for all \( v \))
- D. Projection onto the \( xz \)-plane
- E. Rotation about the \( z \)-axis
- F. Trivial transformation (i.e. \( T(v) = 0 \) for all \( v \))

28. (1 pt) Let \( T: \mathbb{R}^2 \to \mathbb{R}^2 \) be given by \( T(x) = \begin{pmatrix} 5 & -4 \\ 4 & -5 \end{pmatrix} x \). Find the matrix \( M \) of the inverse linear transformation, \( T^{-1} \).

\( M = \begin{pmatrix} \_ \_ \\ \_ \_ \\ \_ \_ \end{pmatrix} \).

29. (1 pt) Find the inverse of the linear transformation

\[ \begin{align*}
y_1 &= 5x_1 - 1x_2 \\
y_2 &= -29x_1 + 6x_2 \\
x_1 &= \_ \_ y_1 + \_ \_ y_2, \\
x_2 &= \_ \_ y_1 + \_ \_ y_2. \\
\end{align*} \]

30. (1 pt) Find the inverse of the linear transformation

\[ \begin{align*}
y_1 &= 5x_1 + 10x_2 - 16x_3 \\
y_2 &= 2x_1 + 5x_2 - 7x_3 \\
y_3 &= x_1 + 2x_2 - 3x_3 \\
x_1 &= \_ \_ y_1 + \_ \_ y_2 + \_ \_ y_3, \\
x_2 &= \_ \_ y_1 + \_ \_ y_2 + \_ \_ y_3, \\
x_3 &= \_ \_ y_1 + \_ \_ y_2 + \_ \_ y_3. \\
\end{align*} \]

31. (1 pt) Find the inverse of the (nonlinear) transformation from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \) given by

\[ \begin{align*}
u &= 4y \\
v &= 4x^3 - 8y \\
x &= \_ \_ \_ \\
y &= \_ \_ \_ \\
\end{align*} \]
1. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_1.pg
Which of the following transformations are linear?

- A. \( T(A) = ASA^{-1} \)
  from \( \mathbb{R}^{2	imes 2} \)
to \( \mathbb{R}^{2	imes 2} \), where \( S = \begin{pmatrix} -7 & 2 \\ 9 & -5 \end{pmatrix} \)
- B. \( T(A) = \det(A) \)
  from \( \mathbb{R}^{4	imes 4} \)
to \( \mathbb{R} \)
- C. \( T(A) = \begin{pmatrix} 7 & 8 \\ -3 & 5 \end{pmatrix} A \)
  from \( \mathbb{R}^{2	imes 3} \)
to \( \mathbb{R}^{2	imes 3} \)
- D. \( T(A) = 2A \)
  from \( \mathbb{R}^{6	imes 4} \)
to \( \mathbb{R}^{6	imes 4} \)
- E. \( T(A) = A \begin{pmatrix} 3 & -9 \\ 5 & 7 \end{pmatrix} - \begin{pmatrix} 3 & 9 \\ 5 & 1 \end{pmatrix} A \)
  from \( \mathbb{R}^{2	imes 2} \)
to \( \mathbb{R}^{2	imes 2} \)
- F. \( T(A) = \text{SAS}^{-1} \)
  from \( \mathbb{R}^{2	imes 2} \)
to \( \mathbb{R}^{2	imes 2} \), where \( S = \begin{pmatrix} 6 & 7 \\ -8 & 0 \end{pmatrix} \)

2. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_2.pg
Which of the following transformations are linear?

- A. \( T(f(t)) = f'(t) + 4f(t) \)
  from \( C^\infty \)
to \( C^\infty \)
- B. \( T(f(t)) = f^6f'(t) \)
  from \( P_2 \)
to \( P_7 \)
- C. \( T(f(t)) = f(-t) \)
  from \( P_3 \)
to \( P_3 \)
- D. \( T(f(t)) = f'(t) + 9f(t) + 7 \)
  from \( C^\infty \)
to \( C^\infty \)
- E. \( T(x + iy) = 6x - iy \)
  from \( C \)
to \( C \)
- F. \( T(x_0, x_1, x_2, \ldots) = (1, x_0, x_1, x_2, \ldots) \)
  from the space of finite sequences into itself

3. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_16.pg
Let \( T : P_3 \rightarrow P_3 \) be the linear transformation satisfying
\( T(1) = 3x^2 + 5, T(x) = 3x + 8, T(x^2) = 2x^2 + x - 6. \)
Find the image of an arbitrary cubic polynomial \( ax^2 + bx + c. \)
\( T(ax^2 + bx + c) = \) 

5. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_17.pg
Let \( T : P_3 \rightarrow P_3 \) be the linear transformation such that \( T(2x^2) = -2x^2 - 2x, T(-0.5x - 4) = -2x^2 + 3x - 2, \) and \( T(5x^2 + 1) = 4x + 3. \)
Find \( T(1), T(x), T(x^2), \) and \( T(ax^2 + bx + c), \) where \( a, b, \) and \( c \)
are arbitrary real numbers.
\( T(1) = \) 
\( T(x) = \) 
\( T(x^2) = \) 
\( T(ax^2 + bx + c) = \)

6. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_18.pg
Let \( V \) be a vector space, and \( T : V \rightarrow V \) a linear transformation such that \( T(5v_1 - 3v_2) = -4v_1 + 5v_2 \) and \( T(-3v_1 + 2v_2) = -2v_1 - 2v_2. \)
Then \( T(v_1) = v_1 + \) 
\( T(v_2) = v_1 + \) 
and \( T(-4v_1 - 2v_2) = v_1 + \) 

8. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_5.pg
Find the matrix \( A \) of the linear transformation
\( T(M) = \begin{pmatrix} 6 & 4 \\ 0 & 3 \end{pmatrix} M \)
from \( U^{2\times 2} \) to \( U^{2\times 2} \), with respect to the basis 
\[ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \]
\( A = \begin{pmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{pmatrix} \)

9. (1 pt) setLinearAlgebra15TransfOfLinSpaces/ur_Ja_15_6.pg
Find the matrix \( A \) of the linear transformation
\( T(M) = \begin{pmatrix} 2 & 9 \\ 0 & 3 \end{pmatrix} M \begin{pmatrix} 2 & 9 \\ 0 & 3 \end{pmatrix}^{-1} \)
Find the matrix $A$ of the linear transformation $T(f(t)) = 6f''(t) + 7f(t)$ from $P_2$ to $P_2$ with respect to the standard basis for $P_2$, $\{1, t, t^2\}$.

$A = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Find the matrix $A$ of the linear transformation $T(f(t)) = f(-1)$ from $P_2$ to $P_2$ with respect to the standard basis for $P_2$, $\{1, t, t^2\}$.

$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Find the matrix $A$ of the linear transformation $T(f(t)) = f(8t + 7)$ from $P_2$ to $P_2$ with respect to the standard basis for $P_2$, $\{1, t, t^2\}$.

$A = \begin{pmatrix} 0 & 8 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Find the matrix $A$ of the linear transformation $T(f(t)) = \int_{-5}^{3} f(t) dt$ from $P_3$ to $\mathbb{R}$ with respect to the standard bases for $P_3$ and $\mathbb{R}$.

$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Find the matrix $A$ of the linear transformation $T(z) = (6 + 3i)z$ from $\mathbb{C}$ to $\mathbb{C}$ with respect to the standard basis $\mathbb{C}$, $\{1, i\}$.

$A = \begin{pmatrix} 6 & 0 \\ 3 & 0 \end{pmatrix}$

Find the matrix $A$ of the linear transformation $T(z) = (7 + 9i)z$ from $\mathbb{C}$ to $\mathbb{C}$ with respect to the basis $\{4 + 4i, 3 + 5i\}$.

$A = \begin{pmatrix} 7 & 0 \\ 9 & 0 \end{pmatrix}$

Let $V$ be the space spanned by the two functions $\cos(t)$ and $\sin(t)$. Find the matrix $A$ of the linear transformation $T(f(t)) = f''(t) + 6f'(t) + 5f(t)$ from $V$ into itself with respect to the basis $\{\cos(t), \sin(t)\}$.

$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Let $V$ be the plane with equation $x_1 - 3x_2 + 5x_3 = 0$ in $\mathbb{R}^3$. Find the matrix $A$ of the orthogonal projection onto the line spanned by the vector $v = \begin{pmatrix} 16 \\ 2 \\ -2 \end{pmatrix}$ with respect to the basis $\{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}\}$.

$A = \begin{pmatrix} 9/16 & 0 & 0 \\ 0 & 2/9 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$

Let $V$ be the plane with equation $x_1 - 4x_2 - 2x_3 = 0$ in $\mathbb{R}^3$. Find the matrix $A$ of the linear transformation $T(x) = \begin{pmatrix} 0 & -4 & 6 \\ 1 & -2 & 1 \\ -2 & 2 & 1 \end{pmatrix}x$ with respect to the basis $\{\begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}\}$.

$A = \begin{pmatrix} 0 & 1 & 0 \\ -4 & 0 & 2 \\ 6 & 1 & 1 \end{pmatrix}$

Let $T : P_3 \rightarrow P_3$ be defined by $T(ax^2 + bx + c) = (4a + b)x^2 + (-3a - 3b + c)x - a$. Find the inverse of $T$.

$T^{-1}(ax^2 + bx + c) = \begin{pmatrix} 4a + b \\ -3a - 3b + c \\ x - a \end{pmatrix}$
1. Consider a linear transformation $T(x) = Ax$ from $\mathbb{R}^2$ to $\mathbb{R}^2$. Suppose for two vectors $v_1$ and $v_2$ in $\mathbb{R}^2$ we have $T(v_1) = 3v_2$ and $T(v_2) = -7v_1$. Find the determinant of the matrix $A$. $\det(A) = \underline{\quad}$

2. Find the determinant of the linear transformation $T(f) = -2f - 6f''$ from $P_2$ to $P_2$. $\det = \underline{\quad}$

3. Find the determinant of the linear transformation $T(f(t)) = f(4t) - 6f(t)$ from $P_2$ to $P_2$. $\det = \underline{\quad}$

4. Find the determinant of the linear transformation $T(z) = (-3 - 5i)z$ from $\mathbb{C}$ to $\mathbb{C}$. $\det = \underline{\quad}$

5. Find the determinant of the linear transformation $T(v) = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} \times v$ from the plane $E$ given by $x - 3y + 5z = 0$ to $E$. $\det = \underline{\quad}$

6. Find the determinant of the linear transformation $T(M) = \begin{pmatrix} 3 & -2 \\ 0 & 9 \end{pmatrix} M$ from the space $V$ of upper triangular $2 \times 2$ matrices to $V$. $\det = \underline{\quad}$

7. Find the determinant of the linear transformation $T(M) = \begin{pmatrix} 4 & 2 \\ 2 & -4 \end{pmatrix} M + M \begin{pmatrix} 4 & 2 \\ 2 & -4 \end{pmatrix}$ from the space $V$ of symmetric $2 \times 2$ matrices to $V$. $\det = \underline{\quad}$

8. Find the determinant of the linear transformation $T(f) = -2f + 9f' - 3f''$ from the space $V$ spanned by $\cos(x)$ and $\sin(x)$ to $V$. $\det = \underline{\quad}$
1. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_8.pg
Let \( x = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \) and \( y = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \). Find the dot product of \( x \) and \( y \).
\( x \cdot y = \)

2. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_9.pg
Let \( x = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \) and \( y = \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix} \). Find the dot product of \( x \) and \( y \).
\( x \cdot y = \)

3. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_10.pg
Find the length of the vector \( x = \begin{pmatrix} -6 \\ -3 \end{pmatrix} \).
\( ||x|| = \)

4. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_11.pg
Find the length of the vector \( x = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \).
\( ||x|| = \)

5. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_12.pg
Let \( x = \begin{pmatrix} -5 \\ -5 \\ -4 \\ -1 \end{pmatrix} \). Find the norm of \( x \) and the unit vector in the direction of \( x \).
\( ||x|| = \)
\( u = \)

6. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_13.pg
Let \( \{ e_1, e_2, e_3, e_4, e_5, e_6 \} \) be the standard basis in \( \mathbb{R}^6 \). Find the length of the vector \( x = 4e_1 - 4e_2 - 2e_3 - 5e_4 - 2e_5 + 5e_6 \).
\( ||x|| = \)

7. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_14.pg
Find the value of \( k \) for which the vectors
\( x = \begin{pmatrix} -3 \\ 2 \\ -3 \\ -5 \end{pmatrix} \) and \( y = \begin{pmatrix} 1 \\ -2 \\ 3 \\ k \end{pmatrix} \) are orthogonal.
\( k = \)

8. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_15.pg
Find the angle \( \alpha \) between the vectors \( \begin{pmatrix} 1 \\ -5 \end{pmatrix} \) and \( \begin{pmatrix} 4 \\ 6 \end{pmatrix} \).
\( \alpha = \)

9. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_16.pg
Find the angle \( \alpha \) between the vectors \( \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix} \) and \( \begin{pmatrix} 4 \\ -3 \end{pmatrix} \).
\( \alpha = \)

10. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_17.pg
Find the orthogonal projection of \( v = \begin{pmatrix} -9 \\ -5 \\ -13 \end{pmatrix} \) onto the subspace \( V \) of \( \mathbb{R}^3 \) spanned by \( \begin{pmatrix} -6 \\ 2 \\ 2 \end{pmatrix} \) and \( \begin{pmatrix} -6 \\ 3 \\ -21 \end{pmatrix} \).
\( \text{proj}_V(v) = \)

11. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_18.pg
Find the orthogonal projection of \( v = \begin{pmatrix} 12 \\ -1 \\ -5 \end{pmatrix} \) onto the subspace \( V \) of \( \mathbb{R}^3 \) spanned by \( \begin{pmatrix} -3 \\ 1 \\ -3 \end{pmatrix} \) and \( \begin{pmatrix} -4 \\ 3 \\ -24 \end{pmatrix} \).
\( \text{proj}_V(v) = \)

12. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_19.pg
Find the orthogonal projection of \( v = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \) onto the subspace \( V \) of \( \mathbb{R}^3 \) spanned by \( \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \), \( \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \), and \( \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \).
\( \text{proj}_V(v) = \)

13. (1 pt) setLinearAlgebra17DotProductRn/ur_Ja_17_20.pg
Find a vector \( v \) perpendicular to the vector \( u = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \).
\( v = \)

\( \text{ARNOLD PIZER} \)
\text{Rochester WeBWorK Problem Library}
14. Find a vector $x$ perpendicular to the vectors $v = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$ and $u = \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix}$. 

15. Find two linearly independent vectors perpendicular to the vector $v = \begin{pmatrix} 1 \\ 3 \\ 9 \\ 1 \end{pmatrix}$.

16. Let $v = \begin{pmatrix} -3 \\ -9 \\ -9 \\ 1 \end{pmatrix}$. Find a basis of the subspace of $\mathbb{R}^4$ consisting of all vectors perpendicular to $v$.

17. Let $v = \begin{pmatrix} -1 \\ 4 \\ -1 \\ 1 \end{pmatrix}$, $u = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}$, and let $W$ be the subspace of $\mathbb{R}^4$ spanned by $v$ and $u$. Find a basis of $W^\perp$.

18. Let $v_1 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{pmatrix}$, and $v_3 = \begin{pmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{pmatrix}$. Find a vector $v_4$ in $\mathbb{R}^4$ such that the vectors $v_1$, $v_2$, $v_3$, and $v_4$ are orthonormal.

19. Let $W$ be the subspace of $\mathbb{R}^3$ spanned by the vectors $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 3 \\ -11 \end{pmatrix}$. Find the matrix $A$ of the orthogonal projection onto $W$.

20. Let $W$ be the subspace of $\mathbb{R}^3$ spanned by the vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 7 \\ 5 \end{pmatrix}$. Find the matrix $A$ of the orthogonal projection onto $W$.

21. Among all the unit vectors $u = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in $\mathbb{R}^3$, find the one for which the sum $x + 7y + 6z$ is minimal. 

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
1. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_1.pg
Find the missing coordinates such that the three vectors form an orthonormal basis for \( \mathbb{R}^3 \):
\[
\begin{pmatrix}
-0.6 \\
-0.8 \\
1
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
-0.6 \\
\_ \\
\_ \\
\end{pmatrix}.
\]

2. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_2.pg
Let \( x = \begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix} \) and \( y = \begin{pmatrix} -3 \\ -1 \\ -2 \end{pmatrix} \).
Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of \( \mathbb{R}^3 \) spanned by \( x \) and \( y \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

3. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_3.pg
Perform the Gram-Schmidt process on the following sequence of vectors.
\[
x = \begin{pmatrix} -4 \\ -8 \\ -8 \end{pmatrix},
y = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix},
z = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix}.
\]
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

4. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_4.pg
Let \( x = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} \) and \( y = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \).
Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of \( \mathbb{R}^3 \) spanned by \( x \) and \( y \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

5. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_5.pg
Let \( x = \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix} \), \( y = \begin{pmatrix} -7 \\ 2 \\ 1 \end{pmatrix} \), and \( z = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \).
Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of \( \mathbb{R}^3 \) spanned by \( x \), \( y \), and \( z \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

6. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_6.pg
Find an orthonormal basis of the plane \( x_1 + 6x_2 - x_3 = 0 \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

7. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_7.pg
Let \( A = \begin{pmatrix} -2 & 1 & 2 \\ -3 & 3 & 3 \end{pmatrix} \).
Find an orthonormal basis of the kernel of \( A \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

8. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_8.pg
Let \( A = \begin{pmatrix} -3 & 9 \\ 2 & -3 \end{pmatrix} \).
Find an orthonormal basis of the image of \( A \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

9. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_9.pg
Let \( A = \begin{pmatrix} 1 & -2 & 1 & 0 \\ -2 & -2 & 4 & -6 \\ 2 & -6 & 4 & -2 \\ 5 & -4 & -1 & 6 \end{pmatrix} \).
Find orthogonal bases of the kernel and image of \( A \).
Basis of the kernel:
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]
Basis of the image:
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

10. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_10.pg
Let \( A = \begin{pmatrix} 1 & 1 & 3 \\ 0 & -6 & -9 \end{pmatrix} \).
Find an orthonormal basis of the row space of \( A \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]

11. (1 pt) setLinearAlgebra18OrthogonalBases/ur_Ja_18_11.pg
Let \( A = \begin{pmatrix} 1 & -4 & 0 \\ -2 & 2 & 6 \\ -3 & 2 & 10 \end{pmatrix} \).
Find an orthonormal basis of the column space of \( A \).
\[
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix},
\begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}.
\]
Let \( A = \begin{pmatrix} -3 & -1 & 0 & 1 \\ -4 & -1 & 1 & -2 \end{pmatrix} \).

Find orthonormal bases of the kernel, row space, and image (column space) of \( A \).

Basis of the kernel: \( \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \).

Basis of the row space: \( \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).

Basis of the image (column space): \( \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).
1. Find the QR factorization of \( M = \begin{pmatrix} 4 & -8 \\ 12 \end{pmatrix} \).

\[ M = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \end{pmatrix} \]

2. Find the QR factorization of \( M = \begin{pmatrix} -3 & 1 \\ -2 & 10 \\ -6 & -12 \end{pmatrix} \).

\[ M = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \end{pmatrix} \]

3. Find the QR factorization of \( M = \begin{pmatrix} 3 & -2 & 1 \\ 6 & 2 & 5 \\ 6 & 8 & -10 \end{pmatrix} \).

\[ M = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \\ \_ & \_ \\ \_ & \_ \end{pmatrix} \]

4. Find the QR factorization of \( M = \begin{pmatrix} -12 & 0 \\ 12 & 8 \\ -12 & -8 \end{pmatrix} \).

\[ M = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \begin{pmatrix} \_ & \_ \end{pmatrix} \]

5. Find the QR factorization of \( M = \begin{pmatrix} -1 & -2 & 3 \\ 1 & 2 & -7 \\ 1 & 4 & 3 \\ 1 & 4 & -1 \end{pmatrix} \).

\[ M = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix} \begin{pmatrix} \_ \end{pmatrix} \]
1. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_1.pg
Find the least-squares solution $x^*$ of the system
\[
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x
\end{pmatrix}
= 
\begin{pmatrix}
-4 \\
6 \\
1
\end{pmatrix}.
\]
x* = \boxed{______}.

2. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_2.pg
Find the least-squares solution $x^*$ of the system
\[
\begin{pmatrix}
2 & -2 \\
-2 & 2 \\
3 & 3
\end{pmatrix}
\begin{pmatrix}
x
\end{pmatrix}
= 
\begin{pmatrix}
7 \\
-13 \\
9
\end{pmatrix}.
\]
x* = \boxed{______}.

3. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_3.pg
Find the least-squares solution $x^*$ of the system
\[
\begin{pmatrix}
2 \\
-1 \\
1
\end{pmatrix}
\begin{pmatrix}
x
\end{pmatrix}
= 
\begin{pmatrix}
3 \\
3 \\
3
\end{pmatrix}.
\]
x* = \boxed{______}.

4. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_4.pg
Find the least-squares solution $x^*$ of the system
\[
\begin{pmatrix}
1 & -1 & -1 \\
1 & 1 & -1 \\
1 & -1 & 1 \\
1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
x
\end{pmatrix}
= 
\begin{pmatrix}
-2 \\
-10 \\
2 \\
-2
\end{pmatrix}.
\]
x* = \boxed{______}.

5. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_5.pg
Fit a linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(-1, -5)$, $(0, 1)$, $(1, 1)$, using least squares.
c_0 = \boxed{______}
c_1 = \boxed{______}

6. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_6.pg
Fit a quadratic function of the form $f(t) = c_0 + c_1 t + c_2 t^2$ to the data points $(0, 6)$, $(1, 12)$, $(2, -4)$, $(3, -2)$, using least squares.
c_0 = \boxed{______}
c_1 = \boxed{______}
c_2 = \boxed{______}

7. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_7.pg
Fit a trigonometric function of the form $f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t)$ to the data points $(0, 13.5)$, $(\frac{\pi}{2}, -0.5)$, $(\pi, 1.5)$, $(\frac{3\pi}{2}, 9.5)$, using least squares.
c_0 = \boxed{______}
c_1 = \boxed{______}
c_2 = \boxed{______}

8. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_8.pg
Let $S(t)$ be the number of daylight hours on the $t$th day of the year in Los Angeles. We are given the following data for $S(t)$:

<table>
<thead>
<tr>
<th>Day</th>
<th>$t$</th>
<th>$S(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>March</td>
<td>70</td>
<td>12</td>
</tr>
<tr>
<td>May</td>
<td>133</td>
<td>13</td>
</tr>
<tr>
<td>July</td>
<td>198</td>
<td>14</td>
</tr>
</tbody>
</table>

We wish to fit a trigonometric function of the form
\[f(t) = a + b \sin \left( \frac{2\pi}{365} t \right) + c \cos \left( \frac{2\pi}{365} t \right)\]
to these data. Find the best approximation of this form, using least squares.
a = \boxed{______}
b = \boxed{______}
c = \boxed{______}

9. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_9.pg
The table below lists the height $h$ (in cm), the age $a$ (in years), the gender $g$ (1=“Male”, 0=“Female”), and the weight $w$ (in kg) of some college students.

<table>
<thead>
<tr>
<th>Height</th>
<th>Age</th>
<th>Gender</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>20</td>
<td>1</td>
<td>84</td>
</tr>
<tr>
<td>172</td>
<td>20</td>
<td>1</td>
<td>74</td>
</tr>
<tr>
<td>169</td>
<td>21</td>
<td>0</td>
<td>66</td>
</tr>
<tr>
<td>164</td>
<td>22</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>158</td>
<td>23</td>
<td>0</td>
<td>54</td>
</tr>
</tbody>
</table>

We wish to fit a linear function of the form
\[f(t) = c_0 + c_1 h + c_2 a + c_3 g\]
to these data. Find the best approximation of this form, using least squares.
c_0 = \boxed{______}
c_1 = \boxed{______}
c_2 = \boxed{______}
c_3 = \boxed{______}

10. (1 pt) setLinearAlgebra20LeastSquares/ur_Ja_20_10.pg
During the summer months Terry makes and sells necklaces on the beach. Terry notices that if he lowers the price, he can sell more necklaces, and if he raises the price than he sells fewer necklaces. The table below shows how the number $n$ of necklaces sold in one day depends on the price $p$ (in dollars).

<table>
<thead>
<tr>
<th>Price</th>
<th>Number of necklaces sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>34</td>
</tr>
<tr>
<td>12</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
</tr>
</tbody>
</table>
(a) Find a linear function of the form
\[ n = c_0 + c_1 p \]
that best fits these data, using least squares.
\[ c_0 = \phantom{0} \]
\[ c_1 = \phantom{0} \]
(b) Find the revenue (number of items sold times the price of each item) as a function of price \( p \).

(c) If the material for each necklace costs Terry 6 dollars, find the profit (revenue minus cost of the material) as a function of price \( p \).
\[ P = \phantom{0} \]
(d) Finally, find the price that will maximize the profit.
\[ p = \phantom{0} \]

\[ R = \phantom{0} \]
Find the norm $||x||$ of $x = \left(1, -\frac{1}{5}, \frac{1}{25}, -\frac{1}{625}, \ldots, (\frac{1}{-5})^n, \ldots\right)$ in $l_2$.

$||x|| = \ldots$

If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ are arbitrary vectors in $M_2(\mathbb{R})$, then the mapping $\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$ defines an inner product in $M_2(\mathbb{R})$.

Use this inner product to determine $\langle A, B \rangle$, $||A||$, $||B||$, and the angle $\alpha_{A,B}$ between $A$ and $B$ for

$A = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -3 \\ 5 & -3 \end{pmatrix}$. 

$\langle A, B \rangle = \ldots$

$||A|| = \ldots$

$||B|| = \ldots$

$\alpha_{A,B} = \ldots$

If $A$ and $B$ are arbitrary $m \times n$ matrices, then the mapping $\langle A, B \rangle = \text{trace}(A^T B)$ defines an inner product in $\mathbb{R}^{m \times n}$.

Use this inner product to find $\langle A, B \rangle$, the norms $||A||$ and $||B||$, and the angle $\alpha_{A,B}$ between $A$ and $B$ for

$A = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & -2 \\ 1 & 2 \end{pmatrix}$. 

$\langle A, B \rangle = \ldots$

$||A|| = \ldots$

$||B|| = \ldots$

$\alpha_{A,B} = \ldots$

If $f(x)$ and $g(x)$ are arbitrary polynomials of degree at most 2, then the mapping $\langle f, g \rangle = f(-2)g(-2) + f(0)g(0) + f(3)g(3)$ defines an inner product in $P_2$.

Use this inner product to find $\langle f, g \rangle$, $||f||$, $||g||$, and the angle $\alpha_{f,g}$ between $f(x)$ and $g(x)$ for

$f(x) = 2x^2 + 6x - 6$ and $g(x) = 4x^2 - 6x + 4$. 

$\langle f, g \rangle = \ldots$

$||f|| = \ldots$

$||g|| = \ldots$

$\alpha_{f,g} = \ldots$

Let $M_1 = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$ and $M_2 = \begin{pmatrix} -4 & 2 \\ 1 & 2 \end{pmatrix}$.

Consider the inner product $\langle A, B \rangle = \text{trace}(A^T B)$ in the vector space $\mathbb{R}^{2 \times 2}$ of $2 \times 2$ matrices. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^{2 \times 2}$ spanned by the matrices $M_1$ and $M_2$. 

$\begin{pmatrix} \ldots & \ldots & \ldots \end{pmatrix}$.

Let $f(x) = 3$, $g(x) = -3x - 4$, and $h(x) = -6x^2 + 8x - 4$. Consider the inner product $\langle p(x), q(x) \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$ in the vector space $P_2$ of polynomials of degree at most 2. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $P_2$ spanned by the polynomials $f(x), g(x),$ and $h(x)$.

Let $f(x) = -5$, $g(x) = -2x + 1$, and $h(x) = 3x^2$.

Consider the inner product $\langle p(x), q(x) \rangle = \int_0^1 p(x)g(x)dx$ in the vector space $C^0[0, 1]$. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $C^0[0, 1]$ spanned by the functions $f(x), g(x),$ and $h(x)$.
1. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_1.pg
Enter a $3 \times 3$ symmetric matrix $A$ that has entries
\[ a_{11} = 4, \quad a_{22} = 2, \quad a_{33} = 0, \quad a_{12} = 5, \quad a_{31} = 1, \quad a_{23} = 3. \]
\[ A = \begin{pmatrix} 4 & 5 & 1 \\ 5 & 2 & 3 \\ 1 & 3 & 0 \end{pmatrix}. \]

2. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_2.pg
Enter a $3 \times 3$ skew-symmetric matrix $A$ that has entries
\[ a_{11} = 5, \quad a_{22} = 2, \quad a_{33} = 1, \quad a_{21} = 4, \quad a_{31} = 3, \quad a_{32} = 0. \]
\[ A = \begin{pmatrix} 5 & -4 & -3 \\ -4 & 2 & -1 \\ -3 & 1 & 1 \end{pmatrix}. \]

3. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_3.pg
Find the eigenvalues and associated unit eigenvectors of the (symmetric) matrix
\[ A = \begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix}. \]
smaller eigenvalue = ____,
associated unit eigenvector = ____. larger eigenvalue = ____,
associated unit eigenvector = ____. The above eigenvectors form an orthonormal eigenbasis for $A$.

4. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_2.pg
Find the eigenvalues and associated unit eigenvectors of the (symmetric) matrix
\[ A = \begin{pmatrix} -15 & 15 \\ 15 & 25 \end{pmatrix}. \]
smaller eigenvalue = ____,
associated unit eigenvector = ____. larger eigenvalue = ____,
associated unit eigenvector = ____. The above eigenvectors form an orthonormal eigenbasis for $A$.

5. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_5.pg
The matrix $M = \begin{pmatrix} 0 & 0 & 4 \\ 0 & -5 & 0 \\ 4 & 0 & 0 \end{pmatrix}$ has two distinct eigenvalues $\lambda_1 < \lambda_2$. Find the eigenvalues and an orthonormal basis for each eigenspace.
$\lambda_1 = ____$, associated unit eigenvector = ____. $\lambda_2 = ____$, associated unit eigenvector = ____. The above eigenvectors form an orthonormal eigenbasis for $M$.

Orthonormal basis: \( \begin{pmatrix} \_ \\ \_ \end{pmatrix} \),
$\lambda_2 = ____$.
Orthonormal basis: \( \begin{pmatrix} \_ \\ \_ \end{pmatrix}, \begin{pmatrix} \_ \end{pmatrix} \).
The above eigenvectors form an orthonormal eigenbasis for $M$.

6. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_4.pg
Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and associated unit eigenvectors of the (symmetric) matrix
\[ M = \begin{pmatrix} -3 & -3 & -3 \\ -3 & -1 & -5 \\ -3 & -5 & -1 \end{pmatrix}. \]
$\lambda_1 = ____$, associated unit eigenvector = ____. $\lambda_2 = ____$, associated unit eigenvector = ____. $\lambda_3 = ____$, associated unit eigenvector = ____. The above eigenvectors form an orthonormal eigenbasis for $M$.

7. (1 pt) setLinearAlgebra22SymmetricMatrices/ur_la22_6.pg
The matrix $M = \begin{pmatrix} 2 & 0 & 0 & -2 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 2 & 0 \\ -2 & 0 & 0 & 2 \end{pmatrix}$ has two distinct eigenvalues $\lambda_1 < \lambda_2$. Find the eigenvalues and an orthonormal basis for each eigenspace.
$\lambda_1 = ____$, associated unit eigenvector = ____. $\lambda_2 = ____$, associated unit eigenvector = ____. The above eigenvectors form an orthonormal eigenbasis for $M$.
1. (1 pt) setLinearAlgebra23QuadraticForms/ur_la_23_1.pg
Write the matrix of the quadratic form
\[ Q(x) = -1x_1^2 - 4x_2^2 - 1x_3^2 + 2x_1x_2 + 8x_1x_3 - 2x_2x_3. \]
\[ A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \]

2. (1 pt) setLinearAlgebra23QuadraticForms/ur_la_23_4.pg
If \[ A = \begin{pmatrix} -5 & -9 \\ -9 & -9 \end{pmatrix} \] and \[ Q(x) = x \cdot Ax, \]
Then \[ Q(e_1) = \text{ and } Q(e_2) = \text{.} \]

3. (1 pt) setLinearAlgebra23QuadraticForms/ur_la_23_5.pg
If \[ A = \begin{pmatrix} 9 & 9 & 7 \\ 9 & 8 & 8 \\ 7 & 8 & 8 \end{pmatrix} \] and \[ Q(x) = x \cdot Ax, \]
Then \[ Q(x_1, x_2, x_3) = x_1^2 + \text{ } x_2^2 + \text{ } x_3^2 + \text{ } x_1x_2 + \text{ } x_1x_3 + \text{ } x_2x_3. \]

4. (1 pt) setLinearAlgebra23QuadraticForms/ur_la_23_2.pg
Find the eigenvalues of the matrix
\[ A = \begin{pmatrix} -0.5 & -0.5 \\ -0.5 & -0.5 \end{pmatrix}. \]
The smaller eigenvalue is \( \lambda_1 = \text{.} \)
and the bigger eigenvalue is \( \lambda_2 = \text{.} \)
Classify the quadratic form \( Q(x) = x^T Ax : \)
- A. \( Q(x) \) is negative definite
- B. \( Q(x) \) is positive definite
- C. \( Q(x) \) is positive semidefinite
- D. \( Q(x) \) is negative semidefinite
- E. \( Q(x) \) is indefinite

5. (1 pt) setLinearAlgebra23QuadraticForms/ur_la_23_3.pg
The matrix
\[ A = \begin{pmatrix} -3.5 & -0.5 & 0 \\ -0.5 & -3.5 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]
has three distinct eigenvalues, \( \lambda_1 < \lambda_2 < \lambda_3, \)
\( \lambda_1 = \text{.} \)
\( \lambda_2 = \text{.} \)
\( \lambda_3 = \text{.} \)
Classify the quadratic form \( Q(x) = x^T Ax : \)
- A. \( Q(x) \) is negative semidefinite
- B. \( Q(x) \) is positive definite
- C. \( Q(x) \) is indefinite
- D. \( Q(x) \) is negative definite
- E. \( Q(x) \) is positive semidefinite
1. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_1.pg
Find the singular values $\sigma_1 \geq \sigma_2$ of
$A = \begin{pmatrix} -8 & 0 \\ 0 & -4 \end{pmatrix}$.
$\sigma_1 =$ 
$\sigma_2 =$

2. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_2.pg
Find the singular values $\sigma_1 \geq \sigma_2$ of
$A = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$.
$\sigma_1 =$ 
$\sigma_2 =$

3. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_3.pg
Find the singular values $\sigma_1 \geq \sigma_2$ of
$A = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$.
$\sigma_1 =$ 
$\sigma_2 =$

4. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_4.pg
Find the singular values $\sigma_1 \geq \sigma_2$ of
$A = \begin{pmatrix} -2 & 1 \\ 0 & 1 \\ 1 & -1 \end{pmatrix}$.
$\sigma_1 =$ 
$\sigma_2 =$

5. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_5.pg
Find the singular values $\sigma_1 \geq \sigma_2 \geq \sigma_3$ of
$A = \begin{pmatrix} -3 & 0 & 6 \\ -6 & 0 & -3 \end{pmatrix}$.
$\sigma_1 =$ 

6. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_6.pg
Find the singular values $\sigma_1 \geq \sigma_2$ of
$A = \begin{pmatrix} 1 & -3 \\ 3 & -1 \end{pmatrix}$.
$\sigma_1 =$ 
$\sigma_2 =$

7. (1 pt) setLinearAlgebra24SingularValues/ur_ja_24_7.pg
Let $A = \begin{pmatrix} 1 & 7 \\ -7 & 1 \\ -7 & 1 \end{pmatrix}$.
A singular value decomposition of $A$ is as follows:
$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{pmatrix}$.
Find the least-squares solution of the linear system
$Ax = b$, where $b = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$.
$x_1 =$ 
$x_2 =$

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Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, © UR
1. (1 pt) setComplexNumbers/ur_cn_1_1.pg

For some practice working with complex numbers:

Calculate

\[(1 + 4i) + (1 - 6i) = \ldots\],
\[(1 + 4i) - (1 - 6i) = \ldots\],
\[(1 + 4i)(1 - 6i) = \ldots\].

The complex conjugate of \((1 + i)\) is \((1 - i)\). In general to obtain the complex conjugate reverse the sign of the imaginary part. (Geometrically this corresponds to finding the "mirror image" point in the complex plane by reflecting through the \(x\)-axis. The complex conjugate of a complex number \(z\) is written with a bar over it: \(\bar{z}\) and read as "z bar".

Notice that if \(z = a + ib\), then
\[(z) (\bar{z}) = |z|^2 = a^2 + b^2\]
which is also the square of the distance of the point \(z\) from the origin. (Plot \(z\) as a point in the "complex" plane in order to see this.)

If \(z = 1 + 4i\) then \(\bar{z} = \ldots\) and \(|z| = \ldots\).

You can use this to simplify complex fractions. Multiply the numerator and denominator by the complex conjugate of the denominator to make the denominator real.

\[\frac{1 + 4i}{1 - 6i} = \ldots + i \ldots\]

Two convenient functions to know about pick out the real and imaginary parts of a complex number.

\(\text{Re}(a + ib) = a\) (the real part (coordinate) of the complex number), and
\(\text{Im}(a + ib) = b\) (the imaginary part (coordinate) of the complex number). \(\text{Re}\) and \(\text{Im}\) are linear functions – now that you know about linear behavior you may start noticing it often.

2. (1 pt) setComplexNumbers/ur_cn_1_2.pg

More on complex numbers. (For additional help check out the appendix in Stewart’s Calculus book. There is an entire appendix of hints for working with complex numbers.)

An apology: The exponents don’t print very well on the screen version of this problem. You can get a better idea of what the notation looks like from the hard copy and/or you can use the "typeset" mode to get a better printing. Unfortunately in typset mode you won’t be able to enter the answers which are within equations.
Enter the complex coordinates of the following points:

A: \[+ \text{ } i\]
B: \[+ \text{ } i\]
C: \[+ \text{ } i\]

Write the following numbers in \(a + bi\) form:
(a) \((-5 + i)^2 = \text{ } + \text{ } i\)
(b) \(\frac{2 - i}{1} = \text{ } + \text{ } i\)
(c) \(\frac{2 - i}{1} = \text{ } + \text{ } i\)

Write the following numbers in \(a + bi\) form:
(a) \((-2 - i)^2 = \text{ } + \text{ } i\)
(b) \(i(\pi - 2i) = \text{ } + \text{ } i\)
(c) \(\frac{-2 + 3i}{i} = \text{ } + \text{ } i\)

Write the following numbers in \(a + bi\) form:
(a) \(\frac{2 + i}{i - (4 - 2i)} = \text{ } + \text{ } i\)
(b) \((i)^2(-2 + i)^2 = \text{ } + \text{ } i\)

Write the following numbers in \(a + bi\) form:
(a) \(\frac{2 + i}{i} = \text{ } + \text{ } i\)
(b) \(\frac{3 + 3i}{2i} = \text{ } + \text{ } i\)
(c) \((-2i)^3 = \text{ } + \text{ } i\)

Write the following numbers in \(a + bi\) form:
(a) \((5 + 4i)(1 + 4i)(5 + 3i) = \text{ } + \text{ } i\)
(b) \((-1 + 4i)^2 - 3i = \text{ } + \text{ } i\)

Calculate the following:
(a) \(i^2 = \text{ }\)
(b) \(i^3 = \text{ }\)
(c) \(i^4 = \text{ }\)
(d) \(i^5 = \text{ }\)
(e) \(i^{10} = \text{ }\)
(f) \(i^0 = \text{ }\)
(g) \(i^{-1} = \text{ }\)
(h) \(i^{-2} = \text{ }\)
(i) \(i^{-3} = \text{ }\)
(j) \(i^{-32} = \text{ }\)

Let \(z = -6 + 7i\). Write the following numbers in \(a + bi\) form:
(a) \(4z = \text{ } + \text{ } i\)
(b) \(\bar{z} = \text{ } + \text{ } i\)
(c) \(\frac{1}{z} = \text{ } + \text{ } i\)

Let \(z = -1 - 3i\). Calculate the following:
(a) \(z^2 + 2i + 1 = \text{ } + \text{ } i\)
Write the following numbers in the polar form:

(a) \( z^2 + iz - (5 + i) = \ldots + \ldots i \),
(b) \( z = \ldots + \ldots i \),
(c) \( \frac{(z - 4)^2}{z + i} = \ldots + \ldots i \).

14. (1 pt) setComplexNumbers/ur.cn.l_15.pg

Solve the following equations for \( z \):

(a) \( iz = 4 - zi \)
(b) \( \frac{z - 1}{z} = 1 - 5i \)
(c) \( (2 - i)z + 8z^2 = 0 \)
(This question has two solutions, one of which is 0, find the other)

\( z = \ldots + \ldots i \).

15. (1 pt) setComplexNumbers/ur.cn.l_16.pg

Calculate:

(a) \( \frac{2 + 3i}{-4 - 4i} = \ldots \)
(b) \( (1 + i)(3 - 4i)(1 - 3i) = \ldots \)
(c) \( \frac{i(3 + 2i)}{(4 - 2i)^2} = \ldots \)
(d) \( \frac{(\pi + i)^{100}}{(\pi - i)^{100}} = \ldots \)

16. (1 pt) setComplexNumbers/ur.cn.l_17.pg

Answer the following questions (T or F):

1. \( \text{Arg } z_{1}z_{2} = \text{Arg } z_{1} + \text{Arg } z_{2}, \text{ if } z_{1} \neq 0, z_{2} \neq 0. \)
2. \( \text{Arg } (0) \text{ is undefined.} \)
3. \( \text{Arg } z = \text{Arg } z + 2\pi k, (k = 0, \pm 1, \pm 2, \pm 3\ldots) \text{ and if } z \neq 0. \)
4. \( \text{Arg } \frac{z}{z} = \text{Arg } z_{1} - \text{Arg } z_{2}, \text{ if } z_{1} \neq 0, z_{2} \neq 0. \)
5. \( \text{Arg } \overline{z} = -\text{Arg } z, \text{ if } z \text{ is not real.} \)

17. (1 pt) setComplexNumbers/ur.cn.l_18.pg

Place the following in order:

(a) \( |z_{2} - |z_{1}| | \),
(b) \( |z_{1} + z_{2} | \),
(c) \( |z_{2} - |z_{1}| | \),
(d) \( |z_{1} + z_{2} | \).

\( \ldots \leq \ldots \leq \ldots \leq \ldots \)

18. (1 pt) setComplexNumbers/ur.cn.l_19.pg

Write the following numbers in the polar form \( re^{i\theta}, 0 \leq \theta < 2\pi: \)

(a) \( \frac{1}{3} \)
(b) \( 7 + 7i \)
(c) \( 9 - 9i \)

19. (1 pt) setComplexNumbers/ur.cn.l_20.pg

Write the following numbers in the polar form \( re^{i\theta}, -\pi < \theta \leq \pi: \)

(a) \( \pi i \)

20. (1 pt) setComplexNumbers/ur.cn.l_21.pg

Write each of the given numbers in the form \( a + bi: \)

(a) \( e^{-\frac{3\pi}{2}} \)
(b) \( e^{\left(1 + \frac{3\pi}{2}\right)} \)
(c) \( e^{\frac{1}{2}} \)

21. (1 pt) setComplexNumbers/ur.cn.l_22.pg

Write each of the given numbers in the form \( a + bi: \)

(a) \( e^{8i} - e^{-8i} \)
(b) \( 2i + \ldots j \)
(c) \( e^{\left(\frac{3\pi}{4}\right)} \)

22. (1 pt) setComplexNumbers/ur.cn.l_25.pg

Write the following numbers in the polar form \( r (\cos \phi + i \sin \phi), 0 \leq \phi < 2\pi. \)

(a) \( 7 \)
(b) \( 9i \)
(c) \( 8 + 4i \)

23. (1 pt) setComplexNumbers/ur.cn.l_36.pg

Let \( z = 6 (\cos 1.5 + i \sin 1.5). \)

Write the following numbers in the polar form \( r (\cos \phi + i \sin \phi), 0 \leq \phi < 2\pi. \)

(a) \( 8\pi \)
(b) \( \bar{z} \)
(c) \( \frac{1}{z} \)

24. (1 pt) setComplexNumbers/ur.cn.l_37.pg

Write the following numbers in the polar form \( r (\cos \phi + i \sin \phi), 0 \leq \phi < 2\pi. \)

(a) \( 8\pi \)
(b) \( \bar{z} \)
(c) \( \frac{1}{z} \)

25. (1 pt) setComplexNumbers/ur.cn.l_38.pg

Write the following numbers in the polar form \( r (\cos \phi + i \sin \phi), 0 \leq \phi < 2\pi. \)

(a) \( 8\pi \)
(b) \( \bar{z} \)
(c) \( \frac{1}{z} \)
24. (1 pt) setComplexNumbers/ur_cn_1_23.pg
Write each of the given numbers in the polar form $re^{\theta}$, $-\pi < \theta \leq \pi$.
(a) $\frac{1 - i}{8}$
   $r = \phantom{0} \theta = \phantom{0}$
(b) $-4\pi(3 + iv\sqrt{3})$
   $r = \phantom{0} \theta = \phantom{0}$
(c) $(1 + i)^3$
   $r = \phantom{0} \theta = \phantom{0}$

25. (1 pt) setComplexNumbers/ur_cn_1_24.pg
Write each of the given numbers in the polar form $re^{\theta}$, $-\pi < \theta \leq \pi$.
(a) $\left( \cos \frac{-2\pi}{9} + i\sin \frac{-2\pi}{9} \right)^3$
   $r = \phantom{0} \theta = \phantom{0}$
(b) $\phantom{0} -\sqrt{3} + i$
   $r = \phantom{0} \theta = \phantom{0}$
(c) $\frac{3e^{(8+i)}}{4i}$
   $r = \phantom{0} \theta = \phantom{0}$

26. (1 pt) setComplexNumbers/ur_cn_1_37.pg
Let $z = 6e^{1.8}$.
Write the following numbers in the polar form $re^{\theta}$, $0 \leq \phi < 2\pi$.
(a) $2z$
   $r = \phantom{0} \phi = \phantom{0}$
(b) $\frac{z}{2}$
   $r = \phantom{0} \phi = \phantom{0}$
(c) $\frac{1}{z}$
   $r = \phantom{0} \phi = \phantom{0}$

27. (1 pt) setComplexNumbers/ur_cn_1_25.pg
Determine which of the following properties of the real exponential function remain true for the complex exponential (i.e., for $x$ replaced by $z$).
Answer T or F:
- 1. $e^z$ is defined for all $x$. T
- 2. $e^z$ is never zero. T
- 3. $e^{\bar{z}}$ is a one-to-one function. F
- 4. $e^{-z} = \frac{1}{e^z}$. T

28. (1 pt) setComplexNumbers/ur_cn_1_26.pg
Which of the following sets are open?
- A. $|z - 2| < 3$
- B. $|z - 1 + i| \leq 3$
- C. $|z - 1| < 3$
- D. $|z - 1 + i| \\ < 3$
- E. $|z - 1 + i| > 3$
- F. $|z - 1 + i| < 3$

29. (1 pt) setComplexNumbers/ur_cn_1_27.pg
Which of the given sets are bounded?
- A. $|z - 1 + i| \leq 3$
- B. $0 < |z - 2| < 3$
- C. $-1 < \text{Im} z \leq 1$
- D. $|\text{Arg } z| < \frac{\pi}{4}$
- E. $|z| \geq 2$
- F. $(\text{Re } z)^2 > 1$

30. (1 pt) setComplexNumbers/ur_cn_1_28.pg
Which of the given sets are bounded?
- A. $-1 < \text{Im } z \leq 1$
- B. $|z| \geq 2$
- C. $0 < |z - 2| < 3$
- D. $(\text{Re } z)^2 > 1$
- E. $|\text{Arg } z| < \frac{\pi}{4}$
- F. $|z - 1 + i| \leq 3$

31. (1 pt) setComplexNumbers/ur_cn_1_29.pg
Which of the given sets are closed regions?
- A. $|z| \geq 2$
- B. $|z - 1 + i| \leq 3$
- C. $-1 < \text{Im } z \leq 1$
- D. $(\text{Re } z)^2 > 1$
- E. $|\text{Arg } z| < \frac{\pi}{4}$
- F. $|z - 1 + i| > 3$

32. (1 pt) setComplexNumbers/ur_cn_1_30.pg
Which of the given sets are domains?
- A. $|z| \geq 2$
- B. $(\text{Re } z)^2 > 1$
- C. $0 < |z - 2| < 3$
- D. $(\text{Re } z)^2 > 1$
- E. $|\text{Arg } z| < \frac{\pi}{4}$
- F. $|z - 1 + i| \leq 3$

33. (1 pt) setComplexNumbers/ur_cn_1_31.pg
Find all the values of the following:
1. $(-256)^\frac{1}{2}$
   Place all answers in the following blank, separated by commas:
2. $1^\frac{1}{2}$
   Place all answers in the following blank, separated by commas:
3. $(-1)^\frac{1}{2}$
   Place all answers in the following blank, separated by commas:
4. $(-1 + \sqrt{3})^\frac{1}{2}$
   Place all answers in the following blank, separated by commas:

34. (1 pt) setComplexNumbers/ur_cn_1_32.pg
Find all the values of the following:
1. $(1 + \sqrt{3})^\frac{1}{2}$
   Place all answers in the following blank, separated by commas:
2. $(i + 1)^\frac{1}{2}$
   Place all answers in the following blank, separated by commas:
(3) \( \left( \frac{6i}{1+i} \right)^{\frac{1}{2}} \)

Place all answers in the following blank, separated by commas:

35. (1 pt) setComplexNumbers/ur.cn.1_33.pg

Solve the following equations for \( z \), find all solutions:

(1) \( 5z^2 + z + 5 = 0 \)

Place all answers in the following blank, separated by commas:

(2) \( z^2 - (3 - 2i)z + 1 - 3i = 0 \)

Place all answers in the following blank, separated by commas:

(3) \( z^2 - 2z + i = 0 \)

Place all answers in the following blank, separated by commas:
1. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-1.pg
Write each of the following functions in the form \( w = u(x, y) + iv(x, y) \):

(1) \( f(z) = 2z^2 + 2z + 3i + 5 \)

(2) \( h(z) = \frac{5z^2}{z^2 + 1} + i \)

(3) \( F(z) = e^{5z} + i \)

2. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-2.pg
Write each of the following functions in the form \( w = u(x, y) + iv(x, y) \):

(1) \( g(z) = \frac{3}{z} + i \)

(2) \( q(z) = \frac{5z^2 + 3}{z - 1} + i \)

(3) \( G(z) = e^z + e^{-z} \)

3. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-3.pg
Decide whether each of the following sequences converges, and if so, type the limit in the answer blank.

If it does not converge, type "DNC":

(1) \( z_n = \frac{4}{n} \)

(2) \( z_n = i(-1)^n \)

(3) \( z_n = \text{Arg}(-1 + \frac{4}{n}) \)

(4) \( z_n = \frac{n(5 + 5i)}{n + 1} \)

(5) \( z_n = (\frac{1 + i}{4})^n \)

(6) \( z_n = \exp(\frac{2\pi n i}{7}) \)

4. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-4.pg
A uniformly charged infinite rod, standing perpendicular to the z-plane at the point \( z_0 \), generates an electric field at every point in the plane. The intensity of this field varies inversely as the distance from \( z_0 \) to the point and is directed along the line from \( z_0 \) to the point.

If three such rods are located at the points \( 3 + 2i, -3 + 2i, \) and 0, find the positions of equilibrium (i.e., the points where the vector sum of the fields is zero). (Hint: \( F(z) = \frac{1}{z - z_0} \))

Enter all answers in the following answer blank, separated by commas:

5. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-5.pg
Find each of the following limits:

(1) \( \lim_{z \to z_0}(z - 7i)^2 = \)

(2) \( \lim_{z \to 3i} \frac{z^2 + 9}{z - 3i} = \)

(3) \( \lim_{\Delta z \to 0} \frac{(z_0 + \Delta z)^2 - z_0^2}{\Delta z} = \)

(Hint: You may use \( \Delta z = z_0 \) in your answer, simply write it as \( z_0 \)).

6. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-6.pg
Find each of the following limits:

(1) \( \lim_{z \to 3} \frac{z^2 + 4}{iz} = \)

(2) \( \lim_{z \to \frac{2}{z} - 1} \frac{z^2 + 1}{z} = \)

(3) \( \lim_{z \to 4 + 2i} |z^2 - 16| = \)

7. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-7.pg
Find each of the following limits:

(1) \( \lim_{z \to 0} e^z = \)

(2) \( \lim_{z \to -2i} e^z - e^{-z} = \)

(3) \( \lim_{z \to i} (z + 4)e^z = \)

(4) \( \lim_{z \to -i} \exp(z^2 + \frac{2}{z + 3i}) = \)

8. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-8.pg
Find the derivatives of the following functions with respect to \( z \):

(1) \( f(z) = 4z^3 + 8z^2 + iz + 3 \)

\( f'(z) = \)

(2) \( f(z) = (z^2 - 6i)^{-7} \)

\( f'(z) = \)

(3) \( f(z) = \frac{z^2 - 9}{iz^4 + 2z + 5i} \)

\( f'(z) = \)

9. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-9.pg
Find the derivatives of the following functions:

(1) \( f(z) = \frac{(z + 1)^3}{(z^2 + 4iz + 1)^7} \)

(2) \( f(z) = 6i(z^3 - 4)^8(z^2 + 5iz)^{100} \)

10. (1 pt) setComplexNumbers2AnalyticFunctions/ur_cn_2-10.pg
For each of the following determine the points at which the function is not analytic:

Enter all answers in the answer blank separated by commas.

(1) \( z = \frac{1}{z} + 2i \)

(2) \( z^2 + 2z \)

(3) \( z^2 + z + 4 \)
11. Use L'Hospital's rule to find $\lim_{z \to i} \frac{1 + i z}{2i + z^2}$.

12. Let $f(z)$ and $g(z)$ be entire functions. Decide which of the following statements are always true.

Answer T or F:

1. $f(z)^3$ is entire.
2. $5f(z) + ig(z)$ is entire.
3. $f(1/z)$ is entire.
4. $g(z^2 + 2)$ is entire.
5. $f(z)g(z)$ is entire.
6. $f(z)/g(z)$ is entire.
7. $g(z)$ is entire.

Find the harmonic conjugate of each harmonic function $u$.

(1) $u = 7y$

(2) $u = 5e^z \sin(y)$

(3) $u = 5xy - 3x + 6y$

Find the harmonic conjugate of each harmonic function $u$.

(1) $u = \sin(x) \cosh(y)$

(2) $u = \ln|z|$ for $\text{Re} z > 0$

(3) $u = \text{Im}(e^z)$

Find a function $\phi(x, y)$ that is harmonic in the infinite vertical strip $\{z : -1 \leq \text{Re} z \leq 3\}$ and takes the value 1 on the left edge and the value 5 on the right edge.

Find a function $\phi(x, y)$ that is harmonic in the region of the first quadrant between the curves $xy = 2$ and $xy = 4$ and takes on the values $-1$ on the lower edge and the value 1 on the upper edge. [Hint: Begin by considering $z^2$.]

17. Find a function $\phi(x, y)$ that is harmonic in the region of the first quadrant between the curves $xy = 2$ and $xy = 4$ and takes on the values $-1$ on the lower edge and the value 1 on the upper edge. [Hint: Begin by considering $z^2$.]

18. Enter T or F depending on whether the statement is true or false.

(You must enter T or F – True and False will not work.)

1. A function $f(z)$ which is analytic on the annulus $r < |z| < R$ can be represented by a Laurent series centered at 0, which converges for all $z$ in the annulus.

2. $f(z) = z^{-1/2}$ can be expanded as a Laurent series convergent in the annulus $0 < |z| < \infty$. It has a simple pole at $z = 0$.

3. If a Laurent series converges in an annulus $r < |z| < R$ then differentiating the Laurent series term by term produces a new Laurent series which converges on an annulus $\frac{r}{2} < |z| < \frac{R}{2}$ but the new annulus might be smaller than the original annulus (i.e. $r < \frac{r}{2}$ and $R > \frac{R}{2}$).

4. A function $f(z)$, analytic in a domain $D$ which contains the annulus $1 < |z| < 4$ can always be expanded in a convergent power series (with no negative exponents) centered at $2i$ and converging in the disk $|z - 2i| < 1$.

5. A function $f(z)$ which is analytic on a domain $D$ can always be expanded in a power series which converges on the smallest disk containing the domain $D$.
1. (1 pt) setAlgebra01RealNumbers/swr1.18.pg
Evaluate the expression $3(-5)(1 - 7 - 2(3))$.

(Your answer cannot be an algebraic expression.)

2. (1 pt) setAlgebra01RealNumbers/sw1.2.9.pg
Use the properties of real numbers to write the expression

$$7(13m)$$

in the form of

$$A \cdot m.$$ 

The number $A = ____$

3. (1 pt) setAlgebra01RealNumbers/sw1.2.11.pg
Use properties of real numbers to write the expression

$$\frac{-8}{2}(4x - 28y)$$

in the form of

$$A \cdot x + B \cdot y.$$ 

The number $A = ____$ and the number $B = ____$

4. (1 pt) setAlgebra01RealNumbers/sw1.2.12.pg
Use properties of real numbers to write the expression

$$(7a)(3b + 2c - 2d)$$

in the form of

$$K \cdot ab + M \cdot ac + N \cdot ad.$$ 

The number $K = ____$.
The number $M = ____$.
The number $N = ____$

5. (1 pt) setAlgebra01RealNumbers/sw1.2.13a.pg
Add the fractions, and reduce your answer.

$$\frac{16}{8} + \frac{16}{8}$$

The reduced answer is ____ / ____

6. (1 pt) setAlgebra01RealNumbers/sw1.2.13b.pg
Add the fractions, and reduce your answer.

$$\frac{16}{2} + \frac{7}{13}$$

The reduced answer is ____ / ____

7. (1 pt) setAlgebra01RealNumbers/sw1.2.14a.pg
Add the fractions, and reduce your answer.

$$\frac{9}{64} + \frac{7}{44}$$

The reduced answer is ____ / ____

8. (1 pt) setAlgebra01RealNumbers/sw1.2.14b.pg
Add the fractions, and reduce your answer.

$$\frac{5}{8} + \frac{20}{11} + 17$$

The reduced answer is ____ / ____

9. (1 pt) setAlgebra01RealNumbers/sw1.2.15a.pg
Add the fractions, and reduce your answer.

$$\frac{10}{35} \div \frac{6}{30}$$

The reduced answer is ____ / ____

10. (1 pt) setAlgebra01RealNumbers/sw1.2.15b.pg
Add the fractions, and reduce your answer.

$$\left( \frac{7}{5} \div \frac{6}{5} \right) - \frac{6}{5}$$

The reduced answer is ____ / ____

11. (1 pt) setAlgebra01RealNumbers/order_of_ops.pg
Evaluate the expression

$$\frac{1}{3}(3 + 5/5^2)$$

NOTE: Your answer cannot be an algebraic expression.

12. (1 pt) setAlgebra01RealNumbers/sw1.2.47a.pg
Evaluate the expression $|72|$.

13. (1 pt) setAlgebra01RealNumbers/sw1.2.47b.pg
Evaluate the expression $| - 80 |$.

14. (1 pt) setAlgebra01RealNumbers/sw1.8.65.pg
Evaluate the expression $|33|$.

15. (1 pt) setAlgebra01RealNumbers/sw1.8.66.pg
Evaluate the expression $| - 165 |$.

16. (1 pt) setAlgebra01RealNumbers/sw1.8.67.pg
Evaluate the expression $|103 - 300 |$.

17. (1 pt) setAlgebra01RealNumbers/ur_ab_8.1.pg
Evaluate the expression $| - (21 - 173) |$.

18. (1 pt) setAlgebra01RealNumbers/sw1.2.49a.pg
Evaluate the expression

$$|| -36 | - | -11 ||.$$ 

19. (1 pt) setAlgebra01RealNumbers/sw1.2.49b.pg
Evaluate the expression

$$-2 \quad | -2 |.$$ 

Your answer is ____

20. (1 pt) setAlgebra01RealNumbers/sw1.2.50a.pg
Evaluate the expression

$$| 20 - | -37 ||.$$
You must get all of the answers correct to receive credit.

21. (1 pt) setAlgebra01RealNumbers/sw1_2_50b.pg
Evaluate the expression
\[-11 - |11 - | -11|].

Your answer is ___.

22. (1 pt) setAlgebra01RealNumbers/lhp1_49-58.pg
This exercise concerns the definition of absolute values. Evaluate the following expressions:
\[| -8| = \text{___} \]
\[-8 - | -4| = \text{___} \]
If \(8 < x\), simplify the expression by removing the absolute sign.
\[|8 - x| = \text{___} \]
If \(y < -4\), evaluate the expression.
\[|y + 4| = \text{___} \]

23. (1 pt) setAlgebra01RealNumbers/sw1_8_73.pg
Evaluate the expression \[\frac{141 - 292}{| -8|}.\] Give your answer in decimal notation correct to three decimal places or give your answer as a fraction.

[NOTE: Your answer can be an algebraic expression. Make sure to include all necessary (, ).]

24. (1 pt) setAlgebra01RealNumbers/sw1_2_17.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false. You must get all of the answers correct to receive credit.

1. \(\pi > 3.1416\)
2. \(-10 < \sqrt{7}\)

25. (1 pt) setAlgebra01RealNumbers/sw1_2_18.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false. You must get all of the answers correct to receive credit. Your answer for the following statement is
\[
\frac{21}{22} < \frac{23}{24}.
\]
Your answer for the following statement is
\[
\frac{20}{21} < \frac{22}{22}.
\]

26. (1 pt) setAlgebra01RealNumbers/sw1_8_77.pg
Find the distance between 475 and 370.

[NOTE: Your answer can be an algebraic expression]

27. (1 pt) setAlgebra01RealNumbers/swr1_1_9-18.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false. You must get all of the answers correct to receive credit.

1. \(-5 < -9\)
2. \(-3 \leq -3\)
3. \(-1 < -1\)

28. (1 pt) setAlgebra01RealNumbers/lhp1_79-82.pg
Use absolute value and inequality notations to describe the following situations. You may use “|” for absolute value sign; leq for \(\leq\) and geq for \(\geq\). E.g. you may use “[|x| geq 5]” for “[|x| \geq 5].” The distance between \(x\) and 29 is no more than 18
\[x\] is at least 9 units from 0
\[x\] is at most 29 units from 0

29. (1 pt) setAlgebra01RealNumbers/sw1_1_21-28.pg
Match the statements defined below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

1. The distance from \(x\) to -1 is less than or equal to 2
2. \(x\) is any real number
3. \(x\) is less than or equal to -1
4. \(x\) is greater than -1
5. \(x\) is greater than or equal to -1

A. \(x \leq -1\)
B. \(|x + 1| \leq 2\)
C. \(-\infty < x < \infty\)
D. \(-1 < x\)
E. \(x \geq -1\)

30. (1 pt) setAlgebra01RealNumbers/swr1_1_39-48.pg
Match the statements defined below with the letters labeling their equivalent intervals. You must get all of the answers correct to receive credit.

1. \(x \in (1, 6)\)
2. \(x \in (1, \infty)\)
3. \(x \in (-\infty, 1]\)
4. \(x \in [1, 6)\)
5. \(x \in (-\infty, 1)\)

A. \(x \leq 1\)
B. \(1 < x < 6\)
C. \(x > 1\)
D. \(x < 1\)
E. \(1 \leq x < 6\)

31. (1 pt) setAlgebra01RealNumbers/lhp1_25-30.pg
Sketch the following sets on a piece of paper and write them in interval notation. Enter the interval in the answer box. You may use “infinity” for \(\infty\) and “-infinity” for \(-\infty\). For example, you may write \((-\infty, 5]\) for the interval \((-\infty, 5].\)

\[\begin{align*}
x \geq 10 & \quad \text{_____} \\
x \leq 12 & \quad \text{_____} \\
x > 21 & \quad \text{_____} \\
x < 8 & \quad \text{_____}
\end{align*}\]

32. (1 pt) setAlgebra01RealNumbers/lhp1_31-34.pg
Sketch the following sets on a piece of paper and write them in interval notation. Enter the interval in the answer box. You may use “infinity” for \(\infty\) and “-infinity” for \(-\infty\). For example, you may write \((-\infty, 5]\) for the interval \((-\infty, 5].\)

\[\begin{align*}
-2 & \leq x \leq 1 \quad \text{_____}
\end{align*}\]
This exercise concerns with operations with inequality and interval notations.

33. (1 pt) setAlgebra01RealNumbers/ur

Match the sets and the inequalities by placing the letter of the inequality next to each set listed below:

1. All $x$ in the interval $(-5, 13]$  
2. $x$ is greater than $-5$  
3. All $x$ in the interval $(-5, 13)$  
4. All $x$ in the interval $[-5, 13]$  
5. $x$ is nonnegative

A. $-5 < x$  
B. $x \geq 0$  
C. $-5 < x \leq 13$  
D. $-5 \leq x < 13$  
E. $-5 < x < 13$

34. (1 pt) setAlgebra01RealNumbers/ur_ab_10_1.png

Match each interval below with set-builder notation for the same interval.

1. $[3, 6)$  
2. $(3, 6)$  
3. $(-\infty, 3)$  
4. $[3, 6]$  
5. $[3, \infty)$

A. $\{x | x \leq 3\}$  
B. $\{x | 3 \leq x\}$  
C. $\{x | 3 < x < 6\}$  
D. $\{x | x \leq 3 \}$  
E. $\{x | 3 \geq x < 6\}$

35. (1 pt) setAlgebra01RealNumbers/ur_ab_8_2.png

The interval described in set-builder notation by the inequality $|3x - 9| < 15$ has interval notation $(a, b)$ for

$a =$  
and  
$b =$

36. (1 pt) setAlgebra01RealNumbers/ur_ab_10_2.png

Let $S = [-6, 3)$, $T = [1, 5]$, and $W = (-\infty, -3)$.  
For each intersection or union, choose the correct notation for the resulting interval.

1. $S \cap W$  
2. $S \cup W$  
3. $T \cap W$  
4. $S \cap T$

A. $[-6, -3)$  
B. $\emptyset$  
C. $(-\infty, 3)$  
D. $[1, 3]$

37. (1 pt) setAlgebra01RealNumbers/ur_ab_10_3.png

Let $S = (-\infty, \infty)$, $T = (-\infty, -3]$, and $W = [-9, -3)$.  
For each intersection or union, choose the correct notation for the resulting interval.

1. $T \cup W$  
2. $S \cap W$  
3. $S \cup W$  
4. $S \cap T$

A. $(-\infty, \infty)$  
B. $(-6, -3)$  
C. $[-9, -\infty)$  
D. $(-\infty, -3]$

38. (1 pt) setAlgebra01RealNumbers/srw1_8_1.png

Match the statements defined below with the letters labeling their equivalent expressions.

You must get all of the answers correct to receive credit.

1. $|x - 3| < 8$  
2. $|x - 3| < \infty$  
3. $|x - 3| \geq 8$  
4. $|x - 3| = 8$  
5. $|x - 3| \leq 8$

A. $x \in (-5, 11)$  
B. $x \in (-5, 11]$  
C. $x \in [-5, 11)$  
D. $x \in (-\infty, \infty)$  
E. $x \in (-\infty, -5] \cup [11, \infty)$

39. (1 pt) setAlgebra01RealNumbers/ur_ab_8_3.png

Let $S$ be the union of the two intervals $(-\infty, -1]$ and $[19, \infty)$.  
Then $S$ can also be described in set-builder notation by the inequality $|x - a| \geq b$ for

$a =$  
and  
$b =$
1. Evaluate the expression $-5^2$.

2. Evaluate the expression $4^3 + 4^4$.

3. Evaluate the expression $2^{-3} \cdot 5^4$.

4. Evaluate the expression $\left( \frac{3}{-3} \right)^2$.

5. Evaluate the expression $\frac{3^0}{3^3}$.

6. Evaluate the expression $\frac{3^2}{3^{-2}}$.

7. Evaluate the expression $\sqrt[4]{\frac{2\cdot3^2}{2^{-3}}}$ equals $2^n$ where $n$ is:

8. The expression $(3a^2b^3c^2)^2 (2a^3b^2c^3)^3$ equals $na^b c^d$ where $n$, the leading coefficient, is: ______ and $r$, the exponent of $a$, is: ______ and $s$, the exponent of $b$, is: ______ and finally $t$, the exponent of $c$, is: ______

9. The expression $\left( \frac{x^3y^2z^4}{x^5y^2z^3} \right)^{-2}$ equals $x^r y^s z^t$ where $r$, the exponent of $x$, is: ______ and $s$, the exponent of $y$, is: ______ and finally $t$, the exponent of $z$, is: ______

10. Find $x$ if $\frac{(3.8)^4 (3.8)^0}{(3.8)^{-1}} = (3.8)^9$.

11. Evaluate the expression $(-2)^6$.


13. Evaluate the expression $(-4)^0$.

14. The expression $4^3 3^{-2}$ equals $n / d$ where the numerator $n$ is ______ and the denominator $d$ is ______.

15. Evaluate the expression $10^5 / 10^3$.

16. Evaluate the expression $(2^3 \cdot 2^2)^2$.

17. The expression $\frac{x^3 (2x)^7}{x^4}$ equals $c x^e$ where the coefficient $c$ is ______ and the exponent $e$ is ______.

18. The expression $x^2 \left( \frac{1}{9} x^2 \right)^5 (54x^{-9})$ equals $c x^e$ where the coefficient $c$ is ______ and the exponent $e$ is ______.

19. The expression $(rs)^{-2} (2s)^4 (2r)^6$ equals $c r^e s^d$ where the coefficient $c$ is ______ the exponent $e$ of $r$ is ______ the exponent of $s$ is ______.

20. The expression $\frac{(6y)^3}{2y^2}$ equals $c y^e$ where the coefficient $c$ is ______ the exponent $e$ of $y$ is ______.
21. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.27.pg
The expression \(\frac{(x^3)^3(x^3)^{-3}}{x^3y^2}\)
equals \(x^c / y^d\) where
the exponent \(c\) of \(x\) is _____, the exponent \(d\) of \(y\) is _____

22. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.29.pg
The expression \(\frac{(x^3y^2z^5)^6}{(x^3z^2)^3}\)
equals \(y^r / x^s\) where
\(r\), the exponent of \(y\), is: _____
\(s\), the exponent of \(z\), is: _____
\(t\), the exponent of \(x\), is: _____
[NOTE: Your answers cannot be algebraic expressions.]

23. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.31.pg
The expression \(\frac{(x^{-3}y^{3}z^{-5})^{-1}}{(y^{-2}z^{-2}x^{6})^{-1}}\)
equals \(z^c / (x^b y^a)\) where
\(r\), the exponent of \(z\), is: _____
\(s\), the exponent of \(x\), is: _____
\(t\), the exponent of \(y\), is: _____
[NOTE: Your answers cannot be algebraic expressions.]

24. (1 pt) setAlgebra02ExponentsRadicals/Test1_5.pg
The expression \((4b^3c^{-6})^{-6}(3b^6d^{-6})^{-1}\) equals \(na^b c^d\)
where \(n\), the leading coefficient, is: _____
and \(r\), the exponent of \(a\), is: _____
and \(s\), the exponent of \(b\), is: _____
and finally \(t\), the exponent of \(c\), is: _____
[NOTE: Your answers cannot be algebraic expressions.]

25. (1 pt) setAlgebra02ExponentsRadicals/Test1_6.pg
The expression \((xy^{-3}z^{-6})^2\) equals \(x^e y^f z^g\)
where \(r\), the exponent of \(x\), is: _____
and \(s\), the exponent of \(y\), is: _____
and finally \(t\), the exponent of \(z\), is: _____
[NOTE: Your answers cannot be algebraic expressions.]

26. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.29.pg
Evaluate the expression \(\sqrt{-32}\)
[NOTE: Your answer cannot be an algebraic expression.]

27. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.90.pg
Evaluate the expression \(\sqrt{18 + 18}\)
[NOTE: Your answer cannot be an algebraic expression.]

28. (1 pt) setAlgebra02ExponentsRadicals/swr1_2.17.pg
Evaluate the expression \(\sqrt{15 \cdot 27}\)
[NOTE: Your answer cannot be an algebraic expression.]

29. (1 pt) setAlgebra02ExponentsRadicals/swr1_2.26.pg
Evaluate the expression \(125^{-4/3}\)
[NOTE: Your answer cannot be an algebraic expression.]

30. (1 pt) setAlgebra02ExponentsRadicals/swr1_2.88.pg
The expression \(\sqrt[3]{x^3y^3} \cdot \sqrt[3]{x^3y^3} \sqrt[3]{x^3}\) \(x^y\)
where \(r\), the exponent of \(x\), is: _____
and \(s\), the exponent of \(y\), is: _____

31. (1 pt) setAlgebra02ExponentsRadicals/swr1_2.88-sol.pg
The expression \(\sqrt[3]{x^3y^3} \cdot \sqrt[3]{x^3y^3} \sqrt[3]{x^3}\) \(x^y\)
where \(r\), the exponent of \(x\), is: _____
and \(s\), the exponent of \(y\), is: _____

32. (1 pt) setAlgebra02ExponentsRadicals/swr1_2.91.pg
The expression \(\sqrt[3]{729x^2}\) \(nx^y\)
where \(n\), the leading coefficient, is: _____
and \(r\), the exponent of \(x\), is: _____

33. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.7a.pg
Evaluate the expression \(\frac{\sqrt[3]{200}}{\sqrt[3]{8}}\)
Your answer is _____

34. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.7b.pg
Evaluate the expression \(\frac{\sqrt[3]{27}}{\sqrt[3]{3}}\)
Your answer is _____

35. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.7c.pg
The expression \(\frac{\sqrt[3]{25}}{\sqrt[3]{64}}\)
equals \(\frac{9}{64}\)
[NOTE: Your answer cannot be an algebraic expression.]

36. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.9a.pg
The expression \(\frac{\sqrt[3]{216}}{\sqrt[3]{343}}\)
equals \(\frac{9}{36}\)

37. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.9b.pg
The expression \(\frac{\sqrt[3]{216}}{\sqrt[3]{343}}\)
equals \(\frac{9}{36}\)

38. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.9c.pg
The expression \(\frac{\sqrt[3]{9}}{\sqrt[3]{36}}\)
equals \(\frac{325}{\sqrt{80}}\)

39. (1 pt) setAlgebra02ExponentsRadicals/swr1_3.11.pg
The expression \(\sqrt[3]{125} - \sqrt[3]{80}\)
If you rationalize the denominator of \( \frac{1}{\sqrt{11} - 2\sqrt{3}} \), then you will get \( \frac{r\sqrt{3} + s}{n} \), where \( r, s, \) and \( n \) are all positive integers (with no common factors).

\[ r = \quad s = \quad n = \]

[NOTE: Your answers cannot be algebraic expressions.]

If you rationalize the denominator of \( \frac{1}{\sqrt{11x + 1} + \sqrt{x} + 1} \), then you will get \( \frac{A}{B} \), where \( A = \quad \) and \( B = \quad \)

If you rationalize the numerator of \( \sqrt{x^2} + 1 \), then you will get \( \frac{A}{B} \), where \( A = \quad \) and \( B = \quad \)

The expression \( x^{2/3} \cdot x^{3/6} \) equals \( x^r \) where \( r, \) the exponent of \( x, \) is: \( \quad \)

The expression \( (9b)^{\sqrt{2}} (6b^{1/6}) \) equals \( nb^{r} \) where \( n, \) the coefficient, is: \( \quad \) \( r, \) the exponent of \( b, \) is: \( \quad \)

The expression \( (c^3d^6)^{-1/6} \) equals \( 1/(c^{r}d^{s}) \) where \( r, \) the exponent of \( c, \) is: \( \quad \) \( s, \) the exponent of \( d, \) is: \( \quad \)

The expression \( (\sqrt[3]{5})^{1/4} \) equals \( y^r \) where \( r, \) the exponent of \( y, \) is: \( \quad \)

The expression \( (3x^4y^{-5/6})^5 (5y^4)^{3/4} \) equals \( nx^r/y^t \) where \( n, \) the coefficient, is: \( \quad \) \( r, \) the exponent of \( x, \) is: \( \quad \) \( t, \) the exponent of \( y, \) is: \( \quad \)

The expression \( (\sqrt[5]{3})^{4/5} \) equals \( na^r/b^t \) where \( n, \) the coefficient, is: \( \quad \) \( a, \) the exponent of \( t, \) is: \( \quad \) \( b, \) the exponent of \( t, \) is: \( \quad \)

The expression \( (9s^{-4}t^3/2) (27s^6t^{-6})^{3/2} \) equals \( nt^a/s^b \) where \( n, \) the coefficient, is: \( \quad \) \( a, \) the exponent of \( t, \) is: \( \quad \) \( b, \) the exponent of \( s, \) is: \( \quad \)

The expression \( \sqrt[4]{x^4} = x^r \) where \( x \) is a non-negative real number. \( r, \) the exponent of \( x, \) is: \( \quad \)

The expression \( \sqrt{x^3y^4} = x^r y^t \) where \( x \) and \( y \) are non-negative real numbers. \( r, \) the exponent of \( x, \) is: \( \quad \) \( s, \) the exponent of \( y, \) is: \( \quad \)

The expression \( \sqrt[6]{a^5b^2} \)
equals \( a^r b^s \) where 
\[ r, \text{ the exponent of } x, \text{ is: } \quad \] 
\[ s, \text{ the exponent of } y, \text{ is: } \quad \]

54. \( \text{(1 pt) setAlgebra02ExponentsRadicals/sw1_3_57a.pg} \)
Rationalize the denominator of expression 
\[ \frac{1}{\sqrt{2}} \]
i.e., write it in the form of 
\[ \frac{\sqrt{a}}{b} \].

Your answer for \( a \) is: \quad \]
Your answer for \( b \) is: \quad \]

55. \( \text{(1 pt) setAlgebra02ExponentsRadicals/sw1_3_57b.pg} \)
Rationalize the denominator of expression 
\[ \sqrt[3]{\frac{x}{5y}} \]
i.e., write the expression in the form of 
\[ \frac{\sqrt[3]{A}}{B} \].

Your answer for \( A \) is: \quad \]
Your answer for \( B \) is: \quad \]

56. \( \text{(1 pt) setAlgebra02ExponentsRadicals/sw1_3_57c.pg} \)
Rationalize the denominator of expression 
\[ \sqrt[3]{\frac{3}{9}} \]
i.e., write it in the form of 
\[ \frac{\sqrt[3]{a}}{b} \].

Your answer for \( a \) is: \quad \]
Your answer for \( b \) is: \quad \]

57. \( \text{(1 pt) setAlgebra02ExponentsRadicals/sw1_3_59a.pg} \)
Rationalize the denominator of expression 
\[ \frac{1}{\sqrt[3]{x}} \]
i.e., write the expression in the form of 
\[ \frac{\sqrt[3]{f(x)}}{g(x)} \].

Your answer for the function \( f(x) \) is: \quad \]
Your answer for the function \( g(x) \) is: \quad \]

58. \( \text{(1 pt) setAlgebra02ExponentsRadicals/sw1_3_59b.pg} \)
Rationalize the denominator of expression 
\[ \frac{1}{\sqrt{x^2}} \]
i.e., write the expression in the form of 
\[ \frac{\sqrt[3]{f(x)}}{g(x)} \].

Your answer for the function \( f(x) \) is: \quad \]
Your answer for the function \( g(x) \) is: \quad \]

59. \( \text{(1 pt) setAlgebra02ExponentsRadicals/sw1_3_59c.pg} \)
Rationalize the denominator of expression 
\[ \frac{1}{\sqrt[3]{x^2}} \]
i.e., write the expression in the form of 
\[ \frac{\sqrt[3]{f(x)}}{g(x)} \].

Your answer for the function \( f(x) \) is: \quad \]
Your answer for the function \( g(x) \) is: \quad \]

60. \( \text{(1 pt) setAlgebra02ExponentsRadicals/Test1_7.pg} \)
The expression \( \sqrt[3]{15625g^2y^2} \) equals \( kx^r y^s \) where \( r, \text{ the exponent of } g, \text{ is: } \quad \] 
and \( s, \text{ the exponent of } y, \text{ is: } \quad \] 
and \( k, \text{ the leading coefficient is: } \quad \]

61. \( \text{(1 pt) setAlgebra02ExponentsRadicals/Test1_10.pg} \)
The expression \( \sqrt[8]{40} \)
equals \( \) \quad \]

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1. (1 pt) setAlgebra03Expressions/sw1_3_1.pg
The expression \(2(2 - 4x) + 6(x - 2)\) equals \(Ax + B\)
where \(A\) equals: 
and \(B\) equals: 
[NOTE: Your answers cannot be algebraic expressions.]

2. (1 pt) setAlgebra03Expressions/sw1_3_6.pg
The expression \(3(5x^2 + 7x + 7) - 7(7x^2 + 7x + 3)\) equals 
\[\boxed{6x^2 + \boxed{7x} + \boxed{9}}\]

3. (1 pt) setAlgebra03Expressions/Test1_11-12.pg
Given 
\[P = 2b^3 + 7b - 6,\]
\[Q = b^2 - 9b + 3,\]
\[R = b^3 + 5\]
Then \(P + Q = \boxed{b^3} + \boxed{b^2} + \boxed{b} + \boxed{b}\)
and \(R(P + Q) = \boxed{b^6} + \boxed{b^5} + \boxed{b^4} + \boxed{b^3} + \boxed{b^2} + \boxed{b} + \boxed{b}\)

4. (1 pt) setAlgebra03Expressions/sw1_3_13.pg
The expression \((2x + 2)(6x - 6)\) equals \(Ax^2 + Bx + C\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 

5. (1 pt) setAlgebra03Expressions/sw1_3_16.pg
The expression \((5t - 4)(6t + 2) + 5t - 4\) equals \(At^2 + Bt + C\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 

6. (1 pt) setAlgebra03Expressions/sw1_3_21.pg
The expression \((4\sqrt{x} + 5\sqrt{y})(4\sqrt{x} - 5\sqrt{y})\) equals \(Ax + By\)
where \(A\) equals: 
and \(B\) equals: 

7. (1 pt) setAlgebra03Expressions/sw1_3_22.pg
The expression \((2x + 5)^2\) equals \(Ax^2 + Bx + C\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 

8. (1 pt) setAlgebra03Expressions/sw1_3_22a.pg
The expression \((x - 6)(x^2 + 4x + 3)\) equals \(Ax^3 + Bx^2 + Cx + D\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 
and \(D\) equals: 

9. (1 pt) setAlgebra03Expressions/sw1_4_1.pg
The expression \((2x^2 + 4x + 2) + (7x^2 - 7x - 3)\) equals 
\[\boxed{\boxed{9x^2} + \boxed{\boxed{5x} + \boxed{1}}}\]

10. (1 pt) setAlgebra03Expressions/sw1_4_3.pg
The expression \((4x^3 + 6x^2 - 5x + 4) - (4x^2 + 2x - 3)\) equals 
\[\boxed{-\boxed{4x^3} + \boxed{\boxed{-3x^2} + \boxed{-3x} + \boxed{-7}}}\]

11. (1 pt) setAlgebra03Expressions/sw1_4_5.pg
The expression \(5(x + 6) - 7(7x - 7)\) equals 
\[\boxed{-\boxed{x} + \boxed{\boxed{3}}}\]

12. (1 pt) setAlgebra03Expressions/sw1_4_9.pg
The expression \(\sqrt{x}(5x - 7\sqrt{x})\) equals \(Ax^{3/2} + Bx\)
where \(A\) equals: 
and \(B\) equals: 

13. (1 pt) setAlgebra03Expressions/sw1_4_13.pg
The expression \((5t - 2)(7t - 5)\) equals \(At^2 + Bt + C\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 

14. (1 pt) setAlgebra03Expressions/sw1_4_15.pg
The expression \((4x + 6)(7x - 2)\) equals \(Ax^2 + Bx + C\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 

15. (1 pt) setAlgebra03Expressions/sw1_4_17.pg
The expression \((2 - 5x)^2\) equals \(Ax^2 + Bx + C\)
where \(A\) equals: 
and \(B\) equals: 
and \(C\) equals: 

16. (1 pt) setAlgebra03Expressions/lhp3_26.pg
The expression \(4(9x^2 - 5x + 9) - (2x^2 + 9x - 8)\) equals 
\[\boxed{\boxed{-6x^2} + \boxed{\boxed{-12x} + \boxed{13}}}\]

17. (1 pt) setAlgebra03Expressions/lhp3_56.pg
The expression \((2x - 4)(9x + 6)\) equals 
\[\boxed{\boxed{\boxed{\boxed{\boxed{\boxed{2x^2}}}}}} + \boxed{\boxed{\boxed{\boxed{\boxed{\boxed{\boxed{-6x}}} + \boxed{\boxed{\boxed{\boxed{\boxed{\boxed{18}}}}}}}}}}\]

18. (1 pt) setAlgebra03Expressions/lhp3_58.pg
The expression \((3x + 2)^2\) equals 
\[\boxed{\boxed{\boxed{\boxed{\boxed{\boxed{9x^2}}}}}} + \boxed{\boxed{\boxed{\boxed{\boxed{\boxed{\boxed{12x}}} + \boxed{\boxed{\boxed{\boxed{\boxed{\boxed{4}}}}}}}}}}\]

19. (1 pt) setAlgebra03Expressions/lhp3_62.pg
The expression \((3x - 9)(3x + 9)\) equals 
\[\boxed{\boxed{\boxed{\boxed{\boxed{\boxed{9x^2}}}}}} + \boxed{\boxed{\boxed{\boxed{\boxed{\boxed{\boxed{-81}}}}}}}

20. (1 pt) setAlgebra03Expressions/Test1_13.pg
The expression \((2\sqrt{x} + 2\sqrt{y})(2\sqrt{x} - 2\sqrt{y})\) equals 

21. (1 pt) setAlgebra03Expressions/Test1_19.pg
A square rug lies in the middle of a rectangular room. There are 6 feet of uncovered floor on 2 sides of the rug and 3 feet of uncovered floor on the other 2 sides. Find a polynomial expression for the area of the room in terms of \(x\), the side length of the rug. The area of the room is 

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1. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_43.png
Factor the polynomial \( x^2 + 7x + 12 \). Your answer can be written as \((x + A)(x + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

2. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_53.png
Factor the polynomial \( x^2 + 7x + 12 \). Your answer can be written as \((x + A)(x + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

3. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_53a.png
Factor the polynomial \( x^2 + 4x - 21 \). Your answer can be written as \((x + A)(x + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

4. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_54.png
Factor the polynomial \( 20x^2 + 33x + 10 \). Your answer can be written as \((5x + B)(Cx + D)\) with \( B, C, \) and \( D \)-integers
where \( B \) equals: 
and \( C \) equals: 
and \( D \) equals: 

5. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_55.png
Factor the polynomial \( x^4 + 11x^2 + 28 \). Your answer can be written as \((x^2 + A)(x^2 + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

6. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_56.png
Factor the polynomial \( t^7 + 7t^6 - 8t^5 \). Your answer can be written as \(t^N(t + A)(t + B)\) where \( A < B \).
\( N \) equals: 
and \( A \) equals: 
and \( B \) equals: 

7. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_57.png
Factor the polynomial \( x^3 - 27 \). Your answer can be written as \((x - A)(x^2 + Bx + C)\)
where \( A \) equals: 
and \( B \) equals: 
and \( C \) equals: 

8. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_41.png
Factor the polynomial \( 7y^3 - 6y^2 \). Your answer can be written as \(y^a(Ay^b + B)\)
where \( a \) equals: 
and \( A \) equals: 
and \( b \) equals: 
and \( B \) equals: 

9. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_43.png
Factor the polynomial \( x^3 + 6x + 5 \). Your answer can be written as \((x + A)(x + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

10. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_45.png
Factor the polynomial \( x^2 - 6x - 16 \). Your answer can be written as \((x + A)(x + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

11. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_47.png
Factor the polynomial \( x^2 - 8x + 12 \). Your answer can be written as \((x + A)(x + B)\) where \( A < B \)
and \( A \) equals: 
and \( B \) equals: 

12. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_49.png
Factor the polynomial \( 10x^2 + 33x + 20 \). Your answer can be written as \((5x + B)(Cx + D)\) with \( B, C, \) and \( D \)-integers
where \( B \) equals: 
and \( C \) equals: 
and \( D \) equals: 

13. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_51.png
Factor the polynomial \( 3x^2 - 12 \). Your answer can be written as \(A(x + B)(x + C)\) with integers \( A, B, C, \) and \( B < C \)
where \( A \) equals: 
and \( B \) equals: 
and \( C \) equals: 

14. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_53.png
Factor the polynomial \( 10x^3 + 19x - 15 \). Your answer can be written as \((5x - B)(Cx + D)\) with \( B, C, \) and \( D \)-integers
where \( B \) equals: 
and \( C \) equals: 
and \( D \) equals: 

15. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_55.png
Factor the polynomial \((x - 1)(x + 4)^2 - (x - 1)^2(x + 4)\). Your answer can be written as \(A(x + B)(x + C)\) with integers \( A, B, \) and \( C \)
where \( A \) equals: 
and \( B \) equals: 
and \( C \) equals: 

16. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_57.png
Factor the polynomial \(y^5(y + 3)^5 + y^6(y + 3)^6\). Your answer can be written as \(y^r(y + A)^s(y^a + By + C)\) with \( r, s, A, B, \) and \( C \)-integers
where \( r \) equals: 
and \( s \) equals: 
and \( A \) equals: 
and \( B \) equals: 
and \( C \) equals: 

17. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_63.png
Factor the polynomial \(4x^2 - 28x + 49\). Your answer can be written as \((Ax - B)^r\) with \( A, B, \) and \( r \) being integers.
\( A \) equals: 
\( B \) equals: 
and \( r \) equals: 

18. (1 pt) setAlgebra04ExpressionsFactoring/sw1_1_69.png
Factor the polynomial \(t^3 + 2r^3 - 15t^2\). Your answer can be written as \(t^N(t + A)(t + B)\) where \( A < B \).
\( N \) equals: 
\( A \) equals: 
and \( B \) equals: 

19. (1 pt) setAlgebra04ExpressionsFactoring/sw1_4_73.pg
Factor the polynomial $x^4 + 5x^2 - 6$. Your answer can be written as $(x + A)(x + B)(x^2 + C)$ with integers $A$, $B$, $C$, and $A < B$ where $A = _____$ $B = _____$ and $C = _____$

20. (1 pt) setAlgebra04ExpressionsFactoring/sw1_4_75.pg
Factor the polynomial $x^4 - 27$. Your answer can be written as $(x - A)(x^3 + Bx + C)$ where $A = _____$ and $B = _____$ and $C = _____$

Factor $9 - 9x^2$
$9 - 9x^2 = (A - Bx)(C + Dx)$
where $A = _____$ $B = _____$ $C = _____$ and $D = _____$

22. (1 pt) setAlgebra04ExpressionsFactoring/lhp4_29-34.pg
Factor the trinomial $16x^2 - 56x + 49$
$16x^2 - 56x + 49 = (Ax - B)^2$
where $A$ is ____ and $B$ is ____

23. (1 pt) setAlgebra04ExpressionsFactoring/lhp4_43-50.pg
Factor the trinomial $x^2 - 13x + 40$
$x^2 - 13x + 40 = (x - A)(x - B)$
where $A = _____$ and $B = _____$ with $A < B$.

24. (1 pt) setAlgebra04ExpressionsFactoring/lhp4_51-54.pg
Factor the trinomial $10x^2 - 39x + 35$
$10x^2 - 39x + 35 = (Ax - B)(Cx - D)$
where $A = _____$ $B = _____$ $C = _____$ and $D = _____$ with $A < C$ and $B < D$.

25. (1 pt) setAlgebra04ExpressionsFactoring/Test1_15.pg
The polynomial $27x^{12} - 125y^{27}$ can be factored into the product of two polynomials, $A \cdot B$ where the degree of $A$ is greater than the degree of $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

26. (1 pt) setAlgebra04ExpressionsFactoring/Test1_16.pg
The polynomial $4x^{12} - 1y^3$ can be factored into the product of two polynomials, $A \cdot B$ where the coefficient of $y$ in $A$ is greater than the coefficient of $y$ in $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

27. (1 pt) setAlgebra04ExpressionsFactoring/Test1_17.pg
The polynomial $25x^6 + 90x^3y^7 + 81y^{14}$ can be factored into the product of two polynomials, $A \cdot B$ where the coefficient of $y$ in $A$ is greater than or equal to the coefficient of $y$ in $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

28. (1 pt) setAlgebra04ExpressionsFactoring/Test1_18.pg
The polynomial $36a^{14}b + 60a^{14}b + 25b^2 - 4c^2$ can be factored into the product of two polynomials, $A \cdot B$ where the coefficient of $c$ in $A$ is less than the coefficient of $c$ in $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

29. (1 pt) setAlgebra04ExpressionsFactoring/factoring_hard.pg
The polynomial $25x^3 + 35x^2 + 35x + 49$ can be factored into the product of two polynomials, $A \cdot B$ where the degree of $A$ is greater than the degree of $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

30. (1 pt) setAlgebra04ExpressionsFactoring/factor_hard2.pg
The polynomial $49x^3 + 294x^2 - 16x - 96$ can be factored into the product of three polynomials, $A \cdot B \cdot C$ where the constant term of $A$ is less than or equal to the constant term of $B$ which is less than or equal to the constant term of $C$. Find $A$, $B$, and $C$.
$A = _____$
$B = _____$
$C = _____$

31. (1 pt) setAlgebra04ExpressionsFactoring/factor_hard3.pg
The polynomial $25x^3 - 16xy^2 + 125x^2 - 80y^2$ can be factored into the product of three polynomials, $A \cdot B \cdot C$ where the coefficient of $y$ in $A$ is less than the coefficient of $y$ in $B$ which is less than the coefficient of $y$ in $C$. Find $A$, $B$, and $C$.
$A = _____$
$B = _____$
$C = _____$

32. (1 pt) setAlgebra04ExpressionsFactoring/factor_hard4.pg
The polynomial $64a^2 - 80ab + 25b^2 - 64$ can be factored into the product of two polynomials, $A \cdot B$ where the constant term in $A$ is less than the constant term in $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

33. (1 pt) setAlgebra04ExpressionsFactoring/factor_hard5.pg
The polynomial $16a^3 + 72a^2b + 81b^2 - 81c^2$ can be factored into the product of two polynomials, $A \cdot B$ where the coefficient of $c$ in $A$ is less than the coefficient of $c$ in $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

34. (1 pt) setAlgebra04ExpressionsFactoring/factor1.pg
The polynomial $8x^3 - 125y^3$ can be factored into the product of two polynomials, $A \cdot B$ where the degree of $A$ is greater than the degree of $B$. Find $A$ and $B$.
$A = _____$
$B = _____$

35. (1 pt) setAlgebra04ExpressionsFactoring/factor2.pg
The polynomial $64x^3 + 72x^2 + 24x + 27$ can be factored into the product of two polynomials, $A \cdot B$ where the degree of $A$ is greater than the degree of $B$. Find $A$ and $B$.
$A = _____$
$B = _____
1. (1 pt) setAlgebra05RationalExpressions/srw1_l_1.pg
Match the expressions below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

\[ \frac{x^2 + 16x + 60}{x^2 + 7x - 30} \]
\[ \frac{x^2 - 6x - 7}{x^2 - 2x - 3} \]
\[ \frac{x^2 + 7x + 6}{x^2 - 6x - 7} \]

A. \( x - 7 \)
B. \( x + 6 \)
C. \( x - 3 \)

2. (1 pt) setAlgebra05RationalExpressions/srw1_l_7.pg
Match the expressions below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

\[ \frac{x^2 - 4}{x^2 + 4 + 4} \]
\[ \frac{x^2 + 4}{x^2 - 4} \]
\[ \frac{x^2 - 8}{x^2 + 2} \]

A. \( x - 2 \)
B. \( x + 2 \)
C. \( x^2 + 2x + 4 \)

3. (1 pt) setAlgebra05RationalExpressions/srw1_l_13.pg
Match the expressions below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

\[ \frac{x^4}{x^6 + 6x + 5} \]
\[ \frac{x^4}{x^6 + 6x + 5} \]
\[ \frac{x^4}{x^6 + 6x + 5} \]
\[ \frac{x^4}{x^6 + 6x + 5} \]

A. \( x^4(x + 1) \)
B. \( \frac{1}{x^4(x + 1)} \)
C. \( \frac{x^4}{x + 1} \)
D. \( \frac{x^4}{x + 1} \)

4. (1 pt) setAlgebra05RationalExpressions/srw1_4_20.pg
Match the expressions below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

\[ \frac{1}{x^4} + \frac{1}{x - 2} \]
\[ \frac{1}{x^4} + \frac{1}{x + 2} \]
\[ \frac{1}{x^4} + \frac{1}{x - 2} \]

A. \( \frac{(x - 2)(x + 7)}{2x + 9} \)
B. \( \frac{(x + 2)(x + 7)}{-9} \)
C. \( \frac{(x - 2)(x + 7)}{2x + 5} \)

5. (1 pt) setAlgebra05RationalExpressions/srw1_4_29.pg
Match the expressions below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

\[ \frac{1}{x - 2} - \frac{1}{x^2 - 4} \]
\[ \frac{1}{x + 2} + \frac{1}{x^2 - 4} \]
\[ \frac{1}{x + 2} + \frac{1}{x^2 + 4} \]

A. \( \frac{x + 1}{x^2 - 4} \)
B. \( \frac{x^2 + x + 6}{(x + 2)(x^2 + 4)} \)
C. \( \frac{x - 1}{x^2 - 4} \)

6. (1 pt) setAlgebra05RationalExpressions/srw1_4_35-37.pg
Match the expressions below with the letters labeling their equivalent expressions. You must get all of the answers correct to receive credit.

\[ \frac{y}{x^2} - \frac{y}{x^2} \]
\[ \frac{x + 2}{x - y^2} \]
\[ \frac{x - y}{x - y} \]

A. \( -xy \)
B. \[ \frac{x^3}{x^2 + y^2} \]
C. \[ \frac{y}{y - 2} \]

7. (1 pt) setAlgebra05RationalExpressions/sw1_4_30.pg
Match the expressions below with the letters labeling their equivalent expressions.
You must get all of the answers correct to receive credit.

1. \[ \sqrt{x + h} + \sqrt{x} \]
2. \[ \frac{h}{\sqrt{x + h} - \sqrt{x}} \]
A. \[ \frac{1}{\sqrt{x + h} - \sqrt{x}} \]
B. \[ \frac{1}{\sqrt{x + h} + \sqrt{x}} \]

8. (1 pt) setAlgebra05RationalExpressions/sw1_4_55-59.pg
Enter a T or an F in each answer space below to indicate whether the corresponding equation is true or false. An equation is true only if it is true for all values of the variables. Disregard values that make denominators 0.
You must get all of the answers correct to receive credit.

1. \[ \frac{42}{75 + x} = \frac{42}{x + 1} \]
2. \[ \frac{x + y}{75} = \frac{1 + y}{75} \]
3. \[ \frac{75 - c}{75} = \frac{1 - 75}{c} \]
4. \[ \frac{42 + a}{42} = \frac{1 + a}{42} \]

9. (1 pt) setAlgebra05RationalExpressions/sw1_4_60-64.pg
Enter a T or an F in each answer space below to indicate whether the corresponding equation is true or false. An equation is true only if it is true for all values of the variables. Disregard values that make denominators 0.
You must get all of the answers correct to receive credit.

1. \[ \frac{x^2 + 1}{x + 1} = x - 1 \]
2. \[ \frac{86}{86b} = \frac{a}{b} \]
3. \[ \frac{86 - 41x + 90x^2}{x} = \frac{86}{x} - 41 + 90x \]
4. \[ \frac{-41a}{b} = \frac{-41a}{b} \]

10. (1 pt) setAlgebra05RationalExpressions/lhp5_15-24.pg
Simplify the expression
\[ \frac{4y^6}{xy - 8y} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)} \]

11. (1 pt) setAlgebra05RationalExpressions/lhp5_25-32.pg
Simplify the expression
\[ \frac{x^2 - 6x + 8}{x^2 - 3x + 2} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)} \]
Your answer for the function \( f(x) \) is : 
Your answer for the function \( g(x) \) is :

12. (1 pt) setAlgebra05RationalExpressions/lhp5_55-56.pg
Simplify the expression
\[ \frac{2x - 3}{x - 2} + \frac{3 - x}{x - 2} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)} \]
Your answer for the function \( f(x) \) is :
Your answer for the function \( g(x) \) is :

13. (1 pt) setAlgebra05RationalExpressions/lhp5_57-58.pg
Simplify the expression
\[ \frac{1}{x - 2} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)} \]
Your answer for the function \( f(x) \) is :
Your answer for the function \( g(x) \) is :

14. (1 pt) setAlgebra05RationalExpressions/lhp5_60.pg
Simplify the expression
\[ \frac{1}{x + 4} - \frac{1}{x + 6} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)} \]
Your answer for the function \( f(x) \) is :
Your answer for the function \( g(x) \) is :

15. (1 pt) setAlgebra05RationalExpressions/sw1_5_3.pg
Simplify the expression
\[ \frac{x^2 - 6x + 8}{x^2 - 7x + 10} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

16. (1 pt) setAlgebra05RationalExpressions/sw1_5_5.pg
Simplify the expression
\[ \frac{y^2 + 4y}{y^2 - 16} \]
and give your answer in the form of
\[ \frac{f(y)}{g(y)}. \]
Your answer for the function \( f(y) \) is: 
Your answer for the function \( g(y) \) is: 

17. (1 pt) setAlgebra05RationalExpressions/sw1_5_7.pg
Simplify the expression
\[ \frac{3x^3 - 2x^2 - 8x}{4x^2 - 12x + 8} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

18. (1 pt) setAlgebra05RationalExpressions/sw1_5_11.pg
Simplify the expression
\[ \frac{x^2 + 5x + 6}{x^2 + 5x + 4} \cdot \frac{x^2 + 6x + 5}{x^2 + 6x + 8} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

19. (1 pt) setAlgebra05RationalExpressions/sw1_5_13.pg
Simplify the expression
\[ \frac{2x^2 + 7x + 3}{x^2 + 3x - 4} \cdot \frac{x^2 + 7x + 12}{2x^2 - 3x + 7} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

20. (1 pt) setAlgebra05RationalExpressions/sw1_5_21.pg
Simplify the expression
\[ \frac{1}{x + 1} - \frac{1}{x + 4} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

21. (1 pt) setAlgebra05RationalExpressions/sw1_5_23.pg
Simplify the expression
\[ \frac{4x}{(x + 1)^2} + \frac{1}{x + 1} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

22. (1 pt) setAlgebra05RationalExpressions/sw1_5_25.pg
Simplify the expression
\[ x + 3 + \frac{4}{x + 3} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

23. (1 pt) setAlgebra05RationalExpressions/sw1_5_27.pg
Simplify the expression
\[ \frac{2}{x^2} + \frac{4}{x^2 + x} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

24. (1 pt) setAlgebra05RationalExpressions/sw1_5_35.pg
Simplify the expression
\[ \frac{1}{x^2 + 2x + 1} - \frac{1}{x^2 - 3x - 4} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

25. (1 pt) setAlgebra05RationalExpressions/sw1_5_39.pg
Simplify the expression
\[ \frac{1}{x + 1} - \frac{1}{x + 4} \]
and give your answer in the form of
\[ \frac{f(x)}{g(x)}. \]
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 
and give your answer in the form of \( \frac{f(c)}{g(c)} \).
Your answer for the function \( f(c) \) is: 
Your answer for the function \( g(c) \) is: 

26. (1 pt) setAlgebra05RationalExpressions/sw1_5_41.pg
Simplify the expression \( \frac{5}{x+1} - \frac{1}{x+1} \)
and give your answer in the form of \( \frac{f(x)}{g(x)} \).
Your answer for the function \( f(x) \) is: 
Your answer for the function \( g(x) \) is: 

27. (1 pt) setAlgebra05RationalExpressions/sw1_5_47.pg
Simplify the expression \( \frac{2 + \pi}{\pi - \frac{2}{h}} \)
and give your answer in the form of \( \frac{A}{B} \).
Your answer for \( A \) is: 
Your answer for \( B \) is: 

28. (1 pt) setAlgebra05RationalExpressions/sw1_5_59.pg
Rationalize the denominator of expression \( \frac{2}{2 + \sqrt{5}} \)

29. (1 pt) setAlgebra05RationalExpressions/Test1_8.pg
The expression \( \sqrt[4]{16h^3s^4} \)
equals \( krs^t \)
where \( r \), the exponent of \( h \), is: 
and \( t \), the exponent of \( s \), is: 
and \( k \), the leading coefficient is: 

30. (1 pt) setAlgebra05RationalExpressions/Test1_9.pg
The expression \( 5\sqrt{\sqrt[5]{55}} \)
equals 

31. (1 pt) setAlgebra05RationalExpressions/Test1_14.pg
The expression \( \left( \frac{4}{17} \right)^3 - 6 \left( \frac{y}{13} \right)^2 \)
equals 

32. (1 pt) setAlgebra05RationalExpressions/Test2_1.pg
Write the following as a simple fraction in lowest terms.

\[ \frac{(-3)^{-4} + (-3)^{-2}}{-3^{-3}} \]

33. (1 pt) setAlgebra05RationalExpressions/Test2_2.pg
In lowest terms, the fraction \( \frac{(a + 5)^4 - (b - 5)^4}{(a + 5)^3 - (b - 5)^3} \)
can be written as \( \frac{A}{B} \) where
\[ A = \quad \text{and} \quad B = \quad \]

34. (1 pt) setAlgebra05RationalExpressions/Test2_3.pg
\[ \frac{9}{x + 8} - \frac{-1}{x - 3} + \frac{-7x + 5}{x^2 + 5x - 24} = \frac{A}{B} \]
where \( A = \quad \text{and} \quad B = \quad \)

35. (1 pt) setAlgebra05RationalExpressions/Test2_4.pg
\[ \frac{x^2 - (8x + 9)}{x^2 + 7x + 8} + \frac{-3x^2 + 9 - 5x}{-8 - 7x - x^2} = \frac{A}{B} \]
where \( A = \quad \text{and} \quad B = \quad \)

36. (1 pt) setAlgebra05RationalExpressions/Test2_5.pg
When written as a simple fraction, without negative exponents, the fraction \( \frac{-5x^{-6} - 5y^{-6}}{(x^2 + y^2)^{y^{-4}}} = \frac{A}{B} \)
where \( A = \quad \text{and} \quad B = \quad \)

37. (1 pt) setAlgebra05RationalExpressions/Test2_12.pg
The total resistance, \( R \), of a particular group is given by the formula:
\[ R = S + \left( \frac{1}{\frac{1}{T} + \frac{1}{U}} \right) \]
This formula can be simplified to the form \( \frac{A}{B} \) where \( A \) and \( B \) contain no fractions.
\[ A = \quad \text{and} \quad B = \quad \]
Suppose that \( S = 18\Omega \), \( T = 55\Omega \) and \( U = 98\Omega \).
Then \( R = \quad \text{Note: Your answer must be a decimal.} \)

38. (1 pt) setAlgebra05RationalExpressions/rational_denominator.pg
If you rationalize the denominator of \( \frac{1}{7x\sqrt{5} - 4y\sqrt{3}} \)
then you will get \( \frac{A}{B} \)
where \( A = \quad \text{and} \quad B = \quad \)
If you rationalize the numerator of\[
\frac{\sqrt{x^2 - 9} \sqrt{x} + 81}{\sqrt{x^3} + 9}
\]
then you will get \( \frac{A}{B} \) where \( A = \) _______ \\
and \( B = \) _______
1. (1 pt) setAlgebra06EqnGraphs/srw1_8_39.png
Determine whether the given points are on the graph of \( y = 2x + 3. \) Enter Yes or No for your answers:
Is \((-4, -5)\) on the graph? ________
Is \((3, 9)\) on the graph? ________
Is \((-2, -2)\) on the graph? ________
Is \((0, -2)\) on the graph? ________

2. (1 pt) setAlgebra06EqnGraphs/srw1_8_43.png
Find the \( x \)- and \( y \)-intercepts of the graph of the equation \( y = x + 2. \)
The \( x \)-intercept(s) have \( x = \) ________
Note: If there is more than one, give a comma separated list. If there are none, type none .
The \( y \)-intercept(s) have \( y = \) ________
Note: If there is more than one, give a comma separated list. If there are none, type none .

3. (1 pt) setAlgebra06EqnGraphs/srw1_8_45.png
Find the \( x \)- and \( y \)-intercepts of the graph of the equation \( y = x^2 + 3x - 54. \)
The \( x \)-intercept(s) have \( x = \) ________
Note: If there is more than one, give a comma separated list. If there are none, type none .
The \( y \)-intercept(s) have \( y = \) ________
Note: If there is more than one, give a comma separated list. If there are none, type none .

4. (1 pt) setAlgebra06EqnGraphs/srw1_8_46.png
For the graph of the equation \( y = 4x + 1 \), draw a sketch of the graph on a piece of paper. Then answer the following questions:
The \( x \)-intercept is : ________
The \( y \)-intercept is : ________
Is the graph symmetric with respect to the \( x \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the \( y \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the origin? Input yes or no here : ________

5. (1 pt) setAlgebra06EqnGraphs/srw1_8_47.png
Find the \( x \)- and \( y \)-intercepts of the graph of the equation \( x^2 + y^2 = 36. \)
The \( x \)-intercepts are : \( x_1 = \) _____, \( x_2 = \) _____ with \( x_1 \leq x_2; \)
The \( y \)-intercepts are : \( y_1 = \) _____, \( y_2 = \) _____ with \( y_1 \leq y_2. \)

6. (1 pt) setAlgebra06EqnGraphs/srw1_8_53.png
For the graph of the equation \( y = x^2 - 9 \), draw a sketch of the graph on a piece of paper. Then answer the following questions:
The \( x \)-intercepts are : \( x_1 = \) _____, \( x_2 = \) _____ with \( x_1 \leq x_2 \).
The \( y \)-intercept is : ________
Is the graph symmetric with respect to the \( x \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the \( y \)-axis? Input yes or no here : ________

7. (1 pt) setAlgebra06EqnGraphs/srw1_8_63.png
For the graph of the equation \( x = y^2 - 1 \), answer the following questions:
The \( x \)-intercept(s) is \( x = \) ________
Note: If there is more than one answer enter them separated by commas. If there are none, enter none .
The \( y \)-intercept(s) is \( y = \) ________
Note: If there is more than one answer enter them separated by commas. If there are none, enter none .
Is the graph symmetric with respect to the \( x \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the \( y \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the origin? Input yes or no here : ________

8. (1 pt) setAlgebra06EqnGraphs/srw1_8_65.png
For the graph of the equation \( x^2y^2 + xy = 6 \), answer the following questions:
Is the graph symmetric with respect to the \( x \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the \( y \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the origin? Input yes or no here : ________

9. (1 pt) setAlgebra06EqnGraphs/srw1_8_67.png
For the graph of the equation \( y = x^3 + 11x \), answer the following questions:
Is the graph symmetric with respect to the \( x \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the \( y \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the origin? Input yes or no here : ________

10. (1 pt) setAlgebra06EqnGraphs/sw2_2_11.png
Find the \( x \)- and \( y \)-intercepts of the graph of the equation \( x^2 + y^2 = 9. \)
The \( x \)-intercepts are : \( x_1 = \) _____, \( x_2 = \) _____ with \( x_1 \leq x_2; \)
The \( y \)-intercepts are : \( y_1 = \) _____, \( y_2 = \) _____ with \( y_1 \leq y_2. \)

11. (1 pt) setAlgebra06EqnGraphs/sw2_2_18.png
For the graph of the equation \( y = 12x + 2 \), draw a sketch of the graph on a piece of paper. Then answer the following questions:
The \( x \)-intercept is : ________
The \( y \)-intercept is : ________
Is the graph symmetric with respect to the \( x \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the \( y \)-axis? Input yes or no here : ________
Is the graph symmetric with respect to the origin? Input yes or no here : ________
Is the graph symmetric with respect to the x-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the y-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the origin? Input yes or no here: ____________

For the graph of the equation $x = y^2 - 4$, answer the following questions:

Note: If there is more than one answer enter them separated by commas.

The y-intercepts are $y = ____________$

Note: If there is more than one answer enter them separated by commas.

Is the graph symmetric with respect to the x-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the y-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the origin? Input yes or no here: ____________

For the graph of the equation $y = x^2 + 10x = -4$ answer the following questions.

Is the equation symmetric with respect to the y-axis? (yes or no)

Is the equation symmetric with respect to the x-axis? (yes or no)

Is the equation symmetric with respect to the origin? (yes or no)

For the equation $-7x^2 + 10y = -4$, answer the following questions.

The x-intercepts have $x = ____________$

Note: If there is more than one answer enter them separated by commas. If there are none, enter none .

The y-intercepts have $y = ____________$

Note: If there is more than one answer enter them separated by commas. If there are none, enter none .

Is the graph symmetric with respect to the x-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the y-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the origin? Input yes or no here: ____________

For the graph of the equation $y = -6|x| + 5$

answer the following questions:

What are the x-intercept(s) written as ordered pair(s)?

Note: If there is more than one write them separated by a comma (i.e.: (1,2),(3,4)). If there are none, type none in the answer blank.

x-intercept(s): ____________

What is the y-intercept written as an ordered pair?

Note: If there is more than one write them separated by a comma (i.e.: (1,2),(3,4)). If there are none, type none in the answer blank.

y-intercept: ____________

Is the graph symmetric with respect to the x-axis? (yes or no)

Is the graph symmetric with respect to the y-axis? (yes or no)

Is the graph symmetric with respect to the origin? (yes or no)

For the graph of the equation $x^3y^2 + x^4y^2 = 7$, answer the following questions:

Is the graph symmetric with respect to the x-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the y-axis? Input yes or no here: ____________
Is the graph symmetric with respect to the origin? Input yes or no here: ____________
1. (1 pt) setAlgebra07PointsCircles/sw2_1_17.pg
Find the midpoint of the segment that joins the points \((-4, -2)\) and \((-5, -5)\).
Input your answer here: (________)

2. (1 pt) setAlgebra07PointsCircles/sw2_1_11.pg
Plot the points \(A(6,0)\), \(B(10,0)\), \(C(9,2)\) and \(D(7,2)\) on a coordinate plane on a piece of paper. Draw the segments \(AB\), \(BC\), \(CD\) and \(DA\).
The quadrilateral \(ABCD\) is commonly called: _____
Its area equals: ______

3. (1 pt) setAlgebra07PointsCircles/sw2_1_17.pg
Sketch the region given by the set \(\{(x,y)\mid 5 \leq x \leq 7, -5 \leq y \leq -2\}\) on a piece of paper.
The area of the region is: ______

4. (1 pt) setAlgebra07PointsCircles/sw2_1_19.pg
Sketch the region given by the set \(\{(x,y)\mid xy < 0\}\) on a piece of paper. Which quadrants of the plane are included in the set?
Input Yes or No at the corresponding space below:
the first quadrant is included: ______
the second quadrant is included: ______
the third quadrant is included: ______
the fourth quadrant is included: ______

Be careful, you only have one chance to enter your answer!!!

5. (1 pt) setAlgebra07PointsCircles/sw2_1_25.pg
Which of the points \(A(6,7)\) or \(B(-5,8)\) is closer to the origin?
Input the corresponding letter \(A\) or \(B\) here: ______

Be careful, you only have one chance to enter your answer!!!

6. (1 pt) setAlgebra07PointsCircles/swr1_9_2.pg
Consider the two points \((5,-3)\) and \((8,10)\). The distance between them is: ______
The \(x\) co-ordinate of the midpoint of the line segment that joins them is: ______
The \(y\) co-ordinate of the midpoint of the line segment that joins them is: ______

7. (1 pt) setAlgebra07PointsCircles/swr1_9_2-sol.pg
Consider the two points \((2,-4)\) and \((7,8)\). The distance between them is: ______
The \(x\) co-ordinate of the midpoint of the line segment that joins them is: ______
The \(y\) co-ordinate of the midpoint of the line segment that joins them is: ______

8. (1 pt) setAlgebra07PointsCircles/swr1_9_4.pg
Consider the two points \((3,-1)\) and \((-4,-3)\). The distance between them is: ______
The \(x\) co-ordinate of the midpoint of the line segment that joins them is: ______
The \(y\) co-ordinate of the midpoint of the line segment that joins them is: ______

9. (1 pt) setAlgebra07PointsCircles/swr1_9_4-sol.pg
Consider the two points \((3,-3)\) and \((-6,-7)\). The distance between them is: ______
The \(x\) co-ordinate of the midpoint of the line segment that joins them is: ______
The \(y\) co-ordinate of the midpoint of the line segment that joins them is: ______

10. (1 pt) setAlgebra07PointsCircles/sApB_1_6.pg
Find the distance between \((2,5)\) and \((-5,1)\).

11. (1 pt) setAlgebra07PointsCircles/sApB_x.pg
Find the perimeter of the triangle with the vertices at \((4,1), (-6,3),\) and \((-4,-3)\).

12. (1 pt) setAlgebra07PointsCircles/sApB_x-sol.pg
Find the perimeter of the triangle with the vertices at \((5,1), (-1,6),\) and \((-3,-6)\).

13. (1 pt) setAlgebra07PointsCircles/midpoint_hard.pg
The midpoint of \(AB\) is at \((1,3)\). If \(A = (7,3)\), find \(B\).
\(B\) is: (______)

14. (1 pt) setAlgebra07PointsCircles/perimeter_of_triangle.pg
Find the perimeter of the triangle with the vertices at \((2,-2), (-3,5),\) and \((-4,-2)\).

15. (1 pt) setAlgebra07PointsCircles/ur_ab_9_2.pg
Find the point \((0,b)\) on the \(y\)-axis that is equidistant from the points \((1,1)\) and \((3,-2)\).
\(b = \) ______

16. (1 pt) setAlgebra07PointsCircles/dist_between_2_pts.pg
Find the distance between \((2,8)\) and \((-6,-2)\).

17. (1 pt) setAlgebra07PointsCircles/Manhattan.pg
Find the Manhattan distance between \((317,249)\) and \((216,-179)\).
The Manhattan distance between the given points is ______

18. (1 pt) setAlgebra07PointsCircles/dist_midpoint.pg
Consider the two points \((0,0)\) and \((-1,6)\). The distance between them is: ______
The \(x\) co-ordinate of the midpoint of the line segment that joins them is: ______
The \(y\) co-ordinate of the midpoint of the line segment that joins them is: ______

19. (1 pt) setAlgebra07PointsCircles/equidist_off_axis.pg
Find the point \((x,y)\) on the line \(y = x\) that is equidistant from the points \((-10,8)\) and \((-5,-1)\).
\(x = \) ______
\(y = \) ______
20. (1 pt) setAlgebra07PointsCircles/equidist_off_axis_hard.pg
Find the point \((x, y)\) on the line \(y = 3x + 3\) that is equidistant from the points \((9, 8)\) and \((-1, -6)\).

\[ \begin{align*}
  x &= \underline{\phantom{000}} \\
  y &= \underline{\phantom{000}} 
\end{align*} \]

21. (1 pt) setAlgebra07PointsCircles/equidist_on_axis.pg
Find the point \((x, y)\) on the x-axis that is equidistant from the points \((-10, -1)\) and \((4, -6)\).

\[ \begin{align*}
  x &= \underline{\phantom{000}} \\
  y &= \underline{\phantom{000}} 
\end{align*} \]

22. (1 pt) setAlgebra07PointsCircles/sw2_2_51.pg
Find an equation of the circle with center \((18, -14)\) and radius 5 in the form of \((x - A)^2 + (y - B)^2 = C\) where \(A, B, C\) are constant. Then

\[ \begin{align*}
  A &= \underline{\phantom{000}} \\
  B &= \underline{\phantom{000}} \\
  C &= \underline{\phantom{000}} 
\end{align*} \]

23. (1 pt) setAlgebra07PointsCircles/sw2_2_53.pg
Find an equation of the circle with center at the origin and passing through \((5, -6)\) in the form of \((x - A)^2 + (y - B)^2 = C\) where \(A, B, C\) are constant. Then

\[ \begin{align*}
  A &= \underline{\phantom{000}} \\
  B &= \underline{\phantom{000}} \\
  C &= \underline{\phantom{000}} 
\end{align*} \]

24. (1 pt) setAlgebra07PointsCircles/sw2_2_61.pg
Find the center and radius of the circle given by the equation

\[ x^2 + y^2 - 2x - 8y + 16 = 0 \]

The center is: \((\underline{\phantom{000}}, \underline{\phantom{000}})\)

The radius is: \(\underline{\phantom{000}}\)

25. (1 pt) setAlgebra07PointsCircles/sw2_2_67.pg
Find the center and radius of the circle given by the equation

\[ x^2 + y^2 + 2x + 10y + 17 = 0 \]

The center is: \((\underline{\phantom{000}}, \underline{\phantom{000}})\)

The radius is: \(\underline{\phantom{000}}\)

26. (1 pt) setAlgebra07PointsCircles/swr1_8_23.pg
Find an equation of the circle with center \((3, -11)\) and radius 6 in the form of \((x - A)^2 + (y - B)^2 = C\) where \(A, B, C\) are constant. Then

\[ \begin{align*}
  A &= \underline{\phantom{000}} \\
  B &= \underline{\phantom{000}} \\
  C &= \underline{\phantom{000}} 
\end{align*} \]

27. (1 pt) setAlgebra07PointsCircles/swr1_8_24.pg
Find an equation of the circle with center at the origin and passing through \((-1, -1)\) in the form of \((x - A)^2 + (y - B)^2 = C\) where \(A, B, C\) are constant. Then

\[ \begin{align*}
  A &= \underline{\phantom{000}} \\
  B &= \underline{\phantom{000}} \\
  C &= \underline{\phantom{000}} 
\end{align*} \]

28. (1 pt) setAlgebra07PointsCircles/swr1_8_81.pg
Find the center and radius of the circle given by the equation

\[ x^2 + y^2 - 2x - 6y + 6 = 0 \]

The center is: \((\underline{\phantom{000}}, \underline{\phantom{000}})\)

The radius is: \(\underline{\phantom{000}}\)

29. (1 pt) setAlgebra07PointsCircles/swr1_8_86.pg
Find the center and radius of the circle given by the equation

\[ x^2 + y^2 + 6x + 8y - 24 = 0 \]

The center is: \((\underline{\phantom{000}}, \underline{\phantom{000}})\)

The radius is: \(\underline{\phantom{000}}\)

30. (1 pt) setAlgebra07PointsCircles/swr1_8_89.pg
Find the area of the region that lies outside the circle

\[ x^2 + y^2 = 4 \]

but inside the circle

\[ x^2 + y^2 - 4y - 12 = 0 \]

Your answer is \(\underline{\phantom{000}}\)

31. (1 pt) setAlgebra07PointsCircles/circle2.pg
Find the standard form for the equation of a circle

\[ (x - h)^2 + (y - k)^2 = r^2 \]

with a diameter that has endpoints \((-7, 3)\) and \((10, 0)\).

\[ h = \underline{\phantom{000}} \]

\[ k = \underline{\phantom{000}} \]

\[ r = \underline{\phantom{000}} \]

32. (1 pt) setAlgebra07PointsCircles/circle3.pg
Find the center \((h, k)\) and the radius \(r\) of the circle

\[ 3x^2 + 2x + 3y^2 - 3y - 10 = 0 \]

\[ h = \underline{\phantom{000}} \]

\[ k = \underline{\phantom{000}} \]

\[ r = \underline{\phantom{000}} \]

33. (1 pt) setAlgebra07PointsCircles/circlenot1.pg
Find the standard form for the equation of a circle \((x - h)^2 + (y - k)^2 = r^2\) with a diameter that has endpoints of \((-2, 1)\) and \((5, -8)\).

\[ h = \underline{\phantom{000}} \]

\[ k = \underline{\phantom{000}} \]

\[ r = \underline{\phantom{000}} \]

34. (1 pt) setAlgebra07PointsCircles/p4.pg
Find an equation of the circle with center at \((-1, 8)\) and passing through \((1, -3)\) in the form of \((x - A)^2 + (y - B)^2 = C\) where \(A, B, C\) are constant. Then

\[ A = \underline{\phantom{000}} \]

\[ B = \underline{\phantom{000}} \]

\[ C = \underline{\phantom{000}} \]

35. (1 pt) setAlgebra07PointsCircles/p5.pg
Find an equation of the circle with center at \((-2, -5)\) that is tangent to the y-axis in the form of \((x - A)^2 + (y - B)^2 = C\) where \(A, B, C\) are constant. Then

\[ A = \underline{\phantom{000}} \]

\[ B = \underline{\phantom{000}} \]

\[ C = \underline{\phantom{000}} \]

36. (1 pt) setAlgebra07PointsCircles/center_radius_from_gen.pg
Find the center and radius of the circle whose equation is

\[ x^2 + 7x + y^2 + 7y + 18 = 0 \]

The center is: \((\underline{\phantom{000}}, \underline{\phantom{000}})\)

The radius is: \(\underline{\phantom{000}}\)
The center of the circle is (__, __).  
The radius of the circle is ____.  
Note: Your answers must be decimals.

37. (1 pt) setAlgebra07PointsCircles/center_radius_from_gen_hard.pg  
Find the center and radius of the circle whose equation is  
$3x^2 + 6x + 3y^2 - 7y - 7 = 0$.  
The center of the circle is (__, __).  
The radius of the circle is ____.  
Note: Your answers must be decimals.

38. (1 pt) setAlgebra07PointsCircles/eqn_from_center_pt.pg  
Complete the equation of the circle centered at (-10, 5) that passes through (8, 13).

39. (1 pt) setAlgebra07PointsCircles/eqn_from_center_radius.pg  
Complete the equation of the circle centered at (1, 5) with radius 19.

40. (1 pt) setAlgebra07PointsCircles/ur_geo_1_5.pg  
Plot the points $A = (-2, 1), B = (-1, 4), C = (-8, 3)$. Notice that these points are vertices of a right triangle (the angle $A$ is 90 degrees).  
Find the distance between $A$ and $B$: ________  
Find the distance between $A$ and $C$: ________  
Find the area of the triangle $ABC$: ________
1. (1 pt) setAlgebra08LinearEqns/sw1_6_11.pg
Solve the equation $-5x + 9 = 2x + 5$.
$x = \underline{\phantom{0000}}$

2. (1 pt) setAlgebra08LinearEqns/sw1_6_13.pg
Solve the equation $\frac{1}{4}y - 9 = \frac{1}{10}y$.
$y = \underline{\phantom{0000}}$

3. (1 pt) setAlgebra08LinearEqns/sw1_6_19.pg
Solve the equation $\frac{-3}{x} = \frac{-7}{3x} - 4$.
$x = \underline{\phantom{0000}}$

4. (1 pt) setAlgebra08LinearEqns/sw1_6_29.pg
Solve the equation $\frac{5}{x + 1} - \frac{3}{2} = \frac{4}{3x + 3}$.
$x = \underline{\phantom{0000}}$

5. (1 pt) setAlgebra08LinearEqns/sw1_6_39.pg
Solve the equation $\frac{3x - 9}{x} - \frac{8}{x - 3}$.
Does the equation have a solution?
Input Yes or No here: \[\underline{\phantom{0000}}\]
If your answer is Yes, input your solution here: $x = \underline{\phantom{0000}}$

6. (1 pt) setAlgebra08LinearEqns/sw1_6_71.pg
Solve the equation $PV = nRT$ for $R$.
Your answer is: \[\underline{\phantom{0000}}\]
Note: The answer is case sensitive. P, V and T are capital letters!

7. (1 pt) setAlgebra08LinearEqns/sw1_6_74.pg
Solve the equation $P = 2l + 2w$ for $w$.
Your answer is: \[\underline{\phantom{0000}}\]
Note: The answer is case sensitive!

8. (1 pt) setAlgebra08LinearEqns/sw3_1_1.pg
Solve the equation $4x - 1 = 2$ algebraically.

9. (1 pt) setAlgebra08LinearEqns/sw3_1_3.pg
Solve the equation $8x - 1 = 4x - 7$ algebraically.
$x = \underline{\phantom{0000}}$

10. (1 pt) setAlgebra08LinearEqns/sw3_1_7.pg
Solve the equation $\frac{9}{x} + \frac{4}{2x} = 6$ algebraically.
$x = \underline{\phantom{0000}}$

11. (1 pt) setAlgebra08LinearEqns/swr1_5_5.pg
Solve the equation $9x + 7 = 4x - 8$.
$x = \underline{\phantom{0000}}$

12. (1 pt) setAlgebra08LinearEqns/swr1_5_14.pg
Solve the equation for $x$.
$5(x + 9) + 2 = -7(x - 5) - 1$
$x = \underline{\phantom{0000}}$

13. (1 pt) setAlgebra08LinearEqns/Test2_10.pg
Solve for $y$.
$-4y + 2(3y + 5) = 6 - 7(-3 - y)$
$y = \underline{\phantom{0000}}$

14. (1 pt) setAlgebra08LinearEqns/Test2_11.pg
Solve for $k$.
$\frac{-3}{4}k + \frac{3}{4} = 9 - \frac{-7}{6}$

$k = \underline{\phantom{0000}}$
1. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_9.pg
The distance (in miles) traveled when driving at a certain speed \( s \) for 42 hours, then driving 8 miles/hour faster for another hour. Express the distance in terms of \( s \).
Your answer is: __________

2. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_11.pg
Express the average age of three sisters in the terms of the age \( a \) of the firstborn (in years) if the second was born 3 years after the first and the third was born 6 years after the second.
Your answer is: __________

3. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_13.pg
An executive in an engineering firm earns a monthly salary plus a Christmas bonus of 6200 dollars. If she earns a total of 85600 dollars per year, what is her monthly salary in dollars?
Your answer is: __________

4. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_17.pg
The oldest child in a family of four children is twice as old as the youngest. The two middle children are 12 and 16 years old. If the average age of the children is 13.75, how old is the youngest child?
Your answer is: __________

5. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_23.pg
A rectangular garden is 25 ft wide. If its area is 1750 ft\(^2\), what is the length of the garden?
Your answer is: __________

6. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_25.pg
Phyllis invested 49000 dollars, a portion earning a simple interest rate of 5 percent per year and the rest earning a rate of 7 percent per year. After one year the total interest earned on these investments was 2790 dollars. How much money did she invest at each rate?
At rate 5 percent: __________
At rate 7 percent: __________

7. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_28.pg
A change purse contains an equal number of pennies, nickels, and dimes. The total value of the coins is 128 cents. How many coins of each type does the purse contain?
Number of pennies: __________

8. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_52.pg
Stan and Hilda can mow the lawn in 35 min if they work together. If Hilda works twice as fast as Stan, how long would it take Stan to mow the lawn alone?
Give your answer in munites here: __________

9. (1 pt) setAlgebra09LinearEqnsModeling/sw3_2_57.pg
Wilma drove at an average speed of 60 mi/h from her home in City A to visit her sister in City B. She stayed in City B 15 hours, and on the trip back averaged 50 mi/h. She returned home 50 hours after leaving. How many miles is City A from City B?
Your answer is: __________

10. (1 pt) setAlgebra09LinearEqnsModeling/sw1_6_11.pg
A cash register contains only five dollar and ten dollar bills. It contains twice as many five’s as ten’s and the total amount of money in the cash register is 560 dollars. How many ten’s are in the cash register?

11. (1 pt) setAlgebra09LinearEqnsModeling/sw1_6_20.pg
After robbing a bank in Dodge City, a robber gallops off at 12 mi/h. 20 minutes later, the marshall leaves to pursue the robber at 15 mi/h. How long (in hours) does it take the marshall to catch up to the robber?

12. (1 pt) setAlgebra09LinearEqnsModeling/sw1_6_25.pg
Two cyclists, 96 miles apart, start riding toward each other at the same time. One cycles 3 times as fast as the other. If they meet 4 hours later, what is the speed (in mi/h) of the faster cyclist?

13. (1 pt) setAlgebra09LinearEqnsModeling/sw1_6_29.pg
What quantity of 75 per cent acid solution must be mixed with a 25 solution to produce 960 mL of a 50 per cent solution?

14. (1 pt) setAlgebra09LinearEqnsModeling/sw1_6_31.pg
The radiator in a car is filled with a solution of 60 per cent antifreeze and 40 per cent water. The manufacturer of the antifreeze suggests that for summer driving, optimal cooling of the engine is obtained with only 50 per cent antifreeze. If the capacity of the radiator is 4.9 liters, how much coolant (in liters) must be drained and replaced with pure water to reduce the antifreeze concentration to 50 per cent?

15. (1 pt) setAlgebra09LinearEqnsModeling/tax.pg
Taxylvania has a tax code that rewards charitable giving. If a person gives \( p \% \) of his income to charity, that person pays \((44 – 1.5p) \% \) tax on the remaining money. For example, if a person gives 10% of his income to charity, he pays 29 % tax on the remaining money. If a person gives 29.3333333333333 % of his income to charity, he pays no tax on the remaining money. A person does not receive a tax refund if he gives more than 29.3333333333333 % of his income to charity. Count Taxula earns $ 64000. What percentage of his income should he give to charity to maximize the money he has after taxes and charitable giving?
The count should give _____% to charity. If the count did receive a tax refund for giving more than 29.3333333333333 % of his income to charity, how much should he give to charity? The count should give _____% to charity.
NOTE: Your answers must be numbers. No arithmetic operations are allowed.
16. (1 pt) setAlgebra09LinearEqnsModeling/c0s1p5.pg
A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 62.300 ft, give the area A of the window in square feet when the width is 11.000 ft. Give the answer to two decimal places.

17. (1 pt) setAlgebra09LinearEqnsModeling/c0s1p6.pg
A Norman window has the shape of a rectangle surmounted by a semicircle. The perimeter is 27.000 ft. Order the widths listed below according to the area of the corresponding Norman window from the lowest area (1) to highest area (5).

You will need to enter the numbers 1 through 5 in the entry blanks below.

1. Width = 5.600 ft.
2. Width = 6.500 ft.
3. Width = 5.700 ft.
4. Width = 5.500 ft.
5. Width = 5.400 ft.

Remark: To be able to order the sizes of the windows you are going to have to calculate the area for all five windows from knowing their widths. Since there are several calculations it will save time to figure out and simplify a formula which calculates the area from the width and the perimeter. This is in contrast to the previous problem where, with only one calculation to make, it wasn’t necessarily worth the effort to find a general formula. You can use that example to check your formula however.

I do this very frequently when I am doing research and solving problems. Work out a special case first. THEN work out a formula for the general case and use the solution to the special case to check the formula.

18. (1 pt) setAlgebra09LinearEqnsModeling/paintingpartners.pg
Mutt and Jeff need to paint a fence. Mutt can do the job alone 2 hours faster than Jeff. If together they work for 31 hours and finish only 6/7 of the job, how long would Jeff need to do the job alone?

Your answer must be a number. No arithmetic operations are allowed.

It would take Jeff ______ hours to do the job alone.

19. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_30.pg
This exercise concerns with modeling with linear equations.
The sum of three consecutive natural numbers is 441, find the numbers.
The three numbers in increasing order are _____, _____, and _____.

20. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_31.pg
This exercise concerns with modeling with linear equations.
One positive number is 5 times another number. The difference between the two numbers is 1076, find the numbers.
The two numbers in increasing order are _____ and _____.

21. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_32.pg
This exercise concerns with modeling with linear equations.
One positive number is one-fifth of another number. The difference between the two numbers is 204, find the numbers.
The two numbers in increasing order are _____ and _____.

22. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_33.pg
What is 55 percent of 280?.
Your answers is: ______

23. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_34.pg
160 is what percent of 550?.
Your answers is: ______ percent.

24. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_35.pg
280 is 50 percent of what number?
Your answers is: ______

25. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_36.pg
Your weekly paycheck is 15 percent less than your coworker’s. Your two paychecks total 815. Find the amount of each paycheck.

Your coworker’s is: ______ and yours is ______.

26. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_37.pg
A rectangular room is 2 times as long as it is wide, and its perimeter is 35 meters. Find the dimension of the room.
The length is: ____ meters and the width is ____ meters.

27. (1 pt) setAlgebra09LinearEqnsModeling/lh1-3_38.pg
Suppose that you are taking a course that has 4 tests. The first three tests are for 100 points each and the fourth test is for 200 points. To get an B in the course, you must have an average of at least 80 percent on the 4 tests. Your scores on the first 3 tests were 78, 78, and 73. What is the minimum score you need on the fourth test to get an B for the course?.

Your answers is: ______.

28. (1 pt) setAlgebra09LinearEqnsModeling/ur_ab6_6_1.pg
A student has scores of 68, 67.5, and 71.5 on his first three tests. He needs an average of at least 70 to earn a grade of C. What is the minimum score that the student needs on the fourth test to ensure a C?

Note: The answer need not be an integer.
Find all real solutions of the equation $x^2 = 64$.
\[ x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 < x_2 \]!!!

Find all real solutions of the equation $x^2 - 216 = 0$.
\[ x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 < x_2 \]!!!

Find all real solutions of the equation $(x - 5)^2 = 9$.
\[ x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 < x_2 \]!!!

How many real solutions does the equation $x^3 = 343$ have?
Input your answer here:
How many real solutions does the equation $x^3 = -343$ have?
Input your answer here:

Solve the equation $x^2 - 3x - 4 = 0$ by factoring.
The solutions are $x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2$.

Solve the equation $x^2 - 10x + 24 = 0$ by factoring.
The solutions are $x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2$.

Solve the equation $3x^2 + 18x + 15 = 0$ by factoring.
The solutions are $x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2$.

Solve the equation $x^2 - 2x - 35 = 0$ by completing the square.
The solutions are $x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2$.

Solve the equation $9x^2 + 6x - 24 = 0$ by completing the square.
The solutions are $x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2$.

Solve the equation $6x^2 - 9x = 0$ by completing the square.
The solutions are $x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2$.

Find all real solutions of equation $4x^2 + 4x - 6 = 0$.
Does the equation have real solutions? Input Yes or No: __
If your answer is Yes, input the solutions:
\[ x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2 \]

Note: Use sqrt(10) or 10**(1/2) for $\sqrt{10}$, etc.

Find all real solutions of equation $3 + 4z + z^2 = 0$.
Does the equation have real solutions? Input Yes or No: __
If your answer is Yes, input the solutions:
\[ z_1 = \quad \text{and} \quad z_2 = \quad \text{with} \quad z_1 \leq z_2 \]

Find all real solutions of equation $4x^2 + 3x + 4 = 0$.
Does the equation have real solutions? Input Yes or No: __
If your answer is Yes, input the solutions:
\[ x_1 = \quad \text{and} \quad x_2 = \quad \text{with} \quad x_1 \leq x_2 \]

A rectangular garden is 5 ft longer than it is wide. Its area is 3300 ft\(^2\). What are its dimensions?

Input your answer here:

A box with a square base and no top is to be made from a square piece of cardboard by cutting 7 in. squares from each corner and folding up the sides. The box is to hold 4375 in\(^3\). How big a piece of cardboard is needed?

Input your answer here:

By completing the square, the expression $x^2 + 6x + 81$ equals __

By completing the square, the expression $x^2 - 4x + 67$ equals __

The equation $x^2 + 9x - 2 = 0$ has two solutions $A$ and $B$ where $A < B$

The equation $4x^2 + 16x + 1 = 0$ has two solutions $A$ and $B$ where $A < B$
23. (1 pt) Algebra10QuadraticEqns/sw1_6.9.pg
The length of a rectangular garden is 10 feet longer than its width. If the garden’s perimeter is 188 feet, what is the area of the garden in square feet?

24. (1 pt) Algebra10QuadraticEqns/ur_ab_6.4.png
A factory is to be built on a lot measuring 270 ft by 360 ft. A local building code specifies that a lawn of uniform width and equal in area to the factory must surround the factory. What must the width of the lawn be? _____
If the dimensions of the factory are A ft by B ft with A < B, then A = _____ and B = _____

25. (1 pt) Algebra10QuadraticEqns/ur_ab_6.5.png
The difference of two positive numbers is 2 and the sum of their squares is 52. Find the numbers.
The bigger number is _____ and the smaller number is _____

26. (1 pt) Algebra10QuadraticEqns/ur_ab_6.6.png
The area of a rectangle is 28, and its perimeter is 22. Find its dimensions and diagonal.
Longer side: _____
Shorter side: _____
Diagonal: _____

27. (1 pt) Algebra10QuadraticEqns/findequation.png
Find b and c so that \( y = 16x^2 + bx + c \) has vertex \((2, -9)\). \( b = \) _____
\( c = \) _____

28. (1 pt) Algebra10QuadraticEqns/area2diagonal.png
The width of a rectangle is 5 less than twice its length. If the area of the rectangle is 172 cm², what is the length of the diagonal? Note: Your answer must be a number. It may not contain any arithmetic operations.
The length of the diagonal is _____ cm.

29. (1 pt) Algebra10QuadraticEqns/area2perimeter.png
Given that the area of an equilateral triangle is 60 cm², find its perimeter.
Note: Your answer must be a number. No arithmetic operations are allowed.
The perimeter of the triangle is _____ cm.

30. (1 pt) Algebra10QuadraticEqns/area2volume.png
The surface area of a cube is 90 cm². What is the volume of the cube?

31. (1 pt) Algebra10QuadraticEqns/maxprofit.png
The Acme Widget Company has found that if widgets are priced at $173, then 48000 will be sold. They have also found that for every increase of $16, there will be 1000 fewer widgets sold. The marginal cost of widgets is $60.55. The fixed costs for the Acme Widget Company are $16000.

If \( x \) represents the price of a widget find the following in terms of \( x \):
The number of widgets that will be sold: _____
The revenue generated by the sale of widgets: _____
The cost of producing just enough widgets to meet demand: _____
The profit from selling widgets: _____
Find the price that will maximize profits from the sale of widgets: _____

32. (1 pt) Algebra10QuadraticEqns/rocket.png
NASA launches a rocket at \( t = 0 \) seconds. Its height, in meters above sea-level, as a function of time is given by \( h(t) = -4.9t^2 + 343t + 395 \).
Assuming that the rocket will splash down into the ocean, at what time does splashdown occur?
The rocket splashes down after _____ seconds.
How high above sea-level does the rocket get at its peak?
The rocket peaks at _____ meters above sea-level.

33. (1 pt) Algebra10QuadraticEqns/wireproblem.png
You have a wire that is 95 cm long. You wish to cut it into two pieces. One piece will be bent into the shape of a square. The other piece will be bent into the shape of a circle. Let A represent the total area of the square and the circle. What is the circumference of the circle when A is a minimum?
The circumference of the circle is _____ cm.

34. (1 pt) Algebra10QuadraticEqns/SA2volume.png
Given that the surface area of a sphere is 209 \( \pi \) cm², find its volume.
Note: Your answer must be a number. No arithmetic operations are allowed.
The volume of the sphere is _____ cm³.
1. (1 pt) setAlgebra11ComplexNumbers/adding.pg
Evaluate the expression \((2 - 3i) + (-2 - 4i)\) and write the result in the form \(a + bi\).
The sum is ________

2. (1 pt) setAlgebra11ComplexNumbers/Multiply.pg
Evaluate the expression \((3 - 7i)(-9 - 3i)\) and write the result in the form \(a + bi\).
The product is ________

3. (1 pt) setAlgebra11ComplexNumbers/Subtract.pg
Evaluate the expression \((5 + 8i) - (-9 - 2i)\) and write the result in the form \(a + bi\).
The difference is ________

4. (1 pt) setAlgebra11ComplexNumbers/Divide.pg
Evaluate the expression
\[
\frac{-1 - 5i}{-2 + 8i}
\]
and write the result in the form \(a + bi\).
The quotient is ________

5. (1 pt) setAlgebra11ComplexNumbers/ur_conj_1.pg
For some practice working with complex numbers:
Calculate
\[
(2 - 5i) + (2 - 3i) = \quad (2 - 5i) - (2 - 3i) = \quad (2 - 5i)(2 - 3i) = \]
The complex conjugate of \((1 + i)\) is \((1 - i)\). In general to obtain the complex conjugate reverse the sign of the imaginary part.
(Geometrically this corresponds to finding the "mirror image" point in the complex plane by reflecting through the \(x\)-axis. The complex conjugate of a complex number \(z\) is written with a bar over it: \(\overline{z}\) and read as "\(z\) bar".
Notice that if \(z = a + ib\), then
\[
(z)(\overline{z}) = |z|^2 = a^2 + b^2
\]
which is also the square of the distance of the point \(z\) from the origin. (Plot \(z\) as a point in the "complex" plane in order to see this.)
If \(z = 2 - 5i\) then \(\overline{z} = \) ________ and \(|z| = \) ________.
You can use this to simplify complex fractions. Multiply the numerator and denominator by the complex conjugate of the denominator to make the denominator real.
\[
\frac{2 - 5i}{2 - 3i} = \quad + i \quad \text{________}
\]
Two convenient functions to know about pick out the real and imaginary parts of a complex number.
\(\text{Re}(a + ib) = a\) (the real part (coordinate) of the complex number), and
\(\text{Im}(a + ib) = b\) (the imaginary part (coordinate) of the complex number). \(\text{Re}\) and \(\text{Im}\) are linear functions – now that you know about linear behavior you may start noticing it often.

6. (1 pt) setAlgebra11ComplexNumbers/ur_conj_2.pg
More on complex numbers. (For additional help check out the appendix in Stewart’s Calculus book. There is an entire appendix of hints for working with complex numbers.)

An apology: The exponents don’t print very well on the screen version of this problem. You can get a better idea of what the notation looks like from the hard copy and/or you can use the "typeset" mode to get a better printing. Unfortunately in typeset mode you won’t be able to enter the answers which are within equations.

The red point represents the complex number \(z_1 = \) ________ and the blue point represents the complex number \(z_2 = \) ________.

We can also write these complex numbers in polar coordinates \((r, \theta)\). The angle is sometimes called the "argument" of the complex number and \(r\) is called the "modulus" or the absolute value of the number.

By comparing Taylor series we find that
\[
e^{i\theta} = \cos(\theta) + i\sin(\theta).
\]
This is a very important and very useful formula. One use is to relate the polar coordinate and cartesian coordinate formulas for the complex number. If \(z\) can be represented by both coordinates \(x + iy\) and by polar coordinates \(r, \theta\) then
\[
re^{i\theta} = r\cos(\theta) + i r\sin(\theta) = x + iy = z.
\]

Represent \(z_1\) (the red point) in polar coordinates (use an angle between \(-\pi\) and \(\pi\)):

Represent \(z_2\) (the blue point) in polar coordinates:

---

\(e^{i\theta} = \cos(\theta) + i\sin(\theta)\).
Using the law of exponents it is really easy to multiply complex numbers represented in polar coordinates – the angles just add!

\[(r_1 e^{i\theta_1})(r_2 e^{i\theta_2}) = r_1 r_2 e^{i(\theta_1 + \theta_2)}.\]

Find \(z_1 \cdot z_2\) using polar coordinates and your answer above:

\[z_1 \cdot z_2 = \text{_____} e^{\text{______}}.\]

Check your answer by doing the standard multiplication and then converting to polar coordinates. Can you plot this number on the graph?

7. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_5.pg
Enter the complex coordinates of the following points:

A: ____ + ____ i.
B: ____ + ____ i.
C: ____ + ____ i.

8. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_6.pg
Enter the complex coordinates of the following points:

A: ____

9. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_7.pg
Write the following numbers in \(a + bi\) form:
(a) \(-3(\frac{i}{2}) = ____ + ____ i.
(b) \((2 - i) - (-2 + 4i) = ____ + ____ i.
(c) \(-\frac{5}{i} = ____ + ____ i.

10. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_8.pg
Write the following numbers in \(a + bi\) form:
(a) \((-3 + i)^2 = ____ + ____ i.
(b) \(-\frac{2 - 2i}{3} = ____ + ____ i.
(c) \(-\frac{2 - 2i}{3} = ____ + ____ i.

11. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_9.pg
Write the following numbers in \(a + bi\) form:
(a) \((-5 + 2i)^2 = ____ + ____ i.
(b) \(i(\pi - 2i) = ____ + ____ i.
(c) \(-\frac{5 + 5i}{i} = ____ + ____ i.

12. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_10.pg
Write the following numbers in \(a + bi\) form:
(a) \(-1 - i = ____ + ____ i.
(b) \(\frac{3i + 2i}{3i} = ____ + ____ i.
(c) \((-3i)^3 = ____ + ____ i.

13. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_11.pg
Write the following numbers in \(a + bi\) form:
(a) \(\left(\frac{2 + i}{i - (-1 - 2i)}\right)^2 = ____ + ____ i.
(b) \((i)^2(-1 + i)^2 = ____ + ____ i.

14. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_12.pg
Write the following numbers in \(a + bi\) form:
(a) \((1 - 4i)(5 + i)(-4 + 3i) = ____ + ____ i.
(b) \((4 - 5i)^2 - 2i = ____ + ____ i.

15. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_13.pg
Calculate the following:
(a) \(i^3 = ____
(b) \(i^4 = ____
(c) \(i^5 = ____
(d) \(i^6 = ____
(e) \(i^7 = ____
(f) \(i^8 = ____
(g) \(i^{-1} = ____
(h) \(i^{-2} = ____
(i) \(i^{-3} = ____
(j) \(i^{-73} = ____

16. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_14.pg
Let \(z = 1 + 3i\). Calculate the following:
(a) \( z^2 + 2z + 1 = \text{______} + \text{______} i. \)
(b) \( z^2 + iz - (1 + i) = \text{______} + \text{______} i. \)
(c) \( \frac{(z - 4)^2}{z + i} = \text{______} + \text{______} i. \)

17. (1 pt) setAlgebra11ComplexNumbers/sw3_4_7.pg
Evaluate the expression \((4 - 5i) + (8 - 3i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

18. (1 pt) setAlgebra11ComplexNumbers/sw3_4_11.pg
Evaluate the expression \((5 + 8i) - (3 - 1i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

19. (1 pt) setAlgebra11ComplexNumbers/sw3_4_15.pg
Evaluate the expression \((-3 - 1i)(1 - 3i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

20. (1 pt) setAlgebra11ComplexNumbers/sw3_4_23.pg
Evaluate the expression \(\frac{4 - 2i}{-1 + 3i}\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

21. (1 pt) setAlgebra11ComplexNumbers/sw3_4_29.pg
Evaluate the expression \(\frac{1 + 3i}{7}\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

22. (1 pt) setAlgebra11ComplexNumbers/sw3_4_35.pg
Evaluate the expression \(i^{121}\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

23. (1 pt) setAlgebra11ComplexNumbers/beth1complex.pg
Evaluate the expression \(\frac{(-2 + 3i)(4i)}{2 - 3i}\) and write the result in the form \(a + bi\).
Then \(a = \text{______}\) and \(b = \text{______}\)

24. (1 pt) setAlgebra11ComplexNumbers/jj1.pg
If we write the following complex number in standard form
\((\sqrt{10} + \sqrt{11}i)(\sqrt{10} - \sqrt{11}i) = a + bi\)
then
\(a = \text{______}\)
\(b = \text{______}\)
Your answers here have to be simplified so that they are just numbers.

25. (1 pt) setAlgebra11ComplexNumbers/sw3_4_13.pg
Evaluate the expression \((2 - 2i) + (3 - 7i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

26. (1 pt) setAlgebra11ComplexNumbers/sw3_4_17.pg
Evaluate the expression \((1 + 7i) - (-3 + 3i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

27. (1 pt) setAlgebra11ComplexNumbers/sw3_4_23.pg
Evaluate the expression \((4 - 3i)(-4 - 3i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

28. (1 pt) setAlgebra11ComplexNumbers/sw3_4_29.pg
Evaluate the expression \((-1 - 1i)\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

29. (1 pt) setAlgebra11ComplexNumbers/sw3_4_35.pg
Evaluate the expression \((\frac{1 + i}{\sqrt{2}})\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

30. (1 pt) setAlgebra11ComplexNumbers/sw3_4_41.pg
Evaluate the expression \(i^{91}\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

31. (1 pt) setAlgebra11ComplexNumbers/sw3_4_43.pg
Evaluate the expression \(\sqrt{-64}\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

32. (1 pt) setAlgebra11ComplexNumbers/sw3_4_45.pg
Evaluate the expression \(\sqrt{-6\sqrt{-486}}\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

33. (1 pt) setAlgebra11ComplexNumbers/sw3_4_47.pg
Evaluate the expression \((4 - \sqrt{-9})(-2 + \sqrt{-9})\) and write the result in the form \(a + bi\).
The real number \(a\) equals \(\text{______}\)
The real number \(b\) equals \(\text{______}\)

34. (1 pt) setAlgebra11ComplexNumbers/sw3_4_49.pg
Evaluate the expression \(\frac{\sqrt{3 + \sqrt{-1}}}{-1 + \sqrt{-4}}\) and write the result in the form \(a + bi\).
35. (1 pt) Write the following numbers in the polar form $r e^{i \theta}$, $-\pi < \theta \leq \pi$:

(a) $\sqrt{2}$

(b) $-2\sqrt{2} - 2i$

(c) $(1 - i)(-\sqrt{3} + i)$

(d) $(\sqrt{2} - 3i)^2$

(e) $3 + 2i$

(f) $-\sqrt{7}(1 + i)$

36. (1 pt) Write each of the given numbers in the polar form $re^{i \theta}$:

(a) $e^{-\pi i}$

(b) $e^{(1 + 2\pi i)}$

(c) $e^{\pi i}$

(d) $e^{\pi i + \pi}$

37. (1 pt) Write each of the given numbers in the polar form $re^{i \theta}$:

(a) $(1 - \sqrt{3})$ is undefined.

(b) $8e^{(\pi + \pi)}$

(c) $e^{\pi i}$

38. (1 pt) Place the following in order:

(a) $|z_2| - |z_1|$

(b) $|z_1 + z_2|$

(c) $|z_1| + |z_2|$

(d) $|z_2 - z_1|$

39. (1 pt) Write the following numbers in the polar form $re^{i \theta}$, $0 \leq \theta < 2\pi$:

(a) $\frac{1}{8}$

(b) $8 + 8i$

(c) $-8 + 8i$

40. (1 pt) Write the following numbers in the polar form $re^{i \theta}$, $-\pi < \theta \leq \pi$:

(a) $\frac{\sqrt{2}}{\sqrt{17}}$ and write the result in the form $a + bi$.

(b) $\sqrt{2} + 2i$

(c) $\sqrt{2} - 3i$

(d) $3 + 2i$

(e) $-\sqrt{7}(1 + i)$

(f) $2 - \sqrt{3}$

41. (1 pt) Solve the following equations for $z$:

(a) $z^2 = 1$.

(b) $z^2 = 2 - 2i$

(c) $(1 - i)(-\sqrt{3} + i)$

(d) $(\sqrt{2} - 3i)^2$

(e) $3 + 2i$

(f) $-\sqrt{7}(1 + i)$

42. (1 pt) The real number $a$ equals ________

The real number $b$ equals ________

43. (1 pt) The real number $a$ equals ________

The real number $b$ equals ________

44. (1 pt) The real number $a$ equals ________

The real number $b$ equals ________

45. (1 pt) The real number $a$ equals ________

The real number $b$ equals ________
\[ r = \frac{4i}{3e^{(6i)}} \Rightarrow 0 = \underline{\text{}} \]

46. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_25.pg
Determine which of the following properties of the real exponential function remain true for the complex exponential (i.e., for \( x \) replaced by \( z \)).

Answer T or F:
1. \( e^z \) is never zero. T
2. \( e^z \) is a one-to-one function. F
3. \( e^{-z} = \frac{1}{e^z} \). T
4. \( e^z \) is defined for all \( x \). T

47. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_26.pg
Which of the following sets are open?
- A. \((\text{Re } z)^2 > 1\)
- B. \(-1 < \text{Im } z \leq 1\)
- C. \(0 < |z - 2| < 3\)
- D. \(|z - 1 + i| \leq 3\)
- E. \(|\text{Arg } z| < \frac{\pi}{4}\)
- F. \(|z| \geq 2\)

48. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_27.pg
Which of the given sets are bounded?
- A. \(0 < |z - 2| < 3\)
- B. \(|\text{Arg } z| < \frac{\pi}{4}\)
- C. \((\text{Re } z)^2 > 1\)
- D. \(-1 < \text{Im } z \leq 1\)
- E. \(|z| \geq 2\)
- F. \(|z - 1 + i| \leq 3\)

49. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_28.pg
Which of the given sets are regions?
- A. \(|z| \geq 2\)
- B. \(0 < |z - 2| < 3\)
- C. \(|z - 1 + i| \leq 3\)
- D. \(|\text{Arg } z| < \frac{\pi}{4}\)
- E. \(-1 < \text{Im } z \leq 1\)
- F. \((\text{Re } z)^2 > 1\)

50. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_29.pg
Which of the given sets are closed regions?
- A. \(0 < |z - 2| < 3\)
- B. \(|z| \geq 2\)
- C. \(-1 < \text{Im } z \leq 1\)
- D. \((\text{Re } z)^2 > 1\)
- E. \(|z - 1 + i| \leq 3\)
- F. \(|\text{Arg } z| < \frac{\pi}{4}\)

51. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_30.pg
Which of the given sets are domains?
- A. \(|z| \geq 2\)
- B. \(0 < |z - 2| < 3\)
- C. \((\text{Re } z)^2 > 1\)
- D. \(|z - 1 + i| \leq 3\)

52. (1 pt) setAlgebra11ComplexNumbers/Sqrt.pg
Find the square root of 9-9i so that the real part of your answer is positive. The square root is \( \underline{\text{}} \)

53. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_31.pg
Find all the values of the following.
1. \((16)\frac{3}{2}\)
Place all answers in the following blank, separated by commas:

2. \(1^\frac{3}{2}\)
Place all answers in the following blank, separated by commas:

3. \(i^\frac{3}{2}\)
Place all answers in the following blank, separated by commas:

54. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_32.pg
Find all the values of the following:
1. \((1 + \sqrt{3}i)^\frac{3}{2}\)
Place all answers in the following blank, separated by commas:

2. \((i-1)^\frac{4}{3}\)
Place all answers in the following blank, separated by commas:

3. \((\frac{3i}{1+i})^\frac{1}{6}\)
Place all answers in the following blank, separated by commas:

55. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_33.pg
Solve the following equations for \( z \), find all solutions:
1. \(5z^2 + z + 3 = 0\)
Place all answers in the following blank, separated by commas:

2. \(z^2 - (3 - 2i)z + 1 - 3i = 0\)
Place all answers in the following blank, separated by commas:

3. \(z^2 - 2z + i = 0\)
Place all answers in the following blank, separated by commas:

56. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_34.pg
Let \( z = -8 - 1i \). Write the following numbers in \( a + bi \) form:
(a) \(-7z = \underline{\text{}} + \underline{\text{}} i\),
(b) \(\bar{z} = \underline{\text{}} + \underline{\text{}} i\),
(c) \(\frac{1}{z} = \underline{\text{}} + \underline{\text{}} i\).

57. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_35.pg
Write the following numbers in the polar form \( r(\cos \phi + i \sin \phi) \),
\(0 \leq \phi < 2\pi\).
(a) \(6\)
\(r = \underline{\text{}} \phi = \underline{\text{}}\)
(b) \(8i\)
58. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_36.png
Let \( z = 6(\cos 2.7 + i \sin 2.7) \).
Write the following numbers in the polar form \( r(\cos \phi + i \sin \phi) \),
\( 0 \leq \phi < 2\pi \).
(a) \( 4z \)
\( r = \) \( \phi = \)
(b) \( \bar{z} \)
\( r = \) \( \phi = \)
(c) \( \frac{1}{z} \)
\( r = \) \( \phi = \)

59. (1 pt) setAlgebra11ComplexNumbers/ur_cn_1_37.png
Let \( z = 9e^{i\theta} \).
Write the following numbers in the polar form \( re^{i\phi}, 0 \leq \phi < 2\pi \).
(a) \( 4z \)
\( r = \) \( \phi = \)
(b) \( \bar{z} \)
\( r = \) \( \phi = \)
(c) \( \frac{1}{z} \)
\( r = \) \( \phi = \)

60. (1 pt) setAlgebra11ComplexNumbers/sw3_4_49.png
Find all solutions of the equation \( x^2 + 2x + 5 = 0 \) and express them in the form \( a + bi \):
First input the solution with \( b < 0 \) here:
The real number \( a \) equals \( \) and the real number \( b \) equals \( \)
Then input the solution with \( b > 0 \) here:
The real number \( a \) equals \( \) and the real number \( b \) equals \( \)

61. (1 pt) setAlgebra11ComplexNumbers/sw3_4_55.png
Find all solutions of the equation \( t + 4 + \frac{2}{t} = 0 \) and express them in the form \( a + bi \):
First input the solution with \( b < 0 \) here:
The real number \( a \) equals \( \) and the real number \( b \) equals \( \)
Then input the solution with \( b > 0 \) here:
The real number \( a \) equals \( \) and the real number \( b \) equals \( \)

62. (1 pt) setAlgebra11ComplexNumbers/beth2complex.png
Find all solutions of the equation \( x^2 + 1x + 6 = 0 \) and express them in the form \( a + bi \):
solutions: \( \)
(Note: If there is more than one solution, enter a comma separated list (i.e.: \( 1+2i,3+4i \)).)

63. (1 pt) setAlgebra11ComplexNumbers/sw3_4_55.png
Find all solutions of the equation \( x^2 - 1x + 6 = 0 \) and express them in the form \( a + bi \):
First input the solution with \( b < 0 \) here:
The real part \( a \) equals \( \) and the imaginary part \( b \) equals \( \)
Then input the solution with \( b > 0 \) here:
The real part \( a \) equals \( \) and the imaginary part \( b \) equals \( \)

64. (1 pt) setAlgebra11ComplexNumbers/sw3_4_61.png
Find all solutions of the equation \( t + 4 + \frac{2}{t} = 0 \) and express them in the form \( a + bi \):
First input the solution with \( b < 0 \) here:
The real part \( a \) equals \( \) and the imaginary part \( b \) equals \( \)
Then input the solution with \( b > 0 \) here:
The real part \( a \) equals \( \) and the imaginary part \( b \) equals \( \)
The real solution of the equation \( x^3 = 125 \) is:

The equation \( x^4 - 81 = 0 \) has two real solutions \( A \) and \( B \) where \( A < B \).
\[ A = \quad \text{and} \quad B = \quad \]

The equation \( 2x^3 - 8x^2 - 3x^2 = 0 \) has three real solutions \( A, B, \) and \( C \) where \( A < B < C \).
\[ A = \quad \text{and} \quad B = \quad \text{and} \quad C = \quad \]

Solve the equation \( x - 3\sqrt{x} - 28 = 0 \) by factoring.  
The only solution is \( x = \quad \)

Solve for \( x \):
\[ \left( \frac{x + 42}{x - 33} \right)^2 - 256 \left( \frac{x + 42}{x - 33} \right) + 14863 = 0 \]
The smaller solution is \( \quad \).  
The larger solution is \( \quad \)

Solve the equation \( \sqrt{10} - x + x = 8 \).  
The only solution is \( x = \quad \)

Solve the equation \( (x - 1)^{\frac{3}{2}} (x - 3) + 1(x - 1)^{\frac{1}{2}} = 0 \)
\[ x = \quad \]

Solve for the only possible solution.  Give your answer to the nearest thousandth.
\[ \sqrt{6x + 7} = \sqrt{7x - 3} \]
\[ x = \quad \]
Does your solution satisfy the equation?  (yes or no) \( \quad \)

Find all possible solutions.  Give your answers in increasing order.  Give your answers to the nearest thousandth.
\[ \sqrt{-9x + 2} - 2 = 6x \]
The smaller possible solution is \( \quad \)  
Is it a solution?  (yes or no) \( \quad \)  
The larger possible solution is \( \quad \)  
Is it a solution?  (yes or no) \( \quad \)

Solve the following equation.
\[ |3x - 9| = 8 \]
Answer: \( \quad \)

Note:  If there is more than one answer, write them separated by commas (e.g., 1, 2).

Solve the following equation.
\[ \quad \]
\[ \frac{1}{|7 - 3x|} = 1 \]

Answer: 

**Note:** If there is more than one answer, write them separated by commas (e.g., 1, 2).

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20. (1 pt) setAlgebra12EqnsOtherTypes/absolutevalue.pg

Solve the following equation for \( x \):

\[ x | x - 5 | = 46x + 5 \]

List the four possible roots in increasing order. Below each possible root, enter ROOT if it is a root or EXTRANEOUS if it is an extraneous root.

\[ , , , \]

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21. (1 pt) setAlgebra12EqnsOtherTypes/volume2SA.pg

Given that the volume of a cylinder is 174, and the radius of the cylinder is twice the height, find the surface area of the cylinder.

Note: Your answer must be a number. No arithmetic operations are allowed.

The surface area of the cylinder is ____ cm\(^3\).
1. (1 pt) setAlgebra13Inequalities/pn2.pg
Express the inequality using interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( x < 8 \)

**Answer:**

2. (1 pt) setAlgebra13Inequalities/pn3.pg
Express the inequality using interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( x \geq -12 \)

**Answer:**

3. (1 pt) setAlgebra13Inequalities/srw1_7_3.pg
The inequality \( 6x + 9 > 5 \) means that \( x \) is greater than \( A \) where \( A \) is ________

**Answer:**

4. (1 pt) setAlgebra13Inequalities/srw1_7_9.pg
Solve the following inequality. Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( 1 - x \geq 15 \)

**Answer:**

5. (1 pt) setAlgebra13Inequalities/srw1_7_13.pg
Solve the following inequality. Write the answer in interval notation. **Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( 3x + 16 \leq 6x + 3 \)

**Answer:**

6. (1 pt) setAlgebra13Inequalities/srw1_7_15.pg
Solve the following inequality. Write the answer in interval notation. **Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( \frac{1}{2}x - 18 > 5 \)

**Answer:**

7. (1 pt) setAlgebra13Inequalities/ur_ab_7_1.pg
Consider the inequality

\( 8 + 12x < 10x + 12 \)

The solution of this inequality consists one or more of the following intervals: \( (-\infty, A) \) and \( (A, \infty) \)

**Find A**

---

For each interval, answer YES or NO to whether the interval is included in the solution.

\( (-\infty, A) \) ______

\( (A, \infty) \) ______

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8. (1 pt) setAlgebra13Inequalities/p1.pg
Solve the following inequality. Write the answer in interval notation. If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \).

\( 1 - x < 4 \)

**Answer:**

---

9. (1 pt) setAlgebra13Inequalities/pn6.pg
Solve the following inequality. Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( -1 - x < 1 \)

**Answer:**

---

10. (1 pt) setAlgebra13Inequalities/p13.pg
Solve the following inequality. Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( 1x - 1 < -2(6x - 2) + 1 \)

**Answer:**

---

11. (1 pt) setAlgebra13Inequalities/pn1.pg
Express the inequality using interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( -7 < x \leq 9 \)

**Answer:**

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12. (1 pt) setAlgebra13Inequalities/srw1_7_19.pg
Solve the following inequality. Write the answer in interval notation. **Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( 17 \leq x + 20 < 22 \)

**Answer:**

---

13. (1 pt) setAlgebra13Inequalities/srw1_7_67.pg
Solve the following inequality. Write the answer in interval notation. **Note:** If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\( 23 \leq \frac{5}{9}(F - 32) \leq 46 \)
14. Solve the following inequality. Write the answer in interval notation.

\[ 2 \leq 7x - 5 < 6 \]

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

15. The inequality \( 2x + 6 \leq x + 7 \leq 3x + 10 \) means that \( x \) is in the closed interval \([A, B]\) where \( A = \) \( \) and \( B = \) \( .

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\[ |2x - 5| \leq 18 \]

\[ |x + 1| - 2 < 3 \]

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

16. Solve the following inequality. Write the answer in interval notation. \( 2x + 6 \leq x + 7 \leq 3x + 10 \) means that \( x \) is in the closed interval \([A, B]\) where \( A = \) \( \) and \( B = \) \( .

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

17. Solve the following inequality. Write the answer in interval notation. \( |x - 6| \leq 4 \) is the same as saying \( x \) is in the closed interval \([A, B]\) where \( A = \) \( \) and \( B = \) \( .

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

18. Solve the following inequality. Write the answer in interval notation. \( |x + 8| \leq 7 \) is the same as saying \( x \) is in the closed interval \([A, B]\) where \( A = \) \( \) and \( B = \) \( .

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

19. Solve the following inequality. Write the answer in interval notation. \( \frac{x - 7}{5} \leq 4 \) is the same as saying \( x \) is in the closed interval \([A, B]\) where \( A = \) \( \) and \( B = \) \( .

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

20. Solve the following inequality. Write the answer in interval notation. \( \frac{x - 6}{4} \leq 6 \) is the same as saying \( x \) is in the closed interval \([A, B]\) where \( A = \) \( \) and \( B = \) \( .

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).

\[ 2x^2 + x \geq 3 \]

Answer: 

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \( -\infty \) as \( -\text{infinity} \).
Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[-2x^2 \leq 24\]

Answer: __________

\[30. (1 \text{ pt}) \text{setAlgebra13Inequalities/p3.pg}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[(x - 2)(x + 3) \leq 0\]

Answer: __________

\[31. (1 \text{ pt}) \text{setAlgebra13Inequalities/ur_ab_2_2.png}\]

Consider the inequality

\[x^2 > 2x + 8\]

The solution of this inequality consists one or more of the following intervals: \((-\infty, A), (A, B), \) and \(B, \infty)\) where \(A < B.\)

Find \(A\) __________

Find \(B\) __________

For each interval, answer YES or NO to whether the interval is included in the solution.

\((-\infty, A)\) __________

\((A, B)\) __________

\((B, \infty)\) __________

\[32. (1 \text{ pt}) \text{setAlgebra13Inequalities/p4.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[x^2 + 2x - 24 > 0\]

Answer: __________

\[33. (1 \text{ pt}) \text{setAlgebra13Inequalities/p11.png}\]

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[x^2 - 1x > 0\]

Answer: __________

\[34. (1 \text{ pt}) \text{setAlgebra13Inequalities/p12.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[-x^2 + 4x \geq 0\]

Answer: __________

\[35. (1 \text{ pt}) \text{setAlgebra13Inequalities/p6.png}\]

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[x^3 - 4x \leq 0\]

Answer: __________

\[36. (1 \text{ pt}) \text{setAlgebra13Inequalities/srw1_7_53.png}\]

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[x^4 > 1x^2\]

Answer: __________

\[37. (1 \text{ pt}) \text{setAlgebra13Inequalities/srw1_7_39.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 9}{x + 12} \geq 0\]

Answer: __________

\[38. (1 \text{ pt}) \text{setAlgebra13Inequalities/p5.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{2 - x}{x - 9} \geq 0\]

Answer: __________

\[39. (1 \text{ pt}) \text{setAlgebra13Inequalities/p14.png}\]

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{1}{x - 7} > 1\]

Answer: __________

\[40. (1 \text{ pt}) \text{setAlgebra13Inequalities/p15.png}\]

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 8}{x - 1} \leq -6\]

Answer: __________

\[41. (1 \text{ pt}) \text{setAlgebra13Inequalities/p17.png}\]

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 1}{x - 2} - \frac{1}{x - 2} > 0\]

Answer: __________

\[42. (1 \text{ pt}) \text{setAlgebra13Inequalities/p12.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 1}{x - 2} + \frac{1}{x - 2} < 1\]

Answer: __________

\[43. (1 \text{ pt}) \text{setAlgebra13Inequalities/p13.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 2}{x - 1} - \frac{1}{x - 1} \geq 0\]

Answer: __________

\[44. (1 \text{ pt}) \text{setAlgebra13Inequalities/p14.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 3}{x - 2} + \frac{1}{x - 2} < 1\]

Answer: __________

\[45. (1 \text{ pt}) \text{setAlgebra13Inequalities/p15.png}\]

Solve the following inequality. Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as \( \text{infinity} \) and \(-\infty \) as \( -\text{infinity} \).

\[\frac{x - 1}{x - 2} - \frac{1}{x - 2} < 0\]

Answer: __________
Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as infinity and \(-\infty\) as -infinity.

\[
\frac{x}{x - 8} > -2
\]

Answer: ______________________

42. (1 pt) setAlgebra13Inequalities/p18.pg

Find \( C \). For each interval, answer YES or NO to whether the interval is included in the solution.

\((-\infty, A)\)_____ \((A, B)\)_____ \((B, C)\)_____ \((C, \infty)\)_____ 

46. (1 pt) setAlgebra13Inequalities/srw177a7.pg

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as infinity and \(-\infty\) as -infinity.

\[
1 + \frac{2}{x + 1} \leq \frac{2}{x}
\]

Answer: ______________________

43. (1 pt) setAlgebra13Inequalities/ur_ab_7_3.pg

Consider the inequality

\[
\frac{x - 6}{x^3(x + 3)} < 0
\]

The solution of this inequality consists of one or more of the following intervals: \((-\infty, A), (A, B), (B, C), \text{and} (C, \infty)\) where \( A < B < C \).

Find \( A \)_____ 
Find \( B \)_____ 
Find \( C \)_____ 

For each interval, answer YES or NO to whether the interval is included in the solution.

\((-\infty, A)\)_____ \((A, B)\)_____ \((B, C)\)_____ \((C, \infty)\)_____ 

47. (1 pt) setAlgebra13Inequalities/srw177b7.pg

Solve the following inequality. Write the answer in interval notation. Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as infinity and \(-\infty\) as -infinity.

\[
\frac{x(x - 8)}{x^2 - 7x - 78} \leq 0
\]

Answer: ______________________

44. (1 pt) setAlgebra13Inequalities/ur_ab_7_4.pg

Consider the inequality

\[
\frac{x + 7}{x + 6} < -2
\]

The solution of this inequality consists one or more of the following intervals: \((-\infty, A), (A, B), \text{and} (B, \infty)\) where \( A < B \).

Find \( A \)_____ 
Find \( B \)_____ 

For each interval, answer YES or NO to whether the interval is included in the solution.

\((-\infty, A)\)_____ \((A, B)\)_____ \((B, \infty)\)_____ 

48. (1 pt) setAlgebra13Inequalities/Inequality.pg

Solve the inequality

\[
\frac{(x - 5)^4(x - 16)^3}{x - 84} \geq 0
\]

Give your answer in interval notation.

\( x \in \)_____ 

Note: Enter your answer without spaces. If you need \(-\infty\), type \(-\inf\). If you need \(\infty\), type \(\inf\). Remember that punctuation is important.

49. (1 pt) setAlgebra13Inequalities/p10.pg

Solve the following inequality. Write the answer in interval notation. If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as "infinity".

\[
\frac{1}{|x - 6|} \geq 4
\]

Answer: ______________________

50. (1 pt) setAlgebra13Inequalities/srw177a99.pg

You arrive in Paris and the forecast is for a low of 15 and a high of 21 degrees Celsius. What is the forecasted low temperature in Fahrenheit? _____

What is the forecasted high temperature in Fahrenheit? _____

51. (1 pt) setAlgebra13Inequalities/srw177b00.pg

Your friend from Paris arrives in New York and the forecast is for a low of 53 and a high of 70 degrees Fahrenheit.

What is the forecasted low temperature in Celsius? _____

What is the forecasted high temperature in Celsius? _____

Note: If the answer includes more than one interval write the intervals separated by the "union" symbol, U. If needed enter \( \infty \) as infinity and \(-\infty\) as -infinity.
52. (1 pt) setAlgebra13Inequalities/sw1_7_73.png
A car rental company offers two plans for renting a car.
Plan A: 30 dollars per day and 17 cents per mile
Plan B: 50 dollars per day with free unlimited mileage
For what range of miles will plan B save you money?
Your answer is that the mileage must be greater than

53. (1 pt) setAlgebra13Inequalities/ih1-7_75.png
Find the interval on the real number line for which the radicand
\[ \sqrt{4x + 1} \]
defines a real number. Write your answer in interval notation.
**Note:** If the answer includes more than one interval write the
intervals separated by the "union" symbol, \( \cup \). If needed enter \( \infty \) as *infinity* and \( -\infty \) as *-infinity*.
For example, you may write \( (-\infty, 5] \) for the interval \( (-\infty, 5] \)
and \( (-\infty, 5) \cup (7, 9) \) for \( (-\infty, 5] \cup (7, 9) \).
Your answer:
1. (1 pt) setAlgebra14Lines/lh2-1_1.pg
Match the Lines L1 (blue), L2 (red) and L3 (green) with the slopes by placing the letter of the slopes next to each set listed below:

1. The slope of line L3
2. The slope of line L1
3. The slope of line L2

A. \( m = 0 \)
B. \( m = -0.8 \)
C. \( m = 1 \)

2. (1 pt) setAlgebra14Lines/lh2-1_5.pg
Find an equation \( y = mx + b \) for the line whose graph is sketched

The slope \( m \) equals ____
The y-intercept \( b \) equals ____

3. (1 pt) setAlgebra14Lines/lh2-1_7.pg
Find an equation \( y = mx + b \) for the line whose graph is sketched

The slope \( m \) equals ____
The y-intercept \( b \) equals ____

4. (1 pt) setAlgebra14Lines/lh2-1_9.pg
Find an equation \( y = mx + b \) for the line whose graph is sketched

The slope \( m \) equals ____
The y-intercept \( b \) equals ____

5. (1 pt) setAlgebra14Lines/sw2_4_11.pg
Find an equation \( y = mx + b \) for the line whose graph is sketched

The slope \( m \) equals ____
The y-intercept \( b \) equals ____
The number $m$ equals ____
The number $b$ equals ____

6. (1 pt) setAlgebra14LINES/SApB_7-10.png
Find the slope of the line through $(4, -3)$ and $(2, 7)$.

7. (1 pt) setAlgebra14LINES/slope_from pts_num.png
Find the slope of the line through $(-8, -6)$ and $(-4, 3)$.

8. (1 pt) setAlgebra14LINES/slope_from pts_var.png
Find the slope of the line passing through the points $(a, -1a - 6)$ and $(a + h, -1(a + 3h) - 6)$.
The slope is ____

9. (1 pt) setAlgebra14LINES/SApB_21-26.png
A line through $(7, -5)$ with a slope of 6 has a $y$-intercept at ____

10. (1 pt) setAlgebra14LINES/sw2_4_17.png
The equation of the line with slope $-1$ that goes through the point $(5, -7)$ can be written in the form $y = mx + b$ where $m$ is: ____
and $b$ is: ________

11. (1 pt) setAlgebra14LINES/sw1_10_7.png
The equation of the line with slope 3 that goes through the point $(5, 3)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

12. (1 pt) setAlgebra14LINES/sw1_10_8.png
The equation of the line with slope 5 that goes through the point $(-6, 7)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

13. (1 pt) setAlgebra14LINES/sw1_10_9.png
The equation of the line with slope $-2$ that goes through the point $(3, -4)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

14. (1 pt) setAlgebra14LINES/srw1_10_10.png
The equation of the line with slope $-2$ that goes through the point $(-7, -5)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

15. (1 pt) setAlgebra14LINES/pts_to_slope_int.png
The equation of the line with slope $-1$ that goes through the point $(-3, -4)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

16. (1 pt) setAlgebra14LINES/sw2_4_21.png
The equation of the line with slope 4 and $y$-intercept $-2$ can be written in the form $y = mx + b$ where the number $m$ is: ______
the number $b$ is: ________

17. (1 pt) setAlgebra14LINES/sw2_4_2.png
The equation of the line that goes through the points $(4, 1)$ and $(10, 6)$ can be written in the form $y = mx + b$ where the slope $m$ is: ______

18. (1 pt) setAlgebra14LINES/sw2_4_5.png
The equation of the line that goes through the points $(-5, -9)$ and $(8, 8)$ can be written in the form $y = mx + b$ where its slope $m$ is: ______

19. (1 pt) setAlgebra14LINES/sw2_4_19.png
The equation of the line that goes through the points $(5, 9)$ and $(10, 10)$ can be written in the form $y = mx + b$ where $m$ is:
and where $b$ is: ________

20. (1 pt) setAlgebra14LINES/sw1_10_11.png
The equation of the line that goes through the points $(5, 5)$ and $(8, 10)$ can be written in the form $y = mx + b$ where $m$ is:
and where $b$ is: ________

21. (1 pt) setAlgebra14LINES/sw1_10_12.png
The equation of the line that goes through the points $(-5, -3)$ and $(7, 9)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

22. (1 pt) setAlgebra14LINES/sw1_10_13.png
The equation of the line that goes through the points $(-3, 7)$ and $(9, -8)$ can be written in the form $y = mx + b$ where $m$ is:
and where $b$ is: ________

23. (1 pt) setAlgebra14LINES/pts_to_slope_int.png
The equation of the line that goes through the points $(-8, 7)$ and $(3, 3)$ can be written in the form $y = mx + b$ where $m$ is: ______
and where $b$ is: ________

24. (1 pt) setAlgebra14LINES/pts_to_gen.png
The equation of the line that goes through the points $(6, 2)$ and $(1, 3)$ can be written in general form $Ax + By + C = 0$ where $A =$ ____
$B =$ ____
The equation of the line with x-intercept 1 and y-intercept 5 can be written in the form $y = mx + b$ where the number $m$ is: ______
The number $b$ is: ______

Find the slope and y-intercept of the line $x + y = -10$. The slope of the line is: ______
The y-intercept of the line is: ______

Find the slope and y-intercept of the line $16x + 19y = 0$. The slope of the line is: ______
The y-intercept of the line is: ______

Find the slope and y-intercept of the line $14x - 19y = 15$. The slope of the line is: ______
The y-intercept of the line is: ______

Find the slope, x-intercept, and y-intercept for the line $8x - 3y + 3 = 0$. The slope is ______
The x-intercept is ______
The y-intercept is ______

Note: Your answers must be decimals.

An equation of a line through (0, 5) which is parallel to the line $y = 2x + 2$ has slope: ______ and y-intercept at: ______

The equation of the line that goes through the point (22, 32) and is parallel to the x-axis can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

The equation of the line that goes through the point (2, 4) and is parallel to the line $4x + 2y = 3$ can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

The equation of the line that goes through the point (3, 1) and is parallel to the line $4x + 3y = 2$ can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

The equation of the line that goes through the point (-7, 6) and is parallel to the line $3x + 2y = 5$ can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

The equation of the line that goes through the point (3, 4) and is parallel to the line going through the points (-2, 2) and (1, 2) can be written in the form $y = mx + b$ where $m$ is: ______ and $b$ is: ______

An equation of a line through (0, 2) which is perpendicular to the line $y = 2x + 2$ has slope: ______ and y-intercept at: ______

The equation of the line that goes through the point (8, 10) and is perpendicular to the line $5x + 4y = 2$ can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

The equation of the line that goes through the point (1, 6) and is perpendicular to the line $4x + 2y = 4$ can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

The equation of the line that goes through the point (8, 6) and is perpendicular to the line $5x + 4y = 2$ can be written in the form $y = mx + b$ where $m$ is: ______ and where $b$ is: ______

An equation of a line through (0, -5) which is parallel to the line $y = 4x + 2$ has slope: ______ and y-intercept at: ______

The equation, in general form, of the line that passes through the point (1, -8) and is parallel to the line $8x + 8y + 7 = 0$ is $Ax + By + C = 0$, where $A =$ ______ $B =$ ______ $C =$ ______

The demand equation for a certain product is given by $p = 112 - 0.055x$, where $p$ is the unit price (in dollars) of the product and $x$ is the number of units produced. The total revenue obtained by producing and selling $x$ units is given by $R = xp$. Determine prices $p$ that would yield a revenue of 5110 dollars. Lowest such price = ______

Highest such price = ______

The line whose equation is $2x - 5y = -10$ goes through the point $(-6, t)$ for $t =$ ______
The line through \((-8, 6)\) and \((11, -8)\) also goes through the point \((t, 0)\) for

\[ t = \underline{\phantom{0}} \]
Express the rule "Multiply by 6, then add 25" as the function 
\[ f(x) = \] 

Express the rule "Subtract 16, then square" as the function 
\[ f(x) = \] 

Given the function \( f(x) = \frac{x}{13} + 8 \) can be expressed in words as "Add 8, then divide by 13". Is this statement true? 
Your answer is (input Yes or No): 

Given the function \( f(x) = 4x^2 - 7 \) can be expressed in words as "Square, multiply by 4, then subtract 7". Is this statement true? 
Your answer is (input Yes or No): 

Given the function \( f(x) = 3x^2 - 7x + 4 \). Calculate the following values: 
\[ f(-2) = \] 
\[ f(-1) = \] 
\[ f(0) = \] 
\[ f(1) = \] 
\[ f(2) = \] 

Let \( f(x) \) be the function \( \frac{x}{x+1} - 1 \). Find the following: 
\[ f(9) = \] 
\[ f(-5) = \] 
\[ f\left(\frac{1}{6}\right) = \] 
\[ f\left(-\frac{1}{8}\right) = \] 

Given the function \( f(x) = 6x^2 + 8x - 2 \). Calculate the following values: 
\[ f(0) = \] 
\[ f(2) = \] 
\[ f(-2) = \] 
\[ f(x+1) = \] 
\[ f(-x) = \] 

Given the function \( f(x) = 8|x - 8| \), calculate the following values: 
\[ f(0) = \] 
\[ f(2) = \] 
\[ f(-2) = \] 
\[ f(x+1) = \] 
\[ f(x^2 + 2) = \] 

Note: In your answer, you may use abs\(g(x)\) for \(|g(x)|\).

Given the function \( f(x) = \begin{cases} 
2x + 1, & \text{if } x \leq -1 \\
x + 7, & \text{if } x > 1 \end{cases} \) 
Calculate the following values: 
\[ f(-8) = \] 
\[ f(-1) = \] 
\[ f(3) = \] 

Given the function \( f(x) = 5x^2 - 4x + 6 \). Calculate the following values: 
\[ f(-2) = \] 
\[ f(-1) = \] 
\[ f(0) = \] 
\[ f(1) = \] 
\[ f(2) = \] 

Given the function \( f(x) = 4x^2 + 6x - 4 \). Calculate the following values: 
\[ f(0) = \] 
\[ f(2) = \] 
\[ f(-2) = \] 
\[ f(x+1) = \] 
\[ f(-x) = \] 

Given the function \( f(x) = 2|x - 7| \). Calculate the following values: 
\[ f(0) = \] 
\[ f(2) = \] 
\[ f(-2) = \] 
\[ f(x+1) = \] 
\[ f(x^2 + 2) = \] 

In your answer, use abs\(g(x)\) for \(|g(x)|\).

Let \( f(x) = 3x^2 + x - 12 \). Find 
\[ f(0) \] 
\[ f(-1) \] 
\[ f(5) \] 
\[ f\left(\sqrt{2}\right) \] 
\[ f\left(1 + \sqrt{2}\right) \] 

Let \( f(x) = 4x^3 + 5x^2 + 5x + 5 \). Find \( f\left(1 + \sqrt{2}\right) \).

Given the function \( f(x) = \begin{cases} 
3x - 5, & \text{if } x < 0 \\
3x - 10, & \text{if } x \geq 0 \end{cases} \)
Calculate the following values:
\[ f(-1) = \]  
\[ f(0) = \]  
\[ f(2) = \]

16. (1 pt setAlgebra15Functions/lh2-2_36.png)
Given the function
\[ f(x) = \begin{cases} 
-3x^2 + 5 & \text{if } x < 1 \\
-6x^2 + 5 & \text{if } x \geq 1 
\end{cases} \]

Calculate the following values:
\[ f(-2) = \]  
\[ f(1) = \]  
\[ f(2) = \]  

17. (1 pt setAlgebra15Functions/faris1.png)
Let
\[ f(x) = \frac{x + 6}{3x - 3}. \]

Compute the following values. If one is not defined, type Undefined.
\[ f(0) = \]  
\[ f(9) = \]  
\[ f(1) = \]  

18. (1 pt setAlgebra15Functions/sw4_1_21.png)
Let \( f(x) = 17. \) Calculate the following values:
\[ f(a) = \]  
\[ f(a + h) = \]  
\[ \frac{f(a + h) - f(a)}{h} = \] for \( h \neq 0 \)

19. (1 pt setAlgebra15Functions/sw4_1_33.png)
Let \( f(x) = 3 - 5x + 6x^2. \) Calculate the following values:
\[ f(a) = \]  
\[ f(a + h) = \]  
\[ \frac{f(a + h) - f(a)}{h} = \] for \( h \neq 0 \)

20. (1 pt setAlgebra15Functions/s0_1_2.png)
Let \( f(x) = 2x^2 + 4x + 4 \) and let \( g(h) = \frac{f(2 + h) - f(2)}{h}. \)

Determine each of the following:
(a) \( g(1) = \)  
(b) \( g(0.1) = \)  
(c) \( g(0.01) = \) 

You will notice that the values that you entered are getting closer and closer to a number \( L. \) This number is called the limit of \( g(h) \) as \( h \) approaches 0 and is also called the derivative of \( f(x) \) at the point when \( x = 2. \) We will see more of this when we get to the calculus textbook.

Enter the value of \( L: \)

21. (1 pt setAlgebra15Functions/s0_1_2a.png)
Let \( f(x) = 5x^2 + 3x + 5 \) and let \( q(h) = \frac{f(3 + h) - f(3)}{h}. \) Then
\[ q(0.01) = \]

22. (1 pt setAlgebra15Functions/sw2_1_25.png)
Given the function \( f(x) = -8 + x^2, \) calculate the following values:
\[ f(x + 1) = \]  
\[ f(x) + f(5) = \]

23. (1 pt setAlgebra15Functions/sw2_1_29.png)
Given the function \( f(x) = 5x - 6, \) calculate the following values:
\[ f(a) = \]  
\[ f(a + h) = \]  
\[ \frac{f(a + h) - f(a)}{h} = \]

24. (1 pt setAlgebra15Functions/sw2_1_33.png)
Given the function \( f(x) = 4 + 2x^2, \) calculate the following values:
\[ f(a) = \]  
\[ f(a + h) = \]  
\[ \frac{f(a + h) - f(a)}{h} = \]

25. (1 pt setAlgebra15Functions/nc1s1p1.png)
Find the domain of this function:
\[ \sqrt[3]{-4 + 2x} \]

(which reads the 3rd root of \(-4 + 2x\)).
The function is defined on the interval from \( \) to \( \)

Use INF for infinity or -INF for minus infinity.

Now find the domain of this function:
\[ \sqrt[4]{-4 + 2x} \]

(which reads the 4th root of \(-4 + 2x\)).
The function is defined on the interval from \( \) to \( \)

26. (1 pt setAlgebra15Functions/s0_1_10.png)
The domain of the function \( f(x) = \frac{33}{7x - 15} \) is all real numbers \( x \) except for \( x \) where \( x \) equals \( \)

27. (1 pt setAlgebra15Functions/s0_1_11.png)
The domain of the function \( f(x) = \sqrt{-5x + 40} \) consists of one or more of the following intervals: \( (-\infty, A] \) and \( [A, \infty). \)

Find \( A \)

For each interval, answer YES or NO to whether the interval is included in the solution.
\( (-\infty, A) \)  
\( (A, \infty) \)

28. (1 pt setAlgebra15Functions/s0_1_11a.png)
The domain of the function \( f(x) = \sqrt[4]{4x - 43} \) is all real numbers in the interval \( [A, \infty) \) where \( A \) equals \( \)

29. (1 pt setAlgebra15Functions/s0_1_18.png)
The domain of the function \( f(x) = \sqrt{-x^2 + 8x - 15} \) consists of one or more of the following intervals: \( (-\infty, A], [A, B] \) and \( [B, \infty) \)

where \( A < B. \)

Find \( A \)
Find $B$ _______

For each interval, answer YES or NO to whether the interval is included in the solution.

$(\infty, A] \quad \quad \quad [A, B] \quad \quad \quad [B, \infty)$

**Exercise 30** (1 pt) setAlgebra15Functions/s0_1_18a.png

The domain of the function $f(x) = \sqrt{24 + 5x - x^2}$ is the closed interval $[A, B]$ where $A =$ ________
and $B =$ ________

**Exercise 31** (1 pt) setAlgebra15Functions/s0_1_18a-sol.png

The domain of the function $f(x) = \sqrt{16 + 6x - x^2}$ is the closed interval $[A, B]$ where $A =$ _______
and $B =$ ________

**Exercise 32** (1 pt) setAlgebra15Functions/p1.png

The domain of the function

$$\frac{1}{\sqrt{4x + 8}}$$

is ________

Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter $-\infty$ as $-\infty$ and $\infty$ as $\infty$.

**Exercise 33** (1 pt) setAlgebra15Functions/p2.png

The domain of the function

$$\sqrt{\frac{10x}{x^2 - 289}}$$

is _______

Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter $-\infty$ as $-\infty$ and $\infty$ as $\infty$.

**Exercise 34** (1 pt) setAlgebra15Functions/p3.png

The domain of the function

$$\sqrt{x(x - 3)}$$

is _______

Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter $-\infty$ as $-\infty$ and $\infty$ as $\infty$.

**Exercise 35** (1 pt) setAlgebra15Functions/ur_fn_1_4.png

Find the domain of the function $f(x) = \sqrt{x^3 - 9x}$. What is the least value of $x$ in the domain?

Least Value = ________

**Exercise 36** (1 pt) setAlgebra15Functions/ur_fn_1_2.png

Find the domain of the function $f(x) = \frac{1}{6x + 3}$. What is the only value of $x$ not in the domain?

Only Value = ________

**Exercise 37** (1 pt) setAlgebra15Functions/ur_fn_1_3.png

Find the domain of the function $f(x) = \sqrt{\frac{1}{x^2 + 10x - 24}}$. What is the greatest value of $x$ not in the domain?

Greatest Value = ________

**Exercise 38** (1 pt) setAlgebra15Functions/ur_fn_1_4.png

Find the domain of the function $f(x) = \sqrt{\frac{7 - 5x}{11 + 3x}}$. What is the greatest value of $x$ in the domain?

Greatest Value = ________

**Exercise 39** (1 pt) setAlgebra15Functions/ur_fn_1_6.png

Define a function $f(x)$ by:

$$f(x) = \begin{cases} 
9 - 10x, & \text{if } x \geq 10 \\
100 - x^2, & \text{if } x < 10
\end{cases}$$

$f(19)$ = ______

$f(6)$ = ______

Looking only at values of $x$ to the left of 10, what would you expect $f(10)$ to be? ______

Looking only at values of $x$ to the right of 10, what would you expect $f(10)$ to be? ______

Now for fun, try graphing $f(x)$ . . .

**Exercise 40** (1 pt) setAlgebra15Functions/ur_fn_1_7.png

For $x < \frac{1}{2}$, the function $f(x) = |2x - 1| + 10$ is equivalent to the function $g(x) = mx + b$ for:

$m =$ ______

and $b =$ ______

Now for fun, try graphing $f(x)$ . . .

**Exercise 41** (1 pt) setAlgebra15Functions/p4.png

Find domain and range of the function

$$6x^2 - 2$$

Domain: ________

Range: ________

Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter $-\infty$ as $-\infty$ and $\infty$ as $\infty$.

**Exercise 42** (1 pt) setAlgebra15Functions/p5.png

Find domain and range of the function

$$5x^2 + 11$$

Domain: ________

Range: ________

Write the answer in interval notation.

**Note:** If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter $-\infty$ as $-\infty$ and $\infty$ as $\infty$.

**Exercise 43** (1 pt) setAlgebra15Functions/p6.png

Find domain and range of the function

$$\sqrt{x + 12}$$
Domain: __________________________
Range: __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

44. (1 pt) setAlgebra15Functions/p7.pg
The domain of the function
\[
\frac{x + 16}{x^2 - 1}
\]
is __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

45. (1 pt) setAlgebra15Functions/p8.pg
Find domain and range of the function
\[13\sqrt{x} - 3\]
Domain: __________________________
Range: __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

46. (1 pt) setAlgebra15Functions/p9.pg
Find the domain of the function
\[(x + 2)(x - 13)\]
Domain: __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

47. (1 pt) setAlgebra15Functions/srw2_1_44.pg
The domain of the function
\[17x^2 + 3\]
is __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

48. (1 pt) setAlgebra15Functions/srw2_1_45.pg
The domain of the function
\[12x + 8, -20 \leq x \leq 19\]
is __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

49. (1 pt) setAlgebra15Functions/srw2_1_49.pg
The domain of the function
\[
\frac{x + 9}{x^2 - 400}
\]
is __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

50. (1 pt) setAlgebra15Functions/srw2_1_53.pg
The domain of the function
\[\sqrt{t - 62}\]
is __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

51. (1 pt) setAlgebra15Functions/srw2_1_55.pg
The domain of the function
\[\sqrt{2x - 22}\]
is __________________________
Write the answer in interval notation.

Note: If the answer includes more than one interval write the intervals separated by the union symbol, U. If needed enter -∞ as - infinity and ∞ as infinity.

52. (1 pt) setAlgebra15Functions/s0_1_77-82.pg
For each of the following functions, decide whether it is even, odd, or neither. Enter E for an EVEN function, O for an ODD function and N for a function which is NEITHER even nor odd.

Note: You will only have four attempts to get this problem right!

1. \[f(x) = x^3 + x^3 + x^3\]
2. \[f(x) = x^{-2}\]
3. \[f(x) = x^2 - 6x^4 + 3x^2\]
4. \[f(x) = -5x^2 - 3x^4 - 2\]

53. (1 pt) setAlgebra15Functions/srw2_2_51.pg
The function \[f(x) = x^{-2}\] is ________ (enter even, odd, or neither).

54. (1 pt) setAlgebra15Functions/srw2_2_53.pg
The function \[f(x) = x^3 + x^3\] is ________ (enter even, odd, or neither).

55. (1 pt) setAlgebra15Functions/srw2_2_55.pg
The function \[f(x) = x^3 - x^3\] is ________ (enter even, odd, or neither).

56. (1 pt) setAlgebra15Functions/srw2_8_9.pg
The domain of the function \[h(x) = (x + 12)^2(2x - 22)^{1/4}\] is

Write the answer in interval notation.
57. (1 pt) setAlgebra15Functions/sw2_2_41_51.pg
Enter Yes or No in each answer space below to indicate whether the equation defines \( y \) as a function of \( x \).

\[
\begin{align*}
1. \quad & 9 + x = y^3 \\
2. \quad & x + 2 = y^2 \\
3. \quad & 2|x| + y = 10 \\
4. \quad & 2x = y^2
\end{align*}
\]

Note: Be careful, You only have TWO chances to get them right.

58. (1 pt) setAlgebra15Functions/beth2.pg
List all real values of \( x \) such that \( f(x) = 0 \). If there are no such real \( x \), type DNE in the answer blank. If there is more than one real \( x \), give a comma separated list (e.g. 1.2).

\[
f(x) = \frac{2x^2 - 2x - 180}{-2x^2 - 68x - 560}
\]

\[x = \]

59. (1 pt) setAlgebra15Functions/beth3.pg
List all real values of \( x \) such that \( f(x) = 0 \). If there are no such real \( x \), type DNE in the answer blank. If there is more than one real \( x \), give a comma separated list (e.g. 1.2).

\[
f(x) = \frac{7}{x - 11} + \frac{11}{x - 14}
\]

\[x = \]

60. (1 pt) setAlgebra15Functions/beth4.pg
List all real values of \( x \) such that \( f(x) = 0 \). If there are no such real \( x \), type DNE in the answer blank. If there is more than one real \( x \), give a comma separated list (e.g. 1.2).

\[
f(x) = 14 + \frac{-15}{x + 10}
\]

\[x = \]

61. (1 pt) setAlgebra15Functions/beth5.pg
List all real values of \( x \) such that \( f(x) = 0 \). If there are no such real \( x \), type DNE in the answer blank. If there is more than one real \( x \), give a comma separated list (e.g. 1.2).

\[
f(x) = \frac{-19x - 17}{-1}
\]

\[x = \]

62. (1 pt) setAlgebra15Functions/beth6.pg
List all real values of \( x \) such that \( f(x) = 0 \). If there are no such real \( x \), type DNE in the answer blank. If there is more than one real \( x \), give a comma separated list (e.g. 1.2).

\[
f(x) = 19x - 13
\]

\[x = \]

63. (1 pt) setAlgebra15Functions/beth7.pg
List all real values of \( x \) such that \( f(x) = 0 \). If there are no such real \( x \), type none in the answer blank. If there is more than one real \( x \), give a comma separated list (e.g. 1.2).

\[
f(x) = -12x^2 + 8x + 17
\]

\[x = \]

64. (1 pt) setAlgebra15Functions/beth8.pg
A company that makes thing-a-ma-bobs has a start up cost of $40801. It costs the company $1.38 to make each thing-a-ma-bob and the company charges $5.66 for each thing-a-ma-bob. Let \( x \) represent the number of thing-a-ma-bobs made.

Write the cost function for this company. \( C(x) = \) 

Write the revenue function for this company. \( R(x) = \) 

Write the profit function for this company. \( P(x) = R(x) - C(x) = \)

What is the minimum number of thing-a-ma-bobs that the company must produce and sell to make a profit? \( \text{answer} = \)

65. (1 pt) setAlgebra15Functions/8.pg
On a remote tropical island, the average life expectancy of women is given by the model

\[y = \sqrt{6000 + 67x - 2.2x^2 + .04x^3},\]

where \( y \) is the "average life expectancy for a woman" since 1980. Thus, \( x \) is the number of years 1980. What is the predicted average life expectancy of a woman in the year 2010?

Average Life Expectancy = 

66. (1 pt) setAlgebra15Functions/box.pg
An open box is to be made from a flat piece of material 10 inches long and 6 inches wide by cutting equal squares of length \( x \) from the corners and folding up the sides.

Write the volume \( V \) of the box as a function of \( x \). Leave it as a product of factors, do not multiply out the factors.

\[V = \]

If we write the domain of the box as an open interval in the form \((a,b)\), then what is \( a = ? \)

\[a = \]

and what is \( b = ? \)

\[b = \]

67. (1 pt) setAlgebra15Functions/jay4.pg
An open box is to be made from a flat square piece of material 8 inches in length and width by cutting equal squares of length \( x \) from the corners and folding up the sides.

Write the volume \( V \) of the box as a function of \( x \). Leave it as a product of factors; you do not have to multiply out the factors.

\[V = \]

If we write the domain of the box as an open interval in the form \((a,b)\), then what is \( a = ? \)

\[a = \]

and what is \( b = ? \)

\[b = \]
68. A company produces very unusual CD’s for which the variable cost is $20 per CD and the fixed costs are $35000. They will sell the CD’s for $100 each. Let x be the number of CD’s produced. Write the total cost C as a function of the number of CD’s produced.

\[ C = \$ \ldots \]

Write the total revenue R as a function of the number of CD’s produced.

\[ R = \$ \ldots \]

Write the total profit P as a function of the number of CD’s produced.

\[ P = \$ \ldots \]

Find the number of CD’s which must be produced to break even. The number of CD’s which must be produced to break even is ________.

69. The altitude of a right triangle is 7 cm. Let h be the length of the hypotenuse and let p be the perimeter of the triangle. Express h as a function of p.

\[ h(p) = \ldots \]

70. At the surface of the ocean, the water pressure is the same as the air pressure above the water, about 15 lb/in². Below the surface the water pressure increases by about 5.04 lb/in² for every 10 ft of descent. Write a function \( f(x) \) which expresses the water pressure in pounds per square inch as a function of the depth in inches below the ocean surface.

\[ f(x) = \ldots \]

At what depth is the pressure 120 lb/in²? Include the units in your answer: ________.
1. (1 pt) setAlgebra16FunctionGraphs/sw4_2_1.pg
For the function $h(x)$ given in the graph

its domain is __________;
its range is __________;
Write the answer in interval notation.
and then enter the corresponding function value in each answer space below:

- 1. $h(3)$
- 2. $h(-2)$
- 3. $h(2)$
- 4. $h(1)$

2. (1 pt) setAlgebra16FunctionGraphs/c4s2p5_7/c4s2p5_7.pg
Enter Yes or No in each answer space below to indicate whether the corresponding curve defines $y$ as a function of $x$. 

- 1. 
- 2. 
- 3. 
- 4. 

- Curve i
- Curve j
- Curve k
- Curve b
Consider the function given in the following graph.

What is its domain? __________
What is its range? __________

Note: Write the answer in interval notation.

Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function.

1. The graph of the line is increasing
2. The graph of the line is decreasing
3. The graph of the line is constant
4. The graph of the line is not the graph of a function

Enter Yes or No in each answer space below to indicate whether the corresponding equation defines \( y \) as a function of \( x \).

Note: Be careful, You only have TWO chances to get them right.

1. \( 9 + x = y^3 \)
2. \( x^2 + 2y = 9 \)
3. \( x + 9 = y^2 \)
4. \( 9x = y^2 \)

Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function.

1. Piecewise function: \( f(x) = x \) if \( x \leq 0 \), and \( f(x) = x + 1 \) if \( x > 0 \)
2. Piecewise function: \( f(x) = 2x + 3 \) if \( x < -1 \), and \( f(x) = 3 - x \) if \( x \geq -1 \)
3. Piecewise function: \( f(x) = 1 - x \) if \( x < -2 \), and \( f(x) = 4 \) if \( x \geq -2 \)
4. Piecewise function: \( f(x) = -1 \) if \( x < 2 \), and \( f(x) = 1 \) if \( x \geq 2 \)

The simplest functions are the linear (or affine) functions — the functions whose graphs are a straight line. They are important because many functions (the so-called differentiable functions) “locally” look like straight lines. (“locally” means that if we zoom in and look at the function at very powerful magnification it will look like a straight line.)

Enter the letter of the graph of the function which corresponds to each statement.

1. The graph of the line is increasing
2. The graph of the line is decreasing
3. The graph of the line is constant
4. The graph of the line is not the graph of a function
8. (1 pt) setAlgebra16FunctionGraphs/hh2-3_30a.pg

Consider the function whose graph is sketched:

Find the intervals over which the function is increasing or decreasing. If the answer includes more than one interval write the intervals separated by the "union" symbol, U. You may use "infinity" for $\infty$ and "-infinity" for $-\infty$. For example, you may write (-infinity, 5] for the interval $(-\infty, 5]$ and (-infinity, 5]U(7,9) for $(-\infty, 5] \cup (7,9)$.

The interval over which the function is increasing:

The interval over which the function is decreasing:

9. (1 pt) setAlgebra16FunctionGraphs/hh2-3_48a.pg

Consider the function whose graph is sketched:

Find the intervals over which the function is increasing or decreasing. If the answer includes more than one interval write the intervals separated by the "union" symbol, U. You may use "infinity" for $\infty$ and "-infinity" for $-\infty$. For example, you may write (-infinity, 5] for the interval $(-\infty, 5]$ and (-infinity, 5]U(7,9) for $(-\infty, 5] \cup (7,9)$.

The interval over which the function is increasing:

The interval over which the function is decreasing:

10. (1 pt) setAlgebra16FunctionGraphs/c0s1p2/c0s1p2.pg

Enter the letter of the graph of the function which corresponds to each statement.

1. The graph of the line is not the graph of a function
2. The graph of the line is constant
3. The graph of the line is increasing
4. The graph of the line is decreasing

Almost any kind of quantitative data can be represented by a graph and most of these graphs represent functions. This is why functions and graphs are the objects analyzed by calculus. The next two problems illustrate data which can be represented by a graph. Match the following descriptions with their graphs below:

1. The graph of the velocity of a car entering a superhighway vs. time.
2. The graph of the velocity of a car as it drives along a city street vs. time.
3. The graph of the distance traveled by a car as it drives along a city street vs. time.
4. The graph of the distance traveled by a car as it enters a superhighway vs. time.

12. (1 pt) setAlgebra16FunctionGraphs/c0s1p4/c0s1p4.pg
Match the following descriptions with their graphs below:

1. The graph of the number of days until next Friday vs. time.
2. The graph of the amount of time until midnight next Friday as a function of time.
3. The graph of the number of days to the nearest Friday (in the future or in the past) as a function of time.
4. The graph of the amount of time to the nearest Friday at midnight vs. time.

13. (1 pt) setAlgebra16FunctionGraphs/c0s1p7/c0s1p7.pg
The following questions concern the profits of firm N. The graph of the profits vs. time is given above. For each of the intervals enter the letters corresponding to the descriptions which describe the behavior of the graph on that interval. (The letters in each answer must be in alphabetical order with no spaces between the letters.)

1. The interval from a to b
2. The interval from b to c
3. The interval from c to d

14. (1 pt) setAlgebra16FunctionGraphs/c0s1p8/c0s1p8.png
The function above represents the velocity of a race car as it travels a linear track. Negative velocities mean the car is backing up.
For each interval, enter all letters whose corresponding statements are true for that interval.

1. The interval from a to b
2. The interval from b to c
3. The interval from c to d
4. The interval from d to e
5. The interval from e to f

A. The firm makes a profit on this interval.
B. The firm registers a loss on this interval.
C. The profit of the firm increases on this interval.
D. The profit of the firm decreases on this interval.
E. Assuming the profits are reinvested in the firm the networth of the company is increasing on this interval.
F. Assuming the profits are reinvested in the firm the networth of the company is decreasing on this interval.
The function above represents the displacement of a toy race car as it travels a linear track. Negative numbers mean the car is behind the starting line, positive numbers mean it is in front. Positive velocities mean it is moving forward, while negative velocities mean it is moving backwards. Remember that a value which changes from -2 to -1 to 0 is increasing!

For each interval, enter all letters whose corresponding statements are true for that interval.

1. The interval from a to b
2. The interval from b to c
3. The interval from c to d
4. The interval from d to e
5. The interval from e to f

A. The car is in front of the starting line on this interval
B. The car is behind the starting line on this interval.
C. The velocity of the car is positive on this interval.
D. The velocity of the car is negative on this interval.
E. The displacement of the car from the starting line is increasing on this interval.
F. The displacement of the car from the starting line is decreasing on this interval.

A 5 gram weight is suspended from a string next to a ruler held vertically. The string is jiggled up and down and the graph of the POSITION of the weight vs. time in seconds is given above. The ruler is calibrated in inches and 0 is in the center of the ruler. Enter the letters for the intervals which correspond to the statements below.

1. The interval from a to b
2. The interval from b to c
3. The interval from c to d
4. The interval from d to e
5. The interval from e to f

A. The weight is moving upward on this interval.
B. The weight is moving downward on this interval.
C. The upward velocity of the weight is increasing on this interval.
D. The upward velocity of the weight is decreasing on this interval.
E. The (signed) distance from the starting point is increasing on this interval.
F. The (signed) distance from the starting point is decreasing on this interval.

The graph indicates the RATE of absorption of carbon dioxide into a body of water. The rate varies with time. Positive quantities mean that the carbon dioxide is being absorbed into solution, while negative quantities mean the carbon dioxide is being released to the air.

For each interval, enter all letters whose corresponding statements are true for that interval.

1. The interval from a to b
2. The interval from b to c
3. The interval from c to d
4. The interval from d to e
5. The interval from e to f

A. Carbon dioxide is being absorbed by the water on this interval.
B. Carbon dioxide is being released from the water on this interval.
C. The rate at which the carbon dioxide is being absorbed is increasing on this interval.
D. The rate at which the carbon dioxide is being absorbed is decreasing on this interval.
E. The total amount of carbon dioxide in the water is increasing on this interval.
18. (1 pt) setAlgebra16FunctionGraphs/c0s1p13/c0s1p13.pg

Answer the questions about the function whose graph is shown above.
Enter the letters for the intervals which correspond to the statements below. The letters for each entry should be in alphabetical order with no spaces.

1. The interval from a to b
2. The interval from b to c
3. The interval from c to d
4. The interval from d to e
5. The interval from e to f

A. The function is increasing on this interval.
B. The function is decreasing on this interval.
C. The slope of the function is increasing on this interval.
D. The slope of the function is decreasing on this interval.
E. The total (signed) area between the graph of the function and the x axis is increasing on this interval.
F. The total (signed) area between the graph of the function and the x axis is decreasing on this interval.

19. (1 pt) setAlgebra16FunctionGraphs/c0s1p14/c0s1p14.pg

Determine which of the following statements are true and which are false. Enter the T or F in front of each statement.

Remember that \( x \in (-1,1) \) is the same as \(-1 < x < 1\) and \( x \in [-1,1] \) means \(-1 \leq x \leq 1\).

1. The function \( f(x) = x^2 \) with domain \( x \in (-3,3) \) has at least one input which produces a largest output value.
2. The function \( f(x) = x^2 \) with domain \( x \in [-3,3] \) has at least one input which produces a largest output value.
3. The function \( \sin(x) \) on the domain \( x \in [-\pi/2, \pi/2] \) has at least one input which produces a smallest output value.
4. The function \( f(x) = x^2 \) with domain \( x \in [-3,3] \) has at least one input which produces a smallest output value.
5. The function \( \sin(x) \) on the domain \( x \in (-\pi/2, \pi/2) \) has at least one input which produces a smallest output value.

20. (1 pt) setAlgebra16FunctionGraphs/c0s5p3.pg

Determine which of the following statements are true and which are false. Enter the T or F in front of each statement.

Remember that \( x \in (-1,1) \) is the same as \(-1 < x < 1\) and \( x \in [-1,1] \) means \(-1 \leq x \leq 1\).

1. The function \( f(x) = x^2 \) with domain \( x \in (-3,3) \) has at least one input which produces a largest output value.
2. The function \( f(x) = x^3 \) with domain \( x \in [-3,3] \) has at least one input which produces a largest output value.
3. The function \( f(x) = x^3 \) with domain \( x \in [-3,3] \) has at least one input which produces a smallest output value.
4. The function \( f(x) = x^3 \) with domain \( x \in (-3, 3) \) has at least one input which produces a smallest output value.

5. The function \( \sin(x) \) on the domain \( x \in (-\pi, \pi) \) has at least one input which produces a smallest output value.

22. (1 pt) setAlgebra16FunctionGraphs/ns1_1_45.png

Write the equation describing the graph above:

\[
f(x) = \begin{array}{ll}
\text{for } x \text{ in the interval } & \\
\text{for } x \text{ in the interval } & 
\end{array}
\]

23. (1 pt) setAlgebra16FunctionGraphs/ns1_1_2.png

Given the graphs of \( f \) (in blue) and \( g \) (in red) to the left answer these questions:

1. What is the value of \( f \) at \(-4\)?
2. For what values of \( x \) is \( f(x) = g(x) \)? Separate answers by spaces (e.g. “5 7”)
3. Estimate the solution of the equation \( g(x) = -4 \)
4. On what interval is the function \( f \) decreasing? (Separate answers by a space: e.g. “-2 4”)

24. (1 pt) setAlgebra16FunctionGraphs/sc_c1s3p2.png

Use a graphing calculator to find the largest value of \( x \) which satisfies \( x^4 - 1.000x + 3.000 = 3.000x^3 + 0.000x^2 \). Give the answer to 2 decimal places.

Remember to calculate the trig functions in radian mode.
Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function. (Click on image for a larger view)

1. Piecewise function: \( f(x) = 1 - x^2 \), if \( x \leq 2 \) and \( f(x) = x \), if \( x > 2 \)
2. Piecewise function: \( f(x) = 2 \), if \( x \leq -1 \) and \( f(x) = x^2 \), if \( x > -1 \)
3. Piecewise function: \( f(x) = 2x + 3 \), if \( x < -1 \) and \( f(x) = 3 - x \), if \( x \geq -1 \)
4. Piecewise function: \( f(x) = 3 \), if \( x < 2 \) and \( f(x) = x - 1 \), if \( x \geq 2 \)

Consider the function shown in the following graph. Where is the function decreasing?

Note: use interval notation to enter your answer.

Enter Yes or No in each answer space below to indicate whether the corresponding equation defines \( y \) as a function of \( x \).

Note: Be careful. You only have TWO chances to get them right.

1. \( x + 4 = y^2 \)
2. \( x^2y + y = 2 \)
3. \( x^2 + 1y = 10 \)
4. \( 3 + x = y^3 \)

Enter Yes or No in each answer space below to indicate whether the corresponding equation defines \( y \) as a function of \( x \).
Note: Be careful, You only have TWO chances to get them right.

31. (1 pt) setAlgebra16FunctionGraphs/4.pg
Enter Yes or No in each answer space below to indicate whether the corresponding equation defines \( y \) as a function of \( x \).

Note: Be careful, You only have TWO chances to get them right.

\[ \begin{align*}
&1. \quad 9x = y^2 \\
&2. \quad x^2y + y = 3 \\
&3. \quad 3 + x = y^3 \\
&4. \quad x + 9 = y^2
\end{align*} \]

32. (1 pt) setAlgebra16FunctionGraphs/5.pg
Enter Yes or No in each answer space below to indicate whether the corresponding equation defines \( y \) as a function of \( x \).

Note: Be careful, You only have TWO chances to get them right.

\[ \begin{align*}
&1. \quad 10 + x = y^3 \\
&2. \quad 2|x| + y = 8 \\
&3. \quad x^2 + 4y = 6 \\
&4. \quad x + 5 = y^2
\end{align*} \]

33. (1 pt) setAlgebra16FunctionGraphs/6.pg
Enter Yes or No in each answer space below to indicate whether the corresponding equation defines \( y \) as a function of \( x \).

Note: Be careful, You only have TWO chances to get them right.

\[ \begin{align*}
&1. \quad 3 + x = y^3 \\
&2. \quad x^2 + 4y = 10 \\
&3. \quad 8x = y^2 \\
&4. \quad 2|x| + y = 8
\end{align*} \]

34. (1 pt) setAlgebra16FunctionGraphs/c1s1p2.pg
Let \( p(x) = 3.5x^{0.9} \). Use a calculator or a graphing program to find the slope of the tangent line to the point \((x, p(x))\) when \( x = 3.6 \). Give the answer to 3 places.

35. (1 pt) setAlgebra16FunctionGraphs/se_c1s3p1.pg
Use a graphing calculator to find the positive value of \( x \) which satisfies \( x = 1.7 \cos(x) \). Give the answer to 2 decimal places. Remember to calculate the trig functions in radian mode.

If you don’t have a graphing calculator you can use the program Xfunctions which is installed on most of the Macintoshes in CLARC (except for the ones in the Mac classrooms). The program is free and you can download it for your own computer – see Mac Software – if you have a Mac. If you have a PC try the CD that came with the textbook – see if that will graph equations for you.
This problem tests calculating new functions from old ones. From the table below calculate the quantities asked for:

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>1</th>
<th>-6</th>
<th>3</th>
<th>-2</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>21</td>
<td>-5</td>
<td>186</td>
<td>-39</td>
<td>6</td>
<td>-258</td>
</tr>
<tr>
<td>g(x)</td>
<td>-21</td>
<td>3</td>
<td>-186</td>
<td>39</td>
<td>-6</td>
<td>258</td>
</tr>
</tbody>
</table>

\[ g(f(-2)) = \] 
\[ (fg)(1) = \] 
\[ (f+g)(-2) = \]

Let \( f(x) = 3x^2 + 5 \) and \( g(x) = 2x^2 + 5x \).
\[ (f+g)(2) = \]

Let \( f(x) = 2x + 5 \) and \( g(x) = 4x^2 + 3x \).
After simplifying, 
\[ (f+g)(x) = \]

Let \( f(x) = 5x + 4 \) and \( g(x) = 2x^2 + 4x \).
\[ (fg)(4) = \]

Let \( f(x) = 3x + 4 \) and \( g(x) = 3x^2 + 5x \).
After simplifying, 
\[ (fg)(x) = \]

This problem gives you some practice identifying how more complicated functions can be built from simpler functions. Let \( f(x) = x^3 + 1 \) and let \( g(x) = x + 1 \). Match the functions defined below with the letters labeling their equivalent expressions.

1. \( g(x^2) \)
2. \( g(x)f(x) \)
3. \( f(g(x)) \)
4. \( f(x^2) \)
   - A. \( 1+x^6 \)
   - B. \( 1+x+x^3+x^4 \)
   - C. \( 1+x^2 \)
   - D. \( 2+3x+3x^2+x^3 \)

Let \( f(x) = x^3 + 9x^2 \) and \( g(x) = 2x^2 - 12 \).
\( f/g \) is undefined at two points A and B where \( A < B \). 
\( A = \), 
\( B = \)

Let \( f(x) = x^3 + 4x^2 \) and \( g(x) = 3x^2 - 6 \).
\( f/g \) is undefined at two points A and B where \( A < B \). 
\( A = \), 
\( B = \)

Given that \( f(x) = \sqrt{x+1} \) and \( g(x) = \sqrt{1-x} \),
\( a) f+g= \) and its domain is \( \)  
\( b) f-g= \) and its domain is \( \)  
\( c) fg= \) and its domain is \( \)  
\( d) f/g= \) and its domain is \( x \neq \) 
Note: If needed enter \( \infty \) as infinity and \( -\infty \) as -infinity.

Given that \( f(x) = x^2 - 3x \) and \( g(x) = x + 14 \), find
\( a) f+g= \) 
\( b) f-g= \) 
\( c) fg= \) 
\( d) f/g= \)

For the function \( f(x) \) and \( g(x) \) given in the graph

\[ (f+g)(4) = \]
\[ (f-g)(0) = \]

Given that \( f(x) = \frac{2}{x-3} \) and \( g(x) = \frac{14}{x+12} \), find
\( a) f+g= \) and its domain is \( \)  
\( b) f-g= \) and its domain is \( \)  
\( c) fg= \) and its domain is \( \)

\[ \text{Note: If needed enter } \infty \text{ as infinity and } -\infty \text{ as -infinity.} \]
Given that $f(x) = x^2 - 5x$ and $g(x) = x + 9$, find

(a) $f + g =$ ________ and its domain is ________

(b) $f - g =$ ________ and its domain is ________

(c) $fg =$ ________ and its domain is ________

(d) $f/g =$ ________ and its domain is ________

**Note:** If the answer includes more than one interval write the intervals separated by the 'union' symbol, U. If needed enter $\infty$ as infinity and $-\infty$ as -infinity.

---

**14.** (1 pt) setAlgebra17FunComposition/sw2_8a.png

Given that $f(x) = \sqrt{7 + x}$ and $g(x) = \sqrt{7 - x}$,

(a) the domain of $f + g$ is ________

(b) the domain of $f - g$ is ________

(c) the domain of $fg$ is ________

(d) one of the intervals (1) $[-7, 7]$, (2) $[-7, 7]$, (3) $[-7, 7]$, or (4) $(-7, 7)$ is the domain of $f/g$.

Which one is the answer? ________ (Input 1, 2, 3, or 4)

---

**15.** (1 pt) setAlgebra17FunComposition/sw2_8b.png

Let $f$ be the linear function (in blue) and let $g$ be the parabolic function (in red) below.

---

**16.** (1 pt) setAlgebra17FunComposition/sw2_8c.png

For the function $f(x)$ and $g(x)$ given in the graph

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**17.** (1 pt) setAlgebra17FunComposition/sw2_8d.png

Find the corresponding function values.

$f(g(0)) =$ ________

$f(g(-3)) =$ ________

---

**18.** (1 pt) setAlgebra17FunComposition/sw2_8e.png

Let $f(x) = 5x + 5$ and $g(x) = 4x^2 + 4x$.

$(f \circ g)(8) =$ ________

---

**19.** (1 pt) setAlgebra17FunComposition/sw2_8f.png

Let $f(x) = 4x + 5$ and $g(x) = 5x^2 + 2x$.

After simplifying, $(f \circ g)(x) =$ ________

---

**20.** (1 pt) setAlgebra17FunComposition/sw2_8g.png

Let $f(x) = 4x + 5$ and $g(x) = 5x^2 + 2x$. Match the statements defined below with the letters labeling their equivalent expressions.

You must get all of the answers correct to receive credit.

1. $g \circ g$
2. $f \circ f$
3. $f \circ g$
4. $g \circ f$
   A. $125x^4 + 100x^3 + 30x^2 + 4x$
   B. $16x + 25$
   C. $20x^2 + 8x + 5$
   D. $80x^2 + 208x + 135$

---

**21.** (1 pt) setAlgebra17FunComposition/sw2_8h.png

Let $f(x) = \frac{1}{3x^3}$, $g(x) = 4x^3$, and $h(x) = 7x^2 + 1$.

Then $f \circ g \circ h(5) =$ ________

---

**22.** (1 pt) setAlgebra17FunComposition/sw2_8i.png

Given that $f(x) = 8x + 3$ and $g(x) = 2 - x^2$, calculate

(a) $f(g(0)) =$ ________

(b) $g(f(0)) =$ ________

---

**23.** (1 pt) setAlgebra17FunComposition/sw2_8j.png

Given that $f(x) = 8x + 3$ and $g(x) = 8 - x^2$, calculate

(a) $f \circ g(-2) =$ ________
24. (1 pt) setAlgebra17FunComposition/sw4_721.pg
Given that \( f(x) = 9x + 6 \) and \( g(x) = 9 - x^2 \), calculate
(a) \( f \circ g(x) = \) 
(b) \( g \circ f(x) = \) 

25. (1 pt) setAlgebra17FunComposition/sw4_721.pg
Given that \( f(x) = 4x - 8 \) and \( g(x) = 2 - x^2 \), calculate
(a) \( f \circ g(x) = \) 
(b) \( g \circ f(x) = \) 

26. (1 pt) setAlgebra17FunComposition/sw4_729.pg
Given that \( f(x) = 7x + 1 \) and \( g(x) = 8x + 6 \), calculate
(a) \( f \circ g(x) = \) its domain is (_______, _______)
(b) \( g \circ f(x) = \) its domain is (_______, _______)
(c) \( f \circ f(x) = \) its domain is (_______, _______)
(d) \( g \circ g(x) = \) its domain is (_______, _______)

**Note:** If needed enter \( \infty \) as infinity and \(-\infty \) as -infinity.

27. (1 pt) setAlgebra17FunComposition/sw4_731.pg
Given that \( f(x) = x^2 + 9 \) and \( g(x) = x - 9 \), calculate
(a) \( f \circ g(x) = \) its domain is (_______, _______)
(b) \( g \circ f(x) = \) its domain is (_______, _______)
(c) \( f \circ f(x) = \) its domain is (_______, _______)
(d) \( g \circ g(x) = \) its domain is (_______, _______)

**Note:** If needed enter \( \infty \) as infinity and \(-\infty \) as -infinity.

28. (1 pt) setAlgebra17FunComposition/sw4_733.pg
Given that \( f(x) = \frac{1}{x} \) and \( g(x) = 9x + 6 \), calculate
(a) \( f \circ g(x) = \) its domain is all real numbers except _______.
(b) \( g \circ f(x) = \) its domain is all real numbers except _______.
(c) \( f \circ f(x) = \) its domain is all real numbers except _______.
(d) \( g \circ g(x) = \) its domain is (_______, _______)

**Note:** If needed enter \( \infty \) as infinity and \(-\infty \) as -infinity.

29. (1 pt) setAlgebra17FunComposition/sw4_735.pg
Given that \( f(x) = |x| \) and \( g(x) = 2x + 8 \), calculate
(a) \( f \circ g(x) = \) its domain is (_______, _______)
(b) \( g \circ f(x) = \) its domain is (_______, _______)
(c) \( f \circ f(x) = \) its domain is (_______, _______)
(d) \( g \circ g(x) = \) its domain is (_______, _______)

**Note:** If needed enter \( \infty \) as infinity and \(-\infty \) as -infinity.

30. (1 pt) setAlgebra17FunComposition/beth3algfun.pg
Given that \( f(x) = 7x - 6 \) and \( g(x) = 6x + 5 \), calculate
(a) \( f \circ g(x) = \)
(b) \( g \circ f(x) = \)
(c) \( f \circ f(x) = \)
(d) \( g \circ g(x) = \)

31. (1 pt) setAlgebra17FunComposition/beth3algfun.pg
Use \( \text{abs}(x) \) for \( |x| \).
Given that \( f(x) = |x| \) and \( g(x) = 8x - 2 \), calculate
(a) \( f \circ g(x) = \)
(b) \( g \circ f(x) = \)

32. (1 pt) setAlgebra17FunComposition/faris1.pg
The number of bacteria in a refrigerated food product is given by \( N(T) = 28T^2 - 143T + 64 \), where \( T > 35 \) is the temperature of the food.
When the food is removed from the refrigerator, the temperature is given by \( T(t) = 3t + 1.2 \), where \( t \) is the time in hours.
Find the composite function \( N(T(t)) = \)
Find the time when the bacteria count reaches 21366
Time Needed =

33. (1 pt) setAlgebra17FunComposition/pcomp2.pg
Given that \( f(x) = x^2 + 8x \) and \( g(x) = x + 4 \), calculate
(a) \( f \circ g(x) = \)
(b) \( g \circ f(x) = \)
(c) \( f \circ f(x) = \)
(d) \( g \circ g(x) = \)

34. (1 pt) setAlgebra17FunComposition/pcomp.pg
Given that \( f(x) = x^2 - 8x \) and \( g(x) = x + 3 \), calculate
(a) \( f \circ g(x) = \)
(b) \( g \circ f(x) = \)
(c) \( f \circ f(x) = \)
(d) \( g \circ g(x) = \)

35. (1 pt) setAlgebra17FunComposition/sw4_817.pg
Given that \( f(x) = 7x + 4 \) and \( g(x) = 8 - x^2 \), calculate
(a) \( f(g(0)) = \)
(b) \( g(f(0)) = \)

36. (1 pt) setAlgebra17FunComposition/sw4_819.pg
Given that \( f(x) = 7x - 8 \) and \( g(x) = 3 - x^2 \), calculate
(a) \( f(g(-2)) = \)
(b) \( g(f(-2)) = \)

37. (1 pt) setAlgebra17FunComposition/sw4_821.pg
Given that \( f(x) = 6x - 5 \) and \( g(x) = 7 - x^2 \), calculate
(a) \( f \circ g(x) = \)
(b) \( g \circ f(x) = \)

38. (1 pt) setAlgebra17FunComposition/sw4_829.pg
Given that \( f(x) = 5x - 2 \) and \( g(x) = 8x + 1 \), calculate
(a) \( f \circ g(x) = \) its domain is (_______, _______)
(b) \( g \circ f(x) = \) its domain is (_______, _______)
(c) \( f \circ f(x) = \) its domain is (_______, _______)
(d) \( g \circ g(x) = \) its domain is (_______, _______)

**Note:** If needed enter \( \infty \) as infinity and \(-\infty \) as -infinity.

39. (1 pt) setAlgebra17FunComposition/sw4_831.pg
Given that \( f(x) = x^2 - 2 \) and \( g(x) = x - 6 \), calculate
(a) \( f \circ g(x) = \) its domain is (_______, _______)
(b) \( g \circ f(x) = \) its domain is (_______, _______)
(c) \( f \circ f(x) = \) its domain is (_______, _______)
(d) \( g \circ g(x) = \) its domain is (_______, _______)

**Note:** If needed enter \( \infty \) as infinity and \(-\infty \) as -infinity.
Given that \( f(x) = \frac{1}{x} \) and \( g(x) = 2x - 8 \), calculate

(a) \( f \circ g(x) = \) ________, its domain is all real numbers except ________

(b) \( g \circ f(x) = \) ________, its domain is all real numbers except ________

(c) \( f \circ f(x) = \) ________, its domain is all real numbers except ________

(d) \( g \circ g(x) = \) ________, its domain is ________ ________

Note: If needed enter \( -\infty \) as \(-\infty\) and \( \infty \) as \( \infty \).

Express the function \( f \) if given that \( f(x) = |x| \) and \( g(x) = 3x - 9 \), calculate

(a) \( f \circ g(x) = \) ________, its domain is ________ ________

(b) \( g \circ f(x) = \) ________, its domain is ________ ________

(c) \( f \circ f(x) = \) ________, its domain is ________ ________

(d) \( g \circ g(x) = \) ________, its domain is ________ ________

Note: If needed enter \( -\infty \) as \(-\infty\) and \( \infty \) as \( \infty \).

If \( f(x) = x^3 + 5 \) and \( g(x) = x - 3 \), then \( h(x) = \sqrt{x} \), find the function \( g(x) \).

Your answer is \( g(x) = \) ________

Express the function \( f(x) = (x + 5)^2 \) in the form \( f \circ g \). If \( f(x) = x^2 \), find the function \( g(x) \).

Your answer is \( g(x) = \) ________

Express the function \( f(x) = \frac{1}{x - 4} \) in the form \( f \circ g \). If \( g(x) = x - 4 \), find the function \( f(x) \).

Your answer is \( f(x) = \) ________

Express the function \( h(x) = (x - 5)^2 \) in the form \( f \circ g \). If \( f(x) = x^2 \), find the function \( g(x) \).

Your answer is \( g(x) = \) ________

Express the function \( h(x) = \frac{1}{x - 2} \) in the form \( f \circ g \). If \( g(x) = x - 2 \), find the function \( f(x) \).

Your answer is \( f(x) = \) ________

A spherical weather balloon is being inflated. The radius of the balloon is increasing at the rate of 6 cm per second. Express the surface area of the balloon as a function of time \( t \) (in seconds).

If needed you can enter \( \pi \) as \( \pi \).

Your answer is \( \) ________

Let \( f(x) = \sqrt{72 - x} \) and \( g(x) = x^2 - x \).

Then the domain of \( f \circ g \) is equal to \([a, b]\) for \( a = \) ________ and \( b = \) ________

Let \( f(x) = \frac{1}{x - 7} \) and \( g(x) = \frac{1}{x - 5} \).

Then the domain of \( f \circ g \) is equal to all reals except for two values, \( a \) and \( b \) with \( a < b \) and \( a = \) ________ and \( b = \) ________

Let \( f(x) = 5x + 3 \) and \( g(x) = 3x^2 + 5x \).

Then \( (f \circ g)(-2) = \) ________

Let \( f(x) = \frac{1}{x - 5} \) and \( g(x) = 5x + 6 \).

Then \( (f \circ g)(4) = \) ________

Let \( f(x) = 2 - \sqrt{x^2 + 1} \) and \( g(x) = x - 6 \).

Then \( (f \circ g)(8) = \) ________

Let \( f(x) = 2x - 5 \) and \( g(x) = x^2 - 4x + 4 \).

Then \( (f \circ g)(x) = \) ________

Let \( f(x) = \frac{1}{x - 4} \) and \( g(x) = \frac{2}{x} + 4 \).

Then \( (g \circ f)(x) = \) ________

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, © UR
1. (1 pt) setAlgebra18FunInverse/ur_inv1.pg
Enter T or F depending on whether the function is one-to-one or not. (You must enter T or F – True and False will not work.)

1. \(d(x) = (3x - 5)^2 + 7\)
2. \(b(x) = 5x^3 - 7x\)
3. \(a(x) = 3x^4 - 3x\)
4. \(c(x) = \frac{x - 3}{3 + x}\)
5. \(e(x) = 3\sqrt{x + 3}\)

2. (1 pt) setAlgebra18FunInverse/srw2_10-7-12a.pg
Enter a Y (for Yes) or an N (for No) in each answer space below to indicate whether the corresponding function is one-to-one or not.

You must get all of the answers correct to receive credit.

1. \(g(t) = 4r^2 + 2\)
2. \(f(t) = 2^t\)
3. \(h(x) = |x| + 2\)
4. \(h(t) = 4t^2 + 2, \quad t \leq 0\)
5. \(f(x) = \sin x, \quad 0 \leq x \leq \pi\)
6. \(k(x) = \cos x, \quad 0 \leq x \leq \pi\)

3. (1 pt) setAlgebra18FunInverse/osu_fn4_1.p
Enter a T or an F in each answer space below to indicate whether or not the given function has an inverse. Unless otherwise indicated, assume the domain of the function is as large as possible. You must get all of the answers correct to receive credit.

1. \(18 \ln(x)\)
2. \(2x^3 - 45x^2 + 324x + 5\) on the interval \([9, \infty)\)
3. \(28x + 5\sin(2x)\)
4. \(2x^3 - 45x^2 + 324x + 5\) on the interval \([0, 9]\)
5. \(9 \sin(x) - 3 \cos(10x)\)
6. \(\ln(x^{18})\)

4. (1 pt) setAlgebra18FunInverse/srw2_10_17.pg
If \(f\) is one-to-one and \(f(-7) = 6\), then
\(f^{-1}(6) = \)__
and \((f(-7))^{-1} = \)__
If \(g\) is one-to-one and \(g(-10) = 5\), then
\(g^{-1}(5) = \)__
and \((g(-10))^{-1} = \)__
If \(h\) is one-to-one and \(h(14) = 15\), then
\(h^{-1}(15) = \)__
and \((h(14))^{-1} = \)__

5. (1 pt) setAlgebra18FunInverse/srw2_10_17a.pg
(a) If \(f\) is one-to-one and \(f(-5) = 7\), then \(f^{-1}(7) = \)__
and \((f(-5))^{-1} = \)__
(b) If \(g\) is one-to-one and \(g(13) = 15\), then \(g^{-1}(15) = \)__
and \((g(13))^{-1} = \)__

6. (1 pt) setAlgebra18FunInverse/mec7.pg
If \(f(x) = 13x - 14\), then
\(f^{-1}(y) = \)__
\(f^{-1}(13) = \)__

7. (1 pt) setAlgebra18FunInverse/srw2_10_20.pg
If \(f(x) = x^2, \quad x \geq 0\), then \(f^{-1}(14) = \)__

8. (1 pt) setAlgebra18FunInverse/srw2_10_21.pg
Let
\(f(x) = 8x + 15\)
\(f^{-1}(x) = \)__

9. (1 pt) setAlgebra18FunInverse/mec1.pg
Let
\(f(x) = 10 - x\)
\(f^{-1}(x) = \)__

10. (1 pt) setAlgebra18FunInverse/ur_fn4_1.p
Let \(f(x) = -5x + 3\). Find \(f^{-1}(x)\).
\(f^{-1}(x) = \)__

Now for fun, verify that \((f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x\)

11. (1 pt) setAlgebra18FunInverse/mec2.pg
Let
\(f(x) = \frac{1}{x + 13}\)
\(f^{-1}(x) = \)__

12. (1 pt) setAlgebra18FunInverse/mec3.pg
Let
\(f(x) = 7 - x^2, \quad x \geq 0\)
\(f^{-1}(x) = \)__

13. (1 pt) setAlgebra18FunInverse/ur_fn4_2.p
Let \(f(x) = \frac{x}{x - 6}\). Find \(f^{-1}(x)\).
\(f^{-1}(x) = \)__

Now for fun, verify that \((f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x\)

14. (1 pt) setAlgebra18FunInverse/mec4.pg
Let
\(f(x) = \frac{x + 4}{x + 8}\)
\(f^{-1}(x) = \)__

15. (1 pt) setAlgebra18FunInverse/ur_fn4_3.p
Let \(f(x) = 2 + \sqrt{x - 2}\). Find \(f^{-1}(x)\).
\(f^{-1}(x) = \)__

Now for fun, verify that \((f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x\)

16. (1 pt) setAlgebra18FunInverse/mec5.pg
Let
\(f(x) = \frac{1}{2}x + 4, \quad 4 \leq x \leq 5\)
The domain of \(f^{-1}\) is the interval \([A, B]\) where \(A = \)__ and \(B = \)__
17. Let \( f(x) = 4 + 1x + 5e^x \)
\[ f^{-1}(9) = \]

18. Find the inverse for each of the following functions.
\[ f(x) = 9x + 6 \]
\[ g(x) = 9x^3 - 15 \]
\[ h(x) = \frac{9}{x + 15} \]
\[ j(x) = \sqrt{x + 9} \]

19. \[ f(x) = \frac{1e^x - 11}{15e^x + 12} \]
\[ f^{-1}(x) = \]

The domain of \( f^{-1}(x) \) is the open interval \((a, b)\), where
\[ a = \]
\[ b = \]

20. Assume that the function \( f \) is a one-to-one function.
(a) If \( f(3) = 2 \), find \( f^{-1}(2) \).
Your answer is

(b) If \( f^{-1}(-8) = -5 \), find \( f(-5) \).
Your answer is

21. If \( f(x) = 5 - 3x \), find \( f^{-1}(5) \).
Your answer is

22. If \( f(x) = x + 7 \) and \( g(x) = x - 7 \),
(a) \( f(g(x)) = \)
(b) \( g(f(x)) = \)
(c) Thus \( g(x) \) is called an \( \) function of \( f(x) \)

23. If \( f(x) = 4x - 9 \) and \( g(x) = \frac{x + 9}{4} \),
(a) \( f(g(x)) = \)
(b) \( g(f(x)) = \)
(c) Thus \( g(x) \) is called an \( \) function of \( f(x) \)

24. Find the inverse function of \( f(x) = 9x + 7 \).
\[ f^{-1}(x) = \]

25. Find the inverse function of \( f(x) = \frac{1}{x + 8} \).
\[ f^{-1}(x) = \]

26. Find the inverse function of \( f(x) = \sqrt{8x + 6} \).
\[ f^{-1}(x) = \]

27. Find the inverse function of \( f(x) = 16 + \sqrt{x} \).
\[ f^{-1}(x) = \]

28. (a) Find the inverse function of \( f(x) = 8x - 6 \).
\[ f^{-1}(x) = \]
(b) The graphs of \( f \) and \( f^{-1} \) are symmetric with respect to the line defined by \( y = \)

29. Below is the graph of a function \( f \):

Graph A
The inverse of the function $f$ is (A, B or C): ____

30. (1 pt) setAlgebra18FunInverse/ur_inv6.pg
Below is the graph of a function $f$: 
The inverse of the function $f$ is (A, B, C or D): ___
A function $f(x)$ is graphed in plane A. It is clearly a 1:1 function, so it must have an inverse.
Enter the color ("red", "green", or "blue") of this inverse function which is graphed in plane B. Use what you know about the graphs of inverse functions rather than algebraic calculations based on what you might guess the function to be.
Color of $f^{-1}$ graph =
Important!! You only have 2 attempts to get this problem right!
1. (1 pt) setAlgebra19FunTransforms/c0s2p1.pg
Relative to the graph of 
\[ y = x^2 \]
the graphs of the following equations have been changed in what way?

1. \[ y = (x/16)^2 \]
2. \[ y = (16x)^2 \]
3. \[ y = (x + 19)^2 \]
4. \[ y = x^2 - 19 \]
   - A. shifted 19 units down
   - B. compressed horizontally by the factor 16
   - C. shifted 19 units left
   - D. stretched horizontally by the factor 16

2. (1 pt) setAlgebra19FunTransforms/c0s2p1b.pg
Relative to the graph of 
\[ y = x^2 \]
the graphs of the following equations have been changed in what way?

1. \[ y = (x + 17)^2 \]
2. \[ y = x^2 + 17 \]
3. \[ y = (17x)^2 \]
4. \[ y = (x - 17)^2 \]
   - A. shifted 17 units up
   - B. compressed horizontally by the factor 17
   - C. shifted 17 units left
   - D. shifted 17 units right

3. (1 pt) setAlgebra19FunTransforms/c0s2p3.pg
Relative to the graph of 
\[ y = x^3 \]
the graphs of the following equations have been changed in what way?

1. \[ y = 2x^3 \]
2. \[ y = (x)^3/8 \]
3. \[ y = x^3 + 2 \]
4. \[ y = x^3 - 2 \]
   - A. stretched horizontally by the factor 2
   - B. shifted 2 units down
   - C. stretched vertically by the factor 2
   - D. shifted 2 units up

4. (1 pt) setAlgebra19FunTransforms/c0s2p2/c0s2p2.pg
This is a graph of the function \( F(x) \):

Enter the letter of the graph below which corresponds to the transformation of the function.

1. \( F(3x) \)
2. \( F(x - 3) \)
3. \( F(x/3) \)
4. \( 5F(x) \)

5. (1 pt) setAlgebra19FunTransforms/scaling.pg
Let \( g \) be the function below.

The domain of \( g(x) \) is of the form \([a, b]\), where \( a \) is \[ \] and \( b \) is \[ \]

The range of \( g(x) \) is of the form \([c, d]\), where \( c \) is \[ \] and \( d \) is \[ \]
Enter the letter of the graph which corresponds to each new function defined below:
1. \( g(x - 2) + 2 \) is ___.
2. \( g(2x) \) is ___.
3. \( 2 + g(-x) \) is ___.
4. \( g(x + 2) - 2 \) is ___.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
</tr>
</tbody>
</table>

For each of the following graph transformations, give the \( x \) and \( y \) coordinates of the point on the new graph which corresponds to the point \( P = (2, 7) \) on the original graph.
Shift Left by 8: (___, ___)
Shrink Vertically by 6: (___, ___)
Flip Horizontally: (___, ___)

For each of the following graph transformations, give the \( x \) and \( y \) coordinates of the point on the new graph which corresponds to the point \( P = (5, -2) \) on the original graph.
Shift Down by 9 and then Flip Vertically: (___, ___)
Flip Vertically and then Shift Down by 9: (___, ___)
Shrink Horizontally by 8 and then Shift Right by 4: (___, ___)
Shift Right by 4 and then Shrink Horizontally by 8: (___, ___)
For each of the following graph transformations, give the $x$ and $y$ coordinates of the point on the new graph which corresponds to the point $P = (-3, 5)$ on the original graph.

Shift Right by 8 and then Flip Horizontally: $(__, __)$
Flip Horizontally and then Shift Right by 8: $(__, __)$
Stretch Vertically by 9 and then Shift Down by 5: $(__, __)$
Shift Down by 5 and then Stretch Vertically by 9: $(__, __)$

Important!! You only have 3 attempts to get this problem right!

1. Stretch Horizontally
2. Shrink Vertically
3. Shift Down
4. Shift Right

A. yellow
B. blue
C. red
D. green

Each of the four graphs in plane B below comes from the original graph in plane A via exactly one transformation. Match each transformation of the original graph in plane A with the color of the graph in plane B which is the result.

Each of the four graphs in plane B comes from the original graph in plane A via exactly one transformation. Match each transformation of the original graph in plane A with the color of the graph in plane B which is the result.
12. (1 pt) setAlgebra19FunTransforms/ur_fn_3.7.png
The graph in plane A is of the function \( f(x) = x^2(x + 3) \).
Match the color of each graph in plane B with the equation that fits it. Use the fact that each graph in B can be obtained from the original by applying just one of the basic transformations which we have learned.

A. yellow  
B. blue  
C. red  
D. green

13. (1 pt) setAlgebra19FunTransforms/ur_fn_3.8.png
The graph in plane A is of the equation \( x = 2^y \).
Match the color of each graph in plane B with the equation that fits it. Use the fact that each graph in B can be obtained from the original by applying just one of the basic transformations which we have learned.
Important!! You only have 3 attempts to get this problem right!

1. $x = -2^y$
2. $x = 2^{-y}$
3. $x = 2^y - 2$
4. $x = 2^{3y}$

A. red
B. blue
C. green
D. yellow

The graph of $f(x) = x^2$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = x^2$ to identify the function $g(x)$:

$g(x) =$

The graph of $f(x) = x^2$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = x^2$ to identify the function $g(x)$:

$g(x) =$

The graph of $f(x) = x^2 - 2$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = x^2 - 2$ to identify the function $g(x)$:

$g(x) =$
The graph of $f(x) = x^3$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = x^3$ to identify the function $g(x)$:

$g(x) =$

The graph of $f(x) = x^3 - 2$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = x^3 - 2$ to identify the function $g(x)$:

$g(x) =$

The graph of $f(x) = x^3$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = x^3$ to identify the function $g(x)$:

$g(x) =$

The graph of $f(x) = |x|$ is sketched in red and the graph of $g(x)$ is sketched in blue. Use the translation rule and $f(x) = |x|$ to identify the function $g(x)$:

$g(x) =$

You may use $\text{abs}(\cdot)$ for $|\cdot|$, e.g., write $\text{abs}(5)$ for $|5|$. 

$g(x) =$
The graph of \( f(x) = |x| - 4 \) is sketched in red and the graph of \( g(x) \) is sketched in blue. Use the translation rule and \( f(x) = |x| - 4 \) to identify the function \( g(x) \):

\[ g(x) = \text{[Expression]} \]

You may use \( \text{abs}(.) \) for \(|.|\), e.g. write \( \text{abs}(5) \) for \( |5| \).

22. (1 pt) setAlgebra19FunTransforms/lh2-4_11c.png

The graph of \( f(x) = \sqrt{x} \) is sketched in red and the graph of \( g(x) \) is sketched in green (click on the graph to see an enlarged image). Use the translation rule and \( f(x) = \sqrt{x} \) to identify the function \( g(x) \):

\[ g(x) = \text{[Expression]} \]

You may use \( \text{sqrt}(.) \) for \( \sqrt{.} \), e.g. write \( \text{sqrt}(5) \) for \( \sqrt{5} \).

24. (1 pt) setAlgebra19FunTransforms/lh2-4_12b.png

The graph of \( f(x) = |x| \) is sketched in red and the graph of \( g(x) \) is sketched in blue. Use the translation rule and \( f(x) = |x| \) to identify the function \( g(x) \):

\[ g(x) = \text{[Expression]} \]

You may use \( \text{abs}(.) \) for \(|.|\), e.g. write \( \text{abs}(5) \) for \( |5| \).

23. (1 pt) setAlgebra19FunTransforms/lh2-4_12a.png

The graph of \( f(x) = \sqrt{x} + 3 \) is sketched in red and the graph of \( g(x) \) is sketched in green. Use the translation rule and \( f(x) = \sqrt{x} + 3 \) to identify the function \( g(x) \):

\[ g(x) = \text{[Expression]} \]

You may use \( \text{sqrt}(.) \) for \( \sqrt{.} \), e.g. write \( \text{sqrt}(5) \) for \( \sqrt{5} \).

25. (1 pt) setAlgebra19FunTransforms/lh2-4_12c.png
The graph of \( f(x) = \sqrt{x} \) is sketched in red and the graph of \( g(x) \) is sketched in green. Use the translation rule and \( f(x) = \sqrt{x} \) to identify the function \( g(x) \);
\[
g(x) = \text{__________} \quad \text{You may use sqrt(.) for } \sqrt{\cdot}, \text{ e.g. write sqrt(5) for } \sqrt{5}.
\]

26. (1 pt) setAlgebra19FunTransforms/lh2-4_23.pg

The graph of \( f(x) = x^2 \) is sketched in black and the graph of \( g(x) \) is sketched in green. Use the translation rule and \( f(x) = \sqrt{x} \) to identify the function \( g(x) \);
\[
g(x) = \text{__________} \quad \text{You may use sqrt(.) for } \sqrt{\cdot}, \text{ e.g. write sqrt(5) for } \sqrt{5}.
\]

27. (1 pt) setAlgebra19FunTransforms/lh2-4_36.pg

The graph of \( f(x) = x^2 \) is sketched in black and it had undergone a series of translations to graphs of functions \( f_1 \) sketched in green, \( f_2 \) sketched in blue, and \( f_3 \) sketched in red. \( f \rightarrow f_1 \rightarrow f_2 \rightarrow f_3 \). Use the translation rule and \( f(x) = x^2 \) to identify the function \( f_1(x) \);
\[
f_1(x) = \text{__________}
\]

Use the translation rule and \( f_1(x) \) to identify the function \( f_2(x) \);
\[
f_2(x) = \text{__________}
\]

Use the translation rule and \( f_2(x) \) to identify the function \( f_3(x) \);
\[
f_3(x) = \text{__________}
\]

28. (1 pt) setAlgebra19FunTransforms/ns1_2_3.pg

Match the functions shown in the graph above with their formulas:

1. \( x^2 + 3 \)
2. \( -x^2 - 2 \)
3. \( x^3 + 3 \)

29. (1 pt) setAlgebra19FunTransforms/beth1algfun.pg

The graph of \( y = f(x) \) is given below:
On a piece of paper sketch the graph of \( y = f(x - 6) \) and determine the new coordinates of points A, B and C.

A =
B =
C =

On a piece of paper sketch the graph of \( y = -f(x) + 5 \) and determine the new coordinates of points A, B and C.

A =
B =
C =

30. (1 pt) setAlgebra19FunTransforms/SRW2_5_11/srw2_5_11.png

Click on image for a larger view

For the function \( f(x) \) given in the graph

Match the following functions with their graphs. Enter the letter of the graph below which corresponds to the function.

1. \( y = f(x) - 2 \)
2. \( y = \frac{1}{2} f(x - 1) \)
3. \( y = f(2x) \)
4. \( y = 2f(x) \)
5. \( y = f(-x) \)
6. \( y = f(x - 2) \)

31. (1 pt) setAlgebra19FunTransforms/Iancel.png

The graph of \( y = x^3 - 6x^2 \) is given below:
Find a formula for each of the transformations whose graphs are given below.

a) 

b) 

The graph of \( y = x^2 \) is given below:

Find a formula for the transformation whose graph is given below.
34. (1 pt) setAlgebra19FunTransforms/p3.pg
The graph of \( y = x^2 \) is given below:

Find a formula for the transformation whose graph is given below.

35. (1 pt) setAlgebra19FunTransforms/p4.pg
The graph of \( y = \sqrt{x} \) is given below:

Find a formula for each of the transformations whose graphs are given below.

Recall that square root is entered as \( \text{sqrt} \).

a)
Find a formula for each of the transformations whose graphs are given below.
Recall that absolute value is entered as abs.
a) 

\[ y = \text{abs}(x) \]

b) 

\[ y = -\text{abs}(x) \]
37. (1 pt) setAlgebra19FunTransforms/p6.pg
The graph of $y = f(x)$ is given below:

On a piece of paper sketch the graph of $y = f\left(\frac{1}{6}x\right)$ and determine the new coordinates of points A, B and C.
A (_______, _____)
B (_______, _____)
C (_______, _____)

38. (1 pt) setAlgebra19FunTransforms/p7.pg
The graph of $y = f(x)$ is given below:

On a piece of paper sketch the graph of $y = f(2x)$ and determine the new coordinates of points A, B and C.
A (_______, _____)
B (_______, _____)
C (_______, _____)

39. (1 pt) setAlgebra19FunTransforms/p8.pg
The graph of $y = f(x)$ is given below:

On a piece of paper sketch the graph of $y = f(-4x)$ and determine the new coordinates of points A, B and C.
A (_______, _____)
B (_______, _____)
C (_______, _____)

40. (1 pt) setAlgebra19FunTransforms/p9.pg
The graph of $y = f(x)$ is given below:

On a piece of paper sketch the graph of $y = -3f(2x)$ and determine the new coordinates of points A, B and C.
A (_______, _____)
B (_______, _____)
C (_______, _____)

41. (1 pt) setAlgebra19FunTransforms/ptransf1.pg
Describe a function $g(x)$ in terms of $f(x)$ if the graph of $g$ is obtained by reflecting the graph of $f$ about the $x$-axis and if it is horizontally stretched by a factor of 8 when compared to the graph of $f$.
$g(x) = Af(Bx) + C$ where
A = _______
42. (1 pt) setAlgebra19FunTransforms/transf2.pg
Describes a function $g(x)$ in terms of $f(x)$ if the graph of $g$ is obtained by shifting the graph of $f$ to the right 7 units and upward 7 units and if it is vertically stretched by a factor of 5 when compared to $f$.

$$g(x) = Af(x + B) + C$$

where

$A =$ __________

$B =$ __________

$C =$ __________

43. (1 pt) setAlgebra19FunTransforms/srw2_5_1.pg
The graph of the function $y = f(x) + 59$ can be obtained from the graph of $y = f(x)$ by one of the following actions:
(a) shifting the graph of $f(x)$ to the right 59 units;
(b) shifting the graph of $f(x)$ to the left 59 units;
(c) vertically stretching the graph of $f(x)$ upward 59 units;
(d) shifting the graph of $f(x)$ downward 59 units;
Your answer is (input a, b, c, or d)

44. (1 pt) setAlgebra19FunTransforms/srw2_5_3.pg
The graph of the function $y = 24f(x)$ can be obtained from the graph of $y = f(x)$ by one of the following actions:
(a) horizontally stretching the graph of $f(x)$ by a factor 24;
(b) horizontally shrinking the graph of $f(x)$ by a factor 24;
(c) vertically stretching the graph of $f(x)$ by a factor 24;
(d) vertically shrinking the graph of $f(x)$ by a factor 24;
Your answer is (input a, b, c, or d)

45. (1 pt) setAlgebra19FunTransforms/srw2_5_5.pg
The graph of the function $y = -44f(x)$ can be obtained from the graph of $y = f(x)$ by one of the following actions:
(a) horizontally stretching the graph of $f(x)$ by a factor 44;
(b) horizontally shrinking the graph of $f(x)$ by a factor 44;
(c) vertically stretching the graph of $f(x)$ by a factor 44;
(d) vertically shrinking the graph of $f(x)$ by a factor 44;
Your answer is (input a, b, c, or d)

Then followed by one of the following actions:
(e) reflecting the resulting graph in x-axis;
(f) reflecting the resulting graph in y-axis;
Your answer is (input e or f)

46. (1 pt) setAlgebra19FunTransforms/srw2_5_7.pg
The graph of the function $y = f(x - 88) + 95$ can be obtained from the graph of $y = f(x)$ by one of the following actions:
(a) shifting the graph of $f(x)$ to the right 88 units;
(b) shifting the graph of $f(x)$ to the left 88 units;
(c) vertically stretching the graph of $f(x)$ by a factor 88;
(d) vertically shrinking the graph of $f(x)$ by a factor 88;
Your answer is (input a, b, c, or d)

Then followed by one of the following actions:
(e) shifting the resulting graph upward 95 units;
(f) shifting the resulting graph downward 95 units;
(g) horizontally stretching the resulting graph by a factor 95;
(h) horizontally shrinking the resulting graph by a factor 1/95;

47. (1 pt) setAlgebra19FunTransforms/srw2_5_9.pg
The graph of the function $y = f(38x)$ can be obtained from the graph of $y = f(x)$ by one of the following actions:
(a) horizontally stretching the graph of $f(x)$ by a factor 38;
(b) horizontally shrinking the graph of $f(x)$ by a factor 1/38;
(c) vertically stretching the graph of $f(x)$ by a factor 38;
(d) vertically shrinking the graph of $f(x)$ by a factor 1/38;
Your answer is (input a, b, c, or d)

48. (1 pt) setAlgebra19FunTransforms/srw2_5_15.pg
(a) The graph of $f(x) = (x + 46)^2$ can be obtained from shifting the graph of $f(x) = x^2$ to the right 46 units.
(b) The graph of $f(x) = x^2 + 46$ can be obtained from shifting the graph of $f(x) = x^2$ upward 46 units.
(c) The graph of $f(x) = 46\sqrt{x}$ can be obtained from the graph of $f(x) = \sqrt{x}$ vertically by a factor 46.
(d) The graph of $f(x) = \sqrt{46x}$ can be obtained from the graph of $f(x) = \sqrt{x}$ horizontally by a factor $\frac{1}{\sqrt{46}}$.

49. (1 pt) setAlgebra19FunTransforms/srw2_5_19.pg
Given $f(x) = x^2$, after performing the following transformations: shift upward 36 units and shift 23 units to the right, the new function $g(x) =$

50. (1 pt) setAlgebra19FunTransforms/srw2_5_23.pg
Given $f(x) = |x|$, after performing the following transformations: shift to the left 14 units, shrink vertically by a factor of $\frac{1}{3}$ and shift downward 13 units, the new function $g(x) =$

Use abs(x) for $|x|$.

51. (1 pt) setAlgebra19FunTransforms/srw2_5_31.pg
The graph of the function $y = 2 + \sqrt{x}$ can be obtained from the graph of $y = \sqrt{x}$ by one of the following actions:
(a) shifting the graph of $f(x)$ downward 2 units;
(b) shifting the graph of $f(x)$ upward 2 units;
(c) horizontally stretching the graph of $f(x)$ by a factor 2;
(d) horizontally shrinking the graph of $f(x)$ by a factor 1/2;
Your answer is (input a, b, c, or d)

52. (1 pt) setAlgebra19FunTransforms/srw2_5_35.pg
The graph of the function $y = 67 + (x + 8)^2$ can be obtained from the graph of $y = x^2$ by one of the following actions:
(a) shifting the graph of $f(x)$ to the right 8 units;
(b) shifting the graph of $f(x)$ to the left 8 units;
(c) vertically stretching the graph of $f(x)$ by a factor 8;
(d) vertically shrinking the graph of $f(x)$ by a factor 8;
Your answer is (input a, b, c, or d)

Then followed by one of the following actions:
(e) shifting the resulting graph upward 67 units;
(f) shifting the resulting graph downward 67 units;
(g) horizontally stretching the resulting graph by a factor 67;
(h) horizontally shrinking the resulting graph by a factor 67;
Your answer is (input e, f, g, or h)
1. (1 pt) setAlgebra20QuadraticFun/lh3-1_1-3.pg

Attention: you are allowed to submit your answer two times only for this problem!

Identify the graphs A (blue), B (red) and C (green):

[] is the graph of the function \( f(x) = (x - 3)^2 \)

[] is the graph of the function \( g(x) = (x + 4)^2 \)

[] is the graph of the function \( h(x) = x^2 - 2 \)

2. (1 pt) setAlgebra20QuadraticFun/lh3-1_4-6.pg

Attention: you are allowed to submit your answer two times only for this problem!

Identify the graphs A (blue), B (red) and C (green):

[] is the graph of the function \( f(x) = -(x - 5)^2 \)

[] is the graph of the function \( g(x) = -(x - 6)^2 - 5 \)

[] is the graph of the function \( h(x) = (x + 3)^2 - 6 \)

3. (1 pt) setAlgebra20QuadraticFun/lh3-1_6-8.pg

Attention: you are allowed to submit your answer two times only for this problem!

Identify the graphs A (blue), B (red) and C (green):

[] is the graph of the function \( f(x) = -x^2 - 3 \)

[] is the graph of the function \( g(x) = x^2 - 2 \)

[] is the graph of the function \( h(x) = x^2 - 2 \)

4. (1 pt) setAlgebra20QuadraticFun/lh3-1_13-16.pg

Consider the Quadratic function \( f(x) = 4x^2 - 36 \).
Its vertex is (_______, ________).
Its x-intercepts are \( x = \) ________.

Note: If there is more than one answer enter them separated by commas.

Its y-intercept is \( y = \) ________.

5. (1 pt) setAlgebra20QuadraticFun/lh3-1_19-20.pg

Consider the Quadratic function \( f(x) = x^2 - 2x - 24 \).
Its vertex is (_______, ________).
Its x-intercepts are \( x = \) ________.

Note: If there is more than one answer enter them separated by commas.

Its y-intercept is \( y = \) ________.

6. (1 pt) setAlgebra20QuadraticFun/lh3-1_23-24.pg

Consider the Quadratic function \( f(x) = -x^2 + 11x - 24 \).
Its vertex is (_______, ________).
Its x-intercepts are \( x = \) ________.

Note: If there is more than one answer enter them separated by commas.

Its y-intercept is \( y = \) ________.

7. (1 pt) setAlgebra20QuadraticFun/lh3-1_25-26.pg

Consider the Quadratic function \( f(x) = 3x^2 - 19x - 14 \).
Its vertex is (_______, ________);
its x-intercepts are \( x = \) ________.
8. Consider the Quadratic function \( f(x) = 3x^2 - 17x - 6 \).
   Its vertex is (_____ , _____).
   Its \( x \)-intercepts are \( x = _____ \).
   Note: If there is more than one answer enter them separated by commas.
   Its \( y \)-intercept is \( y = _____ \).

9. The graph of a quadratic function \( f(x) \) is shown above. It has a vertex at \((-2, -2)\) and passes the point \((0, 2)\). Find the quadratic function.
   \( f(x) = _____ \).

10. The graph of a quadratic function \( f(x) \) is shown above. It has a vertex at \((2, 4)\) and passes the point \((0, 0)\). Find the quadratic function.
    \( f(x) = _____ \).

11. The graph of a quadratic function \( f(x) \) is shown above. It has a vertex at \((-2, 2)\) and passes the point \((0, 0)\). Find the quadratic function.
    \( f(x) = _____ \).

12. The graph of a quadratic function \( f(x) \) is shown above. It has a vertex at \((2, 0)\) and passes the point \((0, 8)\). Find the quadratic function.
    \( f(x) = _____ \).

13. Find all real zeros of the function \( f(x) = x^2 - 25. 
    Zeros are \( x = _____ \).
Note: If there is more than one answer enter them separated by commas.

14. (1 pt) setAlgebra20QuadraticFun/lh3-2_29.pg
Find all real zeros of \( f(x) = x^2 - 3x - 18 \).
Zeros are \( x = \) _______

Note: If there is more than one answer enter them separated by commas.

15. (1 pt) setAlgebra20QuadraticFun/findroots.pg
Find the roots of \( g(k) = (149kx)^2 + 194kx - 131 \)
The smaller root is _______
The larger root is _______

16. (1 pt) setAlgebra20QuadraticFun/findroots2.pg
Find the roots of \( h(t) = 124(k - 28x)^2 - \frac{k}{3} \)
The smaller root is _______
The larger root is _______

17. (1 pt) setAlgebra20QuadraticFun/findroots3.pg
Find the roots of \( h(t) = (190kr)^2 - 198t + 136 \)
The smaller root is _______
The larger root is _______
What positive value of \( k \) will result in exactly one real root? \( k = \) _______

18. (1 pt) setAlgebra20QuadraticFun/findstandard.pg
A quadratic function has its vertex at the point (6, 6). The function passes through the point (2, 9). Find the quadratic and linear coefficients and the constant term of the function.
The quadratic coefficient is _______
The linear coefficient is _______
The constant term is _______

19. (1 pt) setAlgebra20QuadraticFun/findvertex.pg
A quadratic function has its vertex at the point (5, 6). The function passes through the point (2, -5). When written in vertex form, the function is \( f(x) = a(x-h)^2 + k \), where:
\( a = \) _______
\( h = \) _______
\( k = \) _______

20. (1 pt) setAlgebra20QuadraticFun/givencoeff.pg
Write a quadratic function with a linear coefficient of 47, a constant term of 9, and a quadratic term of \( 20b^2 \).
\( f(b) = \) _______

21. (1 pt) setAlgebra20QuadraticFun/givenroots.pg
Write a quadratic function that has roots of 27 and P.
\( f(m) = \) _______

22. (1 pt) setAlgebra20QuadraticFun/sw3_3_111.pg
For the function \( y = (x - 6)(x + 4) \), its \( y \)-intercept is _______
its \( x \)-intercepts are \( x = \) _______
Note: If there is more than one \( x \)-intercept write the \( x \)-values separated by commas.
When \( x \rightarrow \infty \), \( y \rightarrow \) _______
When \( x \rightarrow -\infty \), \( y \rightarrow \) _______

23. (1 pt) setAlgebra20QuadraticFun/sw3_3_69.pg
A box with a square base and no top is to be made from a square piece of cardboard by cutting 3 in. squares from each corner and folding up the sides. The box is to hold 1200 in\(^3\). How big a piece of cardboard is needed?
Your answer is: _______ in. by _______ in.

24. (1 pt) setAlgebra20QuadraticFun/standardform.pg
Given the function \( f(x) = 10x^2 + 460x + 5268 \) find all of the following:
The vertex of the function is _______
Does the function have a minimum or a maximum? (Type minimum or maximum) _______
Find the extreme value of the function. _______
The smallest root is _______
The largest root is _______

25. (1 pt) setAlgebra20QuadraticFun/vertexform.pg
Given the function \( f(x) = 4(x - 27)^2 - 50 \) find all of the following:
The vertex of the function is _______
Does the function have a minimum or a maximum? (Type minimum or maximum) _______
Find the extreme value of the function. _______
The smallest root is _______
The largest root is _______

26. (1 pt) setAlgebra20QuadraticFun/lh3-1_77.pg
A rancher has 224 feet of fencing to enclose two adjacent rectangular corrals. What dimensions will produce the largest total area?
Your answer is: ________ ________ (Enter length and width separated by commas.)
What is the maximum total area?
Your answer is: ________

27. (1 pt) setAlgebra20QuadraticFun/lh3-1_79.pg
The revenue function in terms of the number of units sold \( x \), is given as
\[ R = 270x - 0.5x^2 \]
where \( R \) is the total revenue in dollars. Find the number of units sold \( x \) that produces a maximum revenue?
Your answer is \( x = \) _______
What is the maximum revenue?

28. (1 pt) setAlgebra20QuadraticFun/lh3-1_85.pg
The height \( y \) (in feet) of a ball thrown by a child is
\[ y = -\frac{1}{14}x^2 + 4x + 5 \]
where \( x \) is the horizontal distance in feet from the point at which the ball is thrown.
(a) How high is the ball when it leaves the child’s hand? (Hint: Find \( y \) when \( x = 0 \))
Your answer is \( y = \) _______
(b) What is the maximum height of the ball? _______
(c) How far from the child does the ball strike the ground?
For the function \( y = (x - 5)(x + 5) \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x = \) __________.

**Note:** If there is more than one answer enter them separated by commas. If there are none, enter none.
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = (x - 2)(x + 8)(5x - 2) \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x = \) __________.

**Note:** If there is more than one answer enter them separated by commas. If there are none, enter none.
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = (x - 3)^2(x - 4) \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x = \) __________.

**Note:** If there is more than one answer enter them separated by commas. If there are none, enter none.
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = x^3 - 3x^2 - 4x \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x = \) __________.

**Note:** If there is more than one answer enter them separated by commas. If there are none, enter none.
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = x^3 - 5x^2 - 24x \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x = \) __________.

**Note:** If there is more than one answer enter them separated by commas. If there are none, enter none.
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = (x - 7)(x + 8) \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x = \) __________ and \( x = \) ________ with \( x_1 \leq x_2 \leq x_3 \)
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = (x - 7)(x + 3)(5x - 2) \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x_1 = \) ________, \( x_2 = \) ________, and \( x_3 = \) ________ with \( x_1 \leq x_2 \leq x_3 \)
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = (x - 1)^2(x - 9) \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x_1 = \) ________ and \( x_2 = \) ________ with \( x_1 < x_2 \)
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = x^3 + 1x^2 - 72x^2 \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x_1 = \) ________, \( x_2 = \) ________, and \( x_3 = \) ________ with \( x_1 < x_2 < x_3 \)
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = x^3 - 3x^2 - 40x \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x_1 = \) ________, \( x_2 = \) ________, and \( x_3 = \) ________ with \( x_1 < x_2 < x_3 \)
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

For the function \( y = x^3 - 5x^2 - 24x \),
its y-intercept is \( y = \) ________
its x-intercepts are \( x_1 = \) ________, \( x_2 = \) ________, and \( x_3 = \) ________ with \( x_1 < x_2 < x_3 \)
When \( x \to \infty \), \( y \to \infty \) (Input + or - for the answer)
When \( x \to -\infty \), \( y \to -\infty \) (Input + or - for the answer)

Given the function \( P(x) = 28x^3 - 6x^2 + 2x + 8 \),
\( P(x) \to \) ________ if \( x \to -\infty \),
\( P(x) \to \) ________ if \( x \to \infty \),
If your answer is \( \to -\infty \), input -infinity; if your answer is \( \to \infty \), input infinity.

Given the function \( P(x) = 22x^{10} - 6x^7 + 2x + 10 \),
\( P(x) \to \) ________ if \( x \to -\infty \),
\( P(x) \to \) ________ if \( x \to \infty \),
If your answer is \( \to -\infty \), input -infinity; if your answer is \( \to \infty \), input infinity.

Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function.

1. \( x^6/2 - 2x^4 \)
2. $x^4 + 2x^3$
3. $x(x^2 - 4)$
4. $-x^5 + 5x^3 - 4x$

15. (1 pt) setAlgebra21PolynomialFun/p1.pg
To get a better look at the graph, you can click on it.
The curve above is the graph of a degree 3 polynomial. It goes through the point $(5, -7.2)$. Find the polynomial.
\[ f(x) = \]

16. (1 pt) setAlgebra21PolynomialFun/p2.pg
The curve above is the graph of a degree 4 polynomial. It goes through the point $(5, -320)$. Find the polynomial.
\[ f(x) = \]

17. For the function $f(x) = x^3 - 48x$,
its local maximum is the point: $(______, ____)$;
its local minimum is the point: $(______, ____)$.

18. The polynomial $P(x) = 6x^3 + 6x^2 - 4x$ has ____ local maxima and minima.
19. The polynomial \( P(x) = (x - 4)^5 - 17 \) has_____ local maxima and minima.

20. The polynomial \( P(x) = (x - 3)^6 + 11 \) has_____ local maxima and minima.

21. How many real solutions does the equation \( x^3 = 64 \) have?
   Input your answer here:_____

22. Let \( x \) be the number of units (in thousands) that a company produces and let \( p(x) \) be the profit (in tens of thousands of dollars). The following table gives the profit for different levels of production.

   \[
   \begin{array}{|c|c|c|c|c|c|c|c|c|c|}
   \hline
   x & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 \\
   \hline
   p(x) & -5 & -7.6 & -9.8 & -9.1 & -2.9 & 7.8 & 25.4 & 50.3 & 85.3 & 145.5 \\
   \hline
   \end{array}
   \]

   Use the cubic regression program to find a mathematical model for \( p(x) \).
   \( p(x) = \)_____

23. Classify the following polynomial according to its degree and number of terms:
   \( f(x) = 7x^3 - 7x^2 + 9x - 4 \)
   \( f(x) \) is a [ ] [ ]
   NOTE: You have only one attempt at this problem.

24. Given the following function, describe its end behavior.
   \( f(x) = -9x^9 - 6x^8 + 7x^7 + 3x^6 - 5x^5 + 6x^4 - 5x^3 + 7x^2 + 8x + 9 \)
   To the left, \( f(x) \) [ ]
   To the right, \( f(x) \) [ ]
   NOTE: You have only one attempt at this problem.

25. Evaluate \( f(x) = 4x^3 + 2x^2 - 8x \) when \( x = 4 \).
   \( f(4) = \)_____
   NOTE: Your answer must be a plain number.

26. Given the table below, find a quartic formula for \( g(x) \).

   \[
   \begin{array}{|c|c|c|c|c|}
   \hline
   x & -4 & -2 & 1 & 2 & 4 \\
   \hline
   g(x) & 1669.16 & 62.14 & 11.11 & 129.86 & 2083 \\
   \hline
   \end{array}
   \]
   \( g(x) = \)_____

27. A box without a lid is constructed from a 30 inch by 30 inch piece of cardboard by cutting \( x \) in. squares from each corner and folding up the sides.
   a) Determine the volume of the box as a function of the variable \( x \).
   \( V(x) = \)___________
   b) Use a graphing calculator to approximate the values of \( x \) that produce a volume of 1898.4375.
   Note: There are 3 values of \( x \) that produce the given value but only two of them are acceptable in the context of the problem.
   List the two answers, to at least one decimal place, separated by commas.
   \( x = \)_________
1. (1 pt) setAlgebra22PolynomialDivision/sw5_2_1.png
Find the quotient and remainder using long division for
\[ \frac{x^2 + 5x + 13}{x + 3}. \]
The quotient is __________
The remainder is __________

2. (1 pt) setAlgebra22PolynomialDivision/sw5_2_3.png
Find the quotient and remainder using long division for
\[ \frac{x^3 - 11x^2 + 32x - 19}{x - 4}. \]
The quotient is __________
The remainder is __________

3. (1 pt) setAlgebra22PolynomialDivision/sw5_2_5.png
Find the quotient and remainder using long division for
\[ \frac{x^3 - 6x^2 + 12x}{x^2 - 2x + 2}. \]
The quotient is __________
The remainder is __________

4. (1 pt) setAlgebra22PolynomialDivision/sw5_2_7.png
Find the quotient and remainder using long division for
\[ \frac{2x^3 - 12x^2 + 7x - 23}{2x^2 + 5}. \]
The quotient is __________
The remainder is __________

5. (1 pt) setAlgebra22PolynomialDivision/sw5_2_15.png
Find the quotient and remainder using synthetic division for
\[ \frac{x^3 + 9x^2 + 19x + 12}{x + 2}. \]
The quotient is __________
The remainder is __________

6. (1 pt) setAlgebra22PolynomialDivision/sw5_2_19.png
Find the quotient and remainder using synthetic division for
\[ \frac{x^3 - x^4 + 1x^3 - 1x^2 + 5x - 6}{x - 1}. \]
The quotient is __________
The remainder is __________

7. (1 pt) setAlgebra22PolynomialDivision/sw5_2_23.png
Find the quotient and remainder using synthetic division for
\[ \frac{x^3 - x^4 + 2x^3 - 2x^2 + 6x - 8}{x - 1}. \]
The quotient is __________
The remainder is __________

8. (1 pt) setAlgebra22PolynomialDivision/sw5_2_25.png
Use synthetic division and the Remainder Theorem to evaluate
\[ P(c), \]
where
\[ P(x) = x^2 + 5x + 5, \quad c = -1. \]
The quotient is __________
The remainder is __________

9. (1 pt) setAlgebra22PolynomialDivision/sw5_2_27.png
Use long division and the Remainder Theorem to evaluate \( P(c) \), where
\[ P(x) = x^3 - 4x^2 + 10x - 18, \quad c = 2. \]
The quotient is __________
The remainder is __________

10. (1 pt) setAlgebra22PolynomialDivision/sw5_2_31.png
Use synthetic division and the Remainder Theorem to evaluate
\[ P(c), \]
where
\[ P(x) = x^4 + 7x^3 + 6x^2 + 43x + 8, \quad c = -7. \]
The quotient is __________
The remainder is __________

11. (1 pt) setAlgebra22PolynomialDivision/sw5_2_39.png
Use the Factor Theorem to show that \( x - 1 \) is a factor of
\[ P(x) = x^3 - 8x^2 + 8x - 1. \]
The function value \( P(1) = \quad \)
Thus, \( x - 1 \) is a _______ of \( P(x) \).

12. (1 pt) setAlgebra22PolynomialDivision/sw5_2_41.png
Use the Factor Theorem to show that \( x - 1/2 \) is a factor of
\[ P(x) = 2x^3 - 4x^2 + 4.5x - 1.5. \]
The function value \( P(1/2) = \quad \)
Thus, \( x - 1/2 \) is a _______ of \( P(x) \).

13. (1 pt) setAlgebra22PolynomialDivision/sw5_2_4.png
Find the quotient and remainder using long division for
\[ \frac{x^2 + 9x + 23}{x + 3}. \]
The quotient is __________
The remainder is __________

14. (1 pt) setAlgebra22PolynomialDivision/sw5_2_6.png
Find the quotient and remainder using long division for
\[ \frac{x^3 - 11x^2 + 36x - 34}{x - 4}. \]
The quotient is __________
The remainder is __________
15. Find the quotient and remainder using long division for \( \frac{x^3 - 3x^2 + 6x}{x^2 - 2x + 2} \)

The quotient is __________
The remainder is __________

16. Find the quotient and remainder using synthetic division for \( \frac{x^3 + 4x^2 + 7x + 8}{x + 2} \)

The quotient is __________
The remainder is __________

17. Find the quotient and remainder using synthetic division for \( \frac{x^5 - x^4 + 7x^3 - 7x^2 + 2x - 5}{x - 1} \)

The quotient is __________
The remainder is __________

18. Use synthetic division and the Remainder Theorem to evaluate \( P(c) \), where

\[ P(x) = x^3 - 7x^2 + 14x - 11, \quad c = 2. \]

The quotient is __________
The remainder is __________

19. Use synthetic division and the Remainder Theorem to evaluate \( P(c) \), where

\[ P(x) = x^4 + 7x^3 + 4x^2 + 36x + 60, \quad c = -7. \]

The quotient is __________
The remainder is __________

20. Use the Factor Theorem to show that \( x - c \) is a factor of \( P(x) \) for the given values of \( c \), where

\[ P(x) = x^3 - 4x^2 + 4x - 1, \quad c = 1. \]

The quotient is __________
The remainder is __________

21. Use synthetic division to show that

\[ x = -8 \]

is a root of the equation

\[ x^3 + 3x^2 - 36x + 32 = 0. \]

Then, use the result to factor the polynomial completely into the form

\[(x + A)(x + B)(x + C)\]

Then give the list of values \( A, B, C \) you obtain:

22. Use long division to find the quotient and remainder when

\[ f(x) = 45x^4 + 15x^3 - 15x^2 - 25x + 15 \]

is divided by \( g(x) = 5x^2 + 9 \).

The quotient is __________
The remainder is __________

23. Use long division to find the quotient and remainder when

\[ f(x) = 4x^4 - 7x^3 - 2x^2 - 3x + 5 \]

is divided by \( g(x) = 6x + 9 \).

The quotient is __________
The remainder is __________

24. Use long division to find the quotient and remainder when

\[ f(x) = 5x^5 + 5x^4 - 5x^3 + 3x^2 + 4x + 2 \]

is divided by

\[ g(x) = 3x^2 + 4x - 1. \]

The quotient is __________
The remainder is __________

25. Use synthetic division to find the quotient and remainder when

\[ f(x) = -5x^4 + 7x^3 - 3x^2 + 7x - 6 \]

is divided by \( g(x) = x + 5 \).

The quotient is __________
The remainder is __________
1. (1 pt) setAlgebra23PolynomialZeros/rational_roots.pg
List all possible rational roots for the function
\[ f(x) = 5x^3 + 8x^3 + 5x^2 - 1x + 65. \]
Give your list in increasing order. Beside each possible rational root, type "yes" if it is a root and "no" if it is not a root. Leave any unnecessary answer blanks empty.

Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __
Possible rational root: __ Is it a root? __

2. (1 pt) setAlgebra23PolynomialZeros/describe_graph.pg
Given \( f(x) = (x - 5)(x - 6)(x - 2) \), find the roots in increasing order.

The roots are ___ ___ and ___.
To the left of the first root, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___
Between the first two roots, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___
Between the last two roots, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___
After the last root, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___

3. (1 pt) setAlgebra23PolynomialZeros/describe_graph_a.pg
Given \( f(x) = (x + 6)(x + 3)(x - 9) \), find the roots in increasing order.

The roots are ___ ___ and ___.
To the left of the first root, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___
Between the first two roots, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___
Between the last two roots, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___
After the last root, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___

4. (1 pt) setAlgebra23PolynomialZeros/describe_graph_b.pg
Given \( f(x) = 3(x + 5)^6(x + 4)^3(x - 4)^7 \), find the roots in increasing order.

The roots are ___ ___ and ___.
To the left of the first root, is the graph of \( f(x) \) above or below the x-axis? Answer above or below: ___

5. (1 pt) setAlgebra23PolynomialZeros/roots1.pg
Match the polynomial function to its correct roots.
Place the letter of the list of correct roots next to each function listed below:

1. \( f(x) = x^4 - 16 \)
2. \( f(x) = x^4 + 4x^3 - 16x - 16 \)
3. \( f(x) = x^4 + 4x^3 + 8x^2 + 16x + 16 \)
4. \( f(x) = x^4 - 8x^3 + 24x^2 - 32x + 16 \)
   A. \( x = 2, 2, 2 \)
   B. \( x = -2i, -2i, -2, -2 \)
   C. \( x = -2, -2, -2, -2 \)
   D. \( x = +2i, -2i, -2, -2 \)

6. (1 pt) setAlgebra23PolynomialZeros/jay1.pg
Match the polynomial function to its correct roots.
Place the letter of the list of correct roots next to each function listed below:

1. \( f(x) = 16x^4 + 32x^3 + 24x^2 + 8x + 1 \)
2. \( f(x) = 16x^4 - 16x^3 + 4x - 1 \)
3. \( f(x) = 16x^4 - 32x^3 + 24x^2 - 8x + 1 \)
4. \( f(x) = 4x^4 + 4x^3 + 5x^2 + 4x + 1 \)
   A. \( x = -0.5 \), with multiplicity 4
   B. \( x = 0.5 \), with multiplicity 4
   C. \( x = 0.5 \), with multiplicity 3, \( x = -0.5 \)
   D. \( x = -0.5 \), with multiplicity 2, \( x = i, x = -i \)

7. (1 pt) setAlgebra23PolynomialZeros/jay2.pg
Match the polynomial function to its correct roots.
Place the letter of the list of correct roots next to each function listed below:

1. \( f(x) = x^4 - 18x^2 + 81 \)
2. \( f(x) = x^4 - 6x^3 + 54x - 81 \)
3. \( f(x) = x^4 - 12x^3 + 54x^2 - 108x + 81 \)
4. \( f(x) = x^4 + 12x^3 + 54x^2 + 108x + 81 \)
   A. \( x = -3 \), with multiplicity 4
   B. \( x = 3 \), with multiplicity 4
   C. \( x = 3 \), with multiplicity 2, \( x = -3 \), with multiplicity 2
   D. \( x = -3, x = 3 \), with multiplicity 3

8. (1 pt) setAlgebra23PolynomialZeros/jay3.pg
Match the polynomial function to its correct roots.
Find all rational zeros of the polynomial \( P(x) = x^3 - x^2 - 9x + 9 \).

Its rational zeros are \( x_1 = \) ___ , \( x_2 = \) ___ and \( x_3 = \) ___ with \( x_1 \leq x_2 \leq x_3 \).

Note: If the polynomial has only two rational zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one rational zero, input it at \( x_1 \).

Find all rational zeros of the polynomial \( P(x) = x^4 - 3x^3 - 17x^2 - 3x - 18 \).

Its rational zeros are \( x_1 = \) ___ , \( x_2 = \) ___ , \( x_3 = \) ___ and \( x_4 = \) ___ with \( x_1 \leq x_2 \leq x_3 \leq x_4 \).

Note: If the polynomial has only three rational zeros, input them at \( x_1 \), \( x_2 \) and \( x_3 \); if the polynomial has only two rational zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one rational zero, input it at \( x_1 \).

Find all rational zeros of the polynomial \( P(x) = 4x^4 - 12x^3 - 12x^2 - 12x - 16 \).

Its rational zeros are \( x_1 = \) ___ , \( x_2 = \) ___ , \( x_3 = \) ___ and \( x_4 = \) ___ with \( x_1 \leq x_2 \leq x_3 \leq x_4 \).

Note: If the polynomial has only three rational zeros, input them at \( x_1 \), \( x_2 \) and \( x_3 \); if the polynomial has only two rational zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one rational zero, input it at \( x_1 \).

Find all rational zeros of the polynomial \( P(x) = x^4 - 7x^2 - 18 \).

Its rational zeros are \( x_1 = \) ___ , \( x_2 = \) ___ , \( x_3 = \) ___ and \( x_4 = \) ___ with \( x_1 \leq x_2 \leq x_3 \leq x_4 \).

Note: If the polynomial has only three rational zeros, input them at \( x_1 \), \( x_2 \) and \( x_3 \); if the polynomial has only two rational zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one rational zero, input it at \( x_1 \).

Find all rational zeros of the polynomial \( P(x) = 4x^4 - 4x^3 - 4x^2 - 8x - 8 \).

Its rational zeros are \( x_1 = \) ___ , \( x_2 = \) ___ , \( x_3 = \) ___ and \( x_4 = \) ___ with \( x_1 \leq x_2 \leq x_3 \leq x_4 \).

Note: If the polynomial has only three rational zeros, input them at \( x_1 \), \( x_2 \) and \( x_3 \); if the polynomial has only two rational zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one rational zero, input it at \( x_1 \).

Find all rational zeros of the polynomial \( P(x) = x^4 + x^3 - 4x^2 + 2x - 12 \).

Enter the rational zeros in a comma separated list. If there are none, enter the word none.

For the function \( y = x^5 - 10x^3 + 25x \),
find all distinct real zeros and enter them as a comma separated list. If there are no real zeros, enter the word none. The distinct real zeros are x =

20. (1 pt) setAlgebra23PolynomialZeros/sw5_3_33.pg
Find all the real zeros of the polynomial

\[ P(x) = x^3 - x^2 - 7x + 3. \]

Its real zeros are \( x_1 = \), \( x_2 = \) and \( x_3 = \) with \( x_1 \leq x_2 \leq x_3 \).

Note: If the polynomial has only two real zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one real zero, input it at \( x_1 \).

21. (1 pt) setAlgebra23PolynomialZeros/sw5_3_35.pg
Find all the real zeros of the polynomial

\[ P(x) = x^4 - x^3 - 13x^2 + x + 12. \]

Its real zeros are \( x_1 = \), \( x_2 = \), \( x_3 = \) and \( x_4 = \) with \( x_1 \leq x_2 \leq x_3 \leq x_4 \).

Note: If the polynomial has only three real zeros, input them at \( x_1, x_2 \) and \( x_3 \); if the polynomial has only two real zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one real zero, input it at \( x_1 \).

22. (1 pt) setAlgebra23PolynomialZeros/swr3_3_33.pg
Find all the real zeros of the polynomial

\[ P(x) = x^3 - 2x^2 - 9x + 4. \]

Give them as a comma separated list, and give exact answers - no decimals. The real zeros of \( P(x) \) have \( x = \)

23. (1 pt) setAlgebra23PolynomialZeros/swr3_2_43.pg
\( c = 1 \) is a zero of \( P(x) = x^3 - 14x^2 + 55x - 42 \). Find all other zeros of \( P(x) \).
\( x_1 = \) and \( x_2 = \) with \( x_1 < x_2 \).

24. (1 pt) setAlgebra23PolynomialZeros/swr3_3_35.pg
Find all the real zeros of the polynomial

\[ P(x) = x^4 - x^3 - 7x^2 + x + 6. \]

Give them as a comma separated list, and give exact answers - no decimals. Its real zeros are \( x = \)

25. (1 pt) setAlgebra23PolynomialZeros/swr3_3_37.pg
Find all the real zeros of the polynomial

\[ P(x) = x^4 - 7x^2 - 6x. \]

If there is more than one zero write them separated by commas. Give EXACT answers. No decimals. Its real zeros are \( x = \)

26. (1 pt) setAlgebra23PolynomialZeros/swr3_3_43.pg
Find all the real zeros of the polynomial

\[ P(x) = x^3 - 4x^2 - 13x + 6. \]

Its real zeros are \( x_1 = \), \( x_2 = \) and \( x_3 = \) with \( x_1 \leq x_2 \leq x_3 \).

Note: If the polynomial has only two real zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one real zero, input it at \( x_1 \). Give EXACT answers. No decimals. When \( x \to \infty, P(x) \to \)
When \( x \to -\infty, P(x) \to \)
If your answer is \( \infty \), enter infinity; if your answer is \( -\infty \), enter -infinity.

27. (1 pt) setAlgebra23PolynomialZeros/swr3_3_47.pg
Find all the real zeros of the polynomial

\[ P(x) = x^4 - 4x^3 - x^2 + 4x. \]

Its real zeros are \( x_1 = \), \( x_2 = \), \( x_3 = \) and \( x_4 = \) with \( x_1 \leq x_2 \leq x_3 \leq x_4 \).

Note: If the polynomial has only three real zeros, input them at \( x_1, x_2 \) and \( x_3 \); if the polynomial has only two real zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one real zero, input it at \( x_1 \). Give EXACT answers. No decimals. When \( x \to \infty, P(x) \to \)
When \( x \to -\infty, P(x) \to \)
If your answer is \( \infty \), enter infinity; if your answer is \( -\infty \), enter -infinity.

28. (1 pt) setAlgebra23PolynomialZeros/jay5.pg
For the function \( y = x^3 + 1x^2 - 6x \), find all real zeros. If there is more than one real zero, separate the answers by commas. Also, if you want to enter the square root of a number, like two, enter sqrt(2) The real zeros are \( x = \)

29. (1 pt) setAlgebra23PolynomialZeros/jay6.pg
Find all real zeros of the function \( y = x^3 - 2x^2 - 49x + 98 \). Give your answer as a comma separated list. If there are no real zeros, type none. The real zeros are \( x = \)

30. (1 pt) setAlgebra23PolynomialZeros/jay7.pg
For the function \( y = x^3 - 3x^2 - 28x \), find all real zeros. Note: If there is more than one real zero, separate the answers by commas. Also, if you want to enter the square root of a number, like two, enter sqrt(2). The real zeros are \( x = \)

31. (1 pt) setAlgebra23PolynomialZeros/swr3_3_51.pg
By Descarte’s rule of signs, \( P(x) = x^3 - 4x^2 - 2x - 3 \) has _____ positive real zero(s); and has _____ or _____ negative real zero(s) (please enter the smaller number first).

32. (1 pt) setAlgebra23PolynomialZeros/Descartes.pg
\[ f(x) = x^8 - 14x^7 + 12x^6 + 924x^5 - 10349x^4 + 40114x^3 + 146688x^2 - 1871424x + 6594048 \]
What is the maximum number of positive real roots for \( f(x) \)?

What is the maximum number of negative real roots for \( f(x) \)?

33. (1 pt) setAlgebra23PolynomialZeros/descartes2.pg
Use Descarte’s Rule of Signs to analyze the number of positive
and negative real roots and the number of non-real roots of the function:
\[ h(x) = -2x^7 + 12x^6 - 9x^5 + x^4 + 20x^3 - 10x^2 - 18x + 17 \]
There are at least ____ and at most ____ positive real roots.
There are at least ____ and at most ____ negative real roots.
There are at least ____ and at most ____ non-real roots.

34. (1 pt) setAlgebra23PolynomialZeros/srw3_5_3.pg
Give all of the zeros of the polynomial
\[ P(x) = x^3 + 2x^2 - 3. \]
as a comma separated list.

35. (1 pt) setAlgebra23PolynomialZeros/srw3_5_7.pg
Find all zeros of the polynomial \( P(x) = x^5 - 16 \).
Its zeros are
\[ x_1 = \underline{____} \quad x_2 = \underline{____} \quad x_3 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part and} \]
\[ x_4 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part.} \]

36. (1 pt) setAlgebra23PolynomialZeros/srw3_5_9.pg
Find all zeros of the polynomial \( P(x) = x^6 - 1 \).
Its zeros are
\[ x_1 = \underline{____} \quad x_2 = \underline{____} \quad x_3 = \underline{____} + \underline{____}i \quad \text{with both negative real and imaginary parts,} \]
\[ x_4 = \underline{____} + \underline{____}i \quad \text{with negative real part and positive imaginary part,} \]
\[ x_5 = \underline{____} + \underline{____}i \quad \text{with positive real part and negative imaginary part,} \]
\[ x_6 = \underline{____} + \underline{____}i \quad \text{with both positive real and imaginary parts.} \]

37. (1 pt) setAlgebra23PolynomialZeros/srw3_5_11.pg
Give all of the zeros of \( P(x) = x^2 + 9 \) as a comma separated list.

38. (1 pt) setAlgebra23PolynomialZeros/srw3_5_13.pg
The zeros of \( P(x) = x^3 + 2x + 4 \) are
\[ x_1 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part,} \]
its multiplicity is ____; and
\[ x_2 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part,} \]
its multiplicity is ____.

39. (1 pt) setAlgebra23PolynomialZeros/srw3_5_15.pg
The zeros of \( P(x) = x^3 + 25x \) are
\[ x_1 = \underline{____} \quad x_2 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part,} \]
\[ x_3 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part,} \]
its multiplicity is ____.

40. (1 pt) setAlgebra23PolynomialZeros/srw3_5_21.pg
The zeros of \( P(x) = x^3 + 5x^2 + 9x + 45 \) are
\[ x_1 = \underline{____} \quad x_2 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part,} \]
its multiplicity is ____; and
\[ x_3 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part,} \]
its multiplicity is ____.

41. (1 pt) setAlgebra23PolynomialZeros/srw3_5_27.pg
The zeros of \( P(x) = x^3 + 2x^2 + 1x \) are
\[ x_1 = \underline{____} \quad \text{with multiplicity:} \]
\[ x_2 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part,} \]
\[ x_3 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part,} \]
its multiplicity is ____.

42. (1 pt) setAlgebra23PolynomialZeros/srw3_5_39.pg
The zeros of \( P(x) = x^3 - 3x^2 + 4x - 12 \) are
\[ x_1 = \underline{____} \quad x_2 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part,} \]
\[ x_3 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part.} \]

43. (1 pt) setAlgebra23PolynomialZeros/srw3_5_45.pg
The zeros of \( P(x) = x^3 + 3x^2 + 5x + 3 \) are
\[ x_1 = \underline{____} \quad x_2 = \underline{____} + \underline{____}i \quad \text{with negative imaginary part,} \]
\[ x_3 = \underline{____} + \underline{____}i \quad \text{with positive imaginary part.} \]

44. (1 pt) setAlgebra23PolynomialZeros/FindAllRoots.pg
\[ f(x) = x^6 - 3x^5 + 10x^4 + 20x^3 - 1176x^2 + 4608x^1 - 8640 \]
Given that \(-6i\) and \(2+2i\) are roots of \( f(x) \), find all the other roots.

DO NOT USE THE GIVEN ROOTS IN YOUR ANSWER.
The roots are: ________

45. (1 pt) setAlgebra23PolynomialZeros/FindAllRoots0.pg
\[ f(x) = x^4 + 7x^3 + 69x^2 - 257x^1 - 1940 \]
Given that \(-4+9i\) is roots of \( f(x) \), find all of the roots, giving real roots first, in increasing order. Then give the complex roots so that the imaginary parts are increasing.

The roots are: ________

46. (1 pt) setAlgebra23PolynomialZeros/srw3_5_55.pg
Factor \( P(x) = x^3 + 3x^2 + 5x + 3 \) into linear and irreducible quadratic factors with real coefficients.
Let \( P(x) = (x + a)(x^2 + bx + c) \).
Then
\[ a = \underline{____} \quad b = \underline{____} \quad c = \underline{____} \]

48. (1 pt) setAlgebra23PolynomialZeros/jj1.pg
The polynomial \( f(x) = 7x^3 - 4x^2 + 567x - 324 \) has \( 9i \) as a root. Give all of the roots of \( f \) in a comma-separated list, including the given one.

Roots: ________

49. (1 pt) setAlgebra23PolynomialZeros/find_all_roots.pg
Let \( g(x) = -8x^5 + 106x^4 - 525x^3 + 814x^2 + 238x \).
Given that 5-3i is a root of \( g(x) \) and that \( g(x) \) has at least one rational root, find all the real roots of \( g(x) \) in increasing order. The roots are: \[ \text{Root(s).} \]

Leave any unneeded answer spaces blank.

If a root has multiplicity greater than 1, enter it into your list multiple times.

50. (1 pt) setAlgebra23PolynomialZeros/find_all_roots_2.pg

Let \( g(x) = -60x^6 - 232x^5 - 3800x^4 + 1788x^3 + 17944x^2 + 17424x + 4400 \).

Given that -2-8i is a root of \( g(x) \) and that \( g(x) \) has at least one rational root, find all the real roots of \( g(x) \) in increasing order. The roots are: \[ \text{Root(s).} \]

Leave any unneeded answer spaces blank.

If a root has multiplicity greater than 1, enter it into your list multiple times.

51. (1 pt) setAlgebra23PolynomialZeros/find_positive.pg

Let \( p(x) = x^6 - 8x^5 + 19x^4 - 16x^3 - 29x^2 + 504x - 1071 \).

The polynomial \( p(x) \) has exactly one positive real root. Between what two consecutive integers does it lie?

The positive root is between \[ \text{and } \]

52. (1 pt) setAlgebra23PolynomialZeros/FindPositiveRoot.pg

\[ f(x) = x^5 + 7x^4 - 329x^3 - 2423x^2 - 6612x^1 - 4524 \]

The function \( f(x) \) has only one positive real root. Between what two consecutive integers does the root lie?

The root is between \[ \text{and } \]

53. (1 pt) setAlgebra23PolynomialZeros/GivenRoots.pg

Find an equation with real coefficients for the polynomial that passes through the point \((0.2800000, 0)\) that has the following roots with the given multiplicities.

<table>
<thead>
<tr>
<th>Root</th>
<th>-2</th>
<th>-3</th>
<th>-i-5</th>
<th>-5-3i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicity</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ f(x) = \]

54. (1 pt) setAlgebra23PolynomialZeros/GivenRoots1.pg

Write the equation, in standard form, of a polynomial with real coefficients that has roots at -5, 5 + 4i, and -3 - 4i, and passes through the point \((0, -91)\).

Your answer should include only (decimal) numbers, the letter "\( x \)" and the characters "+", "-", and "\cdot"

\[ f(x) = \]

55. (1 pt) setAlgebra23PolynomialZeros/GivenRootsAndBehavior.pg

Find an equation for \( f(x) \), the polynomial of smallest degree with real coefficients such that \( f(x) \) bounces off of the x-axis at 5, breaks through the x-axis at -5, has complex roots of 2 - 3i and -4i and passes through the point \((0, 74)\).

\[ f(x) = \]

56. (1 pt) setAlgebra23PolynomialZeros/bounds_on_zeros.pg

Determine if -8 is an upper bound, lower bound or no bound for the roots of \( f(x) = 3x^4 + 1x^3 - 1x^2 - 8x - 2 \).

-8 is (a/an) \[ \text{bound for the roots of } f(x) \].

57. (1 pt) setAlgebra23PolynomialZeros/count_zeros.pg

Given the following table of values, what is the minimum number of roots that \( f(x) \) can have?

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>4</td>
<td>-5</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ f(x) \text{ has at least } \text{root(s).} \]

58. (1 pt) setAlgebra23PolynomialZeros/find_real_roots.pg

Write the equation of a polynomial with real coefficients that has roots at \( \pm \sqrt{23} \) and 9 - 9i that passes through the point \((0, 143)\).

\[ y = \]

59. (1 pt) setAlgebra23PolynomialZeros/find_bounds.pg

\[ f(x) = -18x^4 + 15x^3 - 9x^2 + 3x^2 + 17x - 18 \]

What is the smallest positive integer that is an upper bound to all the roots of \( f(x) \)? \[ \text{What is the largest negative integer that is a lower bound to all the roots of } f(x)? \]

60. (1 pt) setAlgebra23PolynomialZeros/factor_checking.pg

Is \((x + 2)\) a factor of \( f(x) = 5x^4 + 7x^3 - 8x^2 + 9x + 3?\)

Answer yes or no: \[ \]

61. (1 pt) setAlgebra23PolynomialZeros/find_degree.pg

Suppose \( p(x) \) is a polynomial with real coefficients that breaks through the x-axis at 26, breaks through the x-axis at -70, and breaks through the x-axis at -30. If \( p(5+3i) = p(-7+7i) = 0 \).

What is the smallest possible degree that \( p(x) \) could have? \[ \text{If } p(x) \text{ has no other roots than those described above, could the degree of } p(x) \text{ be 31? (yes or no) } \]

62. (1 pt) setAlgebra23PolynomialZeros/find_eqn.pg

Write the equation of a polynomial with real coefficients that has roots at \( \pm \sqrt{31} \) and 3 - 8i that passes through the point \((0, 123)\).

\[ y = \]

63. (1 pt) setAlgebra23PolynomialZeros/find_function.pg

Given that \( f(x) \) is a cubic function with zeros at -7 and -2i - 4, find an equation for \( f(x) \) given that \( f(10) = 7 \).

\[ f(x) = \]

64. (1 pt) setAlgebra23PolynomialZeros/find_function.pg

Given that \( f(x) \) is a cubic function with zeros at -6, -1, and 5, find an equation for \( f(x) \) given that \( f(10) = -6 \).

\[ f(x) = \]

65. (1 pt) setAlgebra23PolynomialZeros/quadfromroots1.pg

Enter a quadratic polynomial which has zeros at 6 and 12.

\[ \]

Enter a quadratic polynomial which has zeros at -9 and 5.

\[ \]

Enter a quadratic polynomial which has a \underline{double} root at 8.
66. (1 pt) setAlgebra23PolynomialZeros/quadfromroots2.pg

Enter a quadratic polynomial which has roots at -19/14 and -9.

67. (1 pt) setAlgebra23PolynomialZeros/sw5_2_45.pg

Find a degree 3 polynomial having zeros -5, 1 and 8 and the coefficient of $x^3$ equal 1.
The polynomial is ____________

68. (1 pt) setAlgebra23PolynomialZeros/sw5_2_47.pg

Find a degree 4 polynomial having zeros -5, -2, 3 and 5 and the coefficient of $x^4$ equal 1.
The polynomial is ____________

69. (1 pt) setAlgebra23PolynomialZeros/sw3_2_45.pg

Find a degree 3 polynomial having zeros -1, 3 and 7 and the coefficient of $x^3$ equal 1.
The polynomial is ____________

70. (1 pt) setAlgebra23PolynomialZeros/p3.pg

The polynomial of degree 5, $P(x)$ has leading coefficient 1, has roots of multiplicity 2 at $x = 1$ and $x = 0$, and a root of multiplicity 1 at $x = -1$.
Find a possible formula for $P(x)$.
$P(x) =$ ____________

71. (1 pt) setAlgebra23PolynomialZeros/p4.pg

The polynomial of degree 4, $P(x)$ has a root of multiplicity 2 at $x = 2$ and roots of multiplicity 1 at $x = 0$ and $x = -1$. It goes through the point (5, 108).
Find a formula for $P(x)$.
$P(x) =$ ____________

72. (1 pt) setAlgebra23PolynomialZeros/p5.pg

The polynomial of degree 3, $P(x)$, has a root of multiplicity 2 at $x = 3$ and a root of multiplicity 1 at $x = -4$. The y-intercept is $y = -28.8$.
Find a formula for $P(x)$.
$P(x) =$ ____________

73. (1 pt) setAlgebra23PolynomialZeros/sw3_2_47.pg

Find a degree 4 polynomial having zeros $-8, -2, 2$ and 5 and the coefficient of $x^4$ equal 1.
The polynomial is ____________

74. (1 pt) setAlgebra23PolynomialZeros/srw3_2_49.pg

Find a degree 3 polynomial that has zeros $-1$, 3 and 6 and in which the coefficient of $x^2$ is $-16$.
The polynomial is ____________

75. (1 pt) setAlgebra23PolynomialZeros/srw3_5_21.pg

Find a degree 3 polynomial with coefficient of $x^3$ equal to 1 and zeros $3, -5i$, and $5i$. Simplify your answer so that it has only real numbers as coefficients.
Your answer is ____________

76. (1 pt) setAlgebra23PolynomialZeros/sw3_5_35.pg

A degree 4 polynomial with integer coefficients has zeros $-2 - 4i$ and 1, with 1 a zero of multiplicity 2. If the coefficient of $x^4$ is 1, then the polynomial is ____________

77. (1 pt) setAlgebra23PolynomialZeros/p1.pg

A degree 4 polynomial $P(x)$ with integer coefficients has zeros $4i$ and 4, with 4 being a zero of multiplicity 2. Moreover, the coefficient of $x^4$ is 1. Find the polynomial.
$P(x) =$ ____________

78. (1 pt) setAlgebra23PolynomialZeros/p10.pg

Find a polynomial with integer coefficients, with leading coefficient 1, degree 5, zeros $i$ and $4 - i$, and passing through the origin.
$P(x) =$ ____________

79. (1 pt) setAlgebra23PolynomialZeros/sw3_3_83.pg

A grain silo consists of a cylindrical main section and a hemispherical roof. If the total volume of the silo (including the part inside the roof section) is 15000 ft$^3$ and the cylindrical part is 30 ft tall, what is the radius of the silo?
your answer is ____________

80. (1 pt) setAlgebra23PolynomialZeros/beth1polydiv.pg

A grain silo consists of a cylindrical main section and a hemispherical roof. If the total volume of the silo (including the part inside the roof section) is 15000 ft$^3$ and the cylindrical part is 30 ft tall, what is the radius of the silo?

Note: The following formulas may be useful:

Volume of a Cylinder = $\pi r^2 h$
Volume of a Sphere = $\frac{4}{3} \pi r^3$

Radius = ____________
1. Suppose $r$ varies directly with $t$ and that $r = 48$ when $t = 6$. What is the value of $r$ when $t = 11$?

$$r = \_\_\_\_\_\_\_\_\_$

2. Suppose $p$ varies directly with $q$ and that $p = 54$ when $q = 9$. What is the value of $p$ when $q = 4$?

$$p = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

3. Suppose $z$ varies inversely with $r$ and that $z = 15$ when $t = 7$. What is the value of $z$ when $t = 5$?

$$z = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

4. Suppose $f$ varies inversely with $g$ and that $f = 21$ when $g = 4$. What is the value of $f$ when $g = 7$?

$$f = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

5. Suppose $z$ varies directly with $x$ and inversely with the square of $y$. If $z = 9$ when $x = 3$ and $y = 1$, what is $z$ when $x = 9$ and $y = 9$?

$$z = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

6. Suppose $z$ varies directly with $y$ and directly with the cube of $x$. If $z = 972$ when $x = 3$ and $y = 9$, what is $z$ when $x = 11$ and $y = 1$?

$$z = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

7. At 3:00 PM a man 134 cm tall casts a shadow 141 cm long. At the same time, a tall building nearby casts a shadow 181 m long. How tall is the building? Give your answer in meters. (You may need the fact that 100 cm = 1 m.)

$$r = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

8. Suppose $p$ varies inversely as the cube root of $q$. If $p = 10$ when $q = 2$, what is $p$ if $q = 13$?

$$p = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

9. Suppose $p$ varies jointly as the square of $q$ and the cube root of $r$. If $p = 12$ when $q = 12$ and $r = 13$, what is $p$ if $q = 5$ and $r = 11$?

$$p = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

10. Suppose $p$ varies directly as the cube of $q$. If $p = 15$ when $q = 10$, what is $p$ if $q = 1$?

$$p = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

11. If $q$ varies jointly as $t$ and $p$ and inversely as $r$, then find an equation for $q$ if $q = 5$ when $t = -9$, $p = 3$, and $r = 5$.

$$q = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

12. If $q$ varies jointly as $t$ and the square root of $p$ and inversely as $r$, then find an equation for $q$ if $q = 5$ when $p = 8$, $t = 7$, and $r = 6$.

$$q = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

13. Suppose $p$ varies directly as the cube of $q$. If $p = 5$ when $q = 2$, what is $p$ if $q = 14$?

$$p = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

14. Suppose $p$ varies jointly as the cube root of $q$ and the cube root of $r$. If $p = 6$ when $q = 9$ and $r = 9$, what is $p$ if $q = 14$ and $r = 5$?

$$p = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

15. If $r$ varies jointly as $q$ and $t$ and inversely as $p$, then find an equation for $r$ if $r = -2$ when $q = -6$, $t = -3$, and $p = 8$.

$$r = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

16. Match each equation with the way in which $r$ varies with respect to $t$ in that equation.

1. $25r^4v^2 = 100$

2. $4rt = \frac{4s^2}{v^2}$

3. $ts^2r^3 = 48v^2$

4. $18 = \frac{4s^2}{v^2}$

5. $\frac{s^2}{r^2} = \frac{4}{s^2}$

6. $\frac{s^2}{v^2} = 2vs$

A. inversely with $t$

B. inversely with the square of $t$

C. directly with the cube root of $t$

D. inversely with the cube root of $t$

E. directly with the square of $t$

F. directly with the square root of $t$
17. For each power function, choose (by letter) the graph which most closely resembles the graph of that function. You may always assume that the constant of variation $k$ is positive.

Warning: You have only 4 attempts at this problem so make them count!

- $y = kx^2$
- $y = kx^3$
- $y = kx^{1.05}$
- $y = kx^8$

18. State sales tax $y$ is directly proportional to retail price $x$. An item that sells for 158 dollars has a sales tax of 10.22 dollars. Find a mathematical model that gives the amount of sales tax $y$ in terms of the retail price $x$.

Your answer is $y = $ .

What is the sales tax on a 360 dollars purchase.
Your answer is: 

19. The stopping distance $d$ of an automobile is directly proportional to the square of its speed $v$. A car required 35 feet to stop when its speed was 60 miles per hour. Find a mathematical model that gives the stopping distance $d$ in terms of its speed $v$.

Your answer is $d = $ .

Estimate the stopping distance if the brakes are applied when the car is traveling at 50 miles per hour.
Your answer is: 

20. A company has found that the demand for its product varies inversely as the price of the product. When the price $x$ is 3.25 dollars, the demand $y$ is 450 units. Find a mathematical model that gives the demand $y$ in terms of the price $x$ in dollars.

Your answer is $y = $ .

Approximate the demand when the price is 8 dollars.
Your answer is: 

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, @UR
The root(s) of \( f(x) \), in increasing order, is/are: \( \), \( \), \( \).

\( f(x) \) has hole(s) when \( x \) is: \( \), \( \), \( \).

\( f(x) \) has vertical asymptotes when \( x \) is: \( \), \( \), \( \).

\( f(x) \) has a horizontal asymptote at \( y = \) ________.

---

5. (1 pt) setAlgebra25RationalFun/FindInfo3.pg

Leave any unneeded answer spaces blank.

\[
 f(x) = \frac{4x^3 - 16x^2 - 45x - 18}{-1x^3 - 1x^2 + 32x + 60}
\]

The domain of the function \( f(x) \), in interval notation from left to right, is: \( \) ______\( \cup \) ______\( \cup \) ______.

(Type -inf for \(-\infty\) and inf for \(\infty\) Do not use any spaces in your answer. Don’t forget to use parentheses.

The root(s) of \( f(x) \), in increasing order, is/are: \( \), \( \), \( \).

\( f(x) \) has hole at the point: \( \), \( \).

\( f(x) \) has vertical asymptotes when \( x \) is: \( \), \( \), \( \).

\( f(x) \) has a horizontal asymptote at \( y = \) ________.

---

6. (1 pt) setAlgebra25RationalFun/cross_OA.pg

Find the equation of the non-vertical asymptote.

\( y = \) ________.

Does \( f(x) \) intersect its non-vertical asymptote? (yes or no) ________

What is the smallest value of \( x \) at which \( f(x) \) intersects its non-vertical asymptote? (Leave this question blank if you answered no above.) ________

---

7. (1 pt) setAlgebra25RationalFun/cross_asymptote.pg

\[
 f(x) = \frac{-4x^4 + 36x^3 + 2x^2 - 1x - 8}{1x^4 - 9x^3 + 7x^2 + 7x - 9}
\]

What is the equation of the horizontal asymptote? \( y = \) ________.

Does the graph of \( f(x) \) intersect its horizontal asymptote? (yes or no) ________

At what \( x \)-values does \( f(x) \) intersect its horizontal asymptote? Give your answers in increasing order. If \( f(x) \) does not intersect its horizontal asymptote, leave this question blank. ________ , ________
1. (1 pt) setAlgebra26PartialFraction/srw8_9_1.pg  
The partial fraction decomposition of \( \frac{13}{(x-1)(x+2)} \) can be written in the form of \( \frac{f(x)}{x-1} + \frac{g(x)}{x+2} \).  
The possible answers for \( f(x) \) and \( g(x) \) are (a) A, a constant, or (b) Ax+B, a linear function.  
\( f(x) \) is in the form of (input a or b) _____ and \( g(x) \) is in the form of (input a or b) _____.

2. (1 pt) setAlgebra26PartialFraction/srw8_9_5.pg  
The partial fraction decomposition of \( \frac{x^2+12}{(x-3)(x^2+4)} \) can be written in the form of \( \frac{f(x)}{x-3} + \frac{g(x)}{x^2+4} \).  
The possible answers for \( f(x) \) and \( g(x) \) are (a) A, a constant, or (b) Ax+B, a linear function.  
\( f(x) \) is in the form of (input a or b) _____ and \( g(x) \) is in the form of (input a or b) _____.

3. (1 pt) setAlgebra26PartialFraction/srw8_9_9.pg  
How many fraction terms are there in the partial fraction decomposition of \( \frac{x^3+x^2+22}{(2x-5)(x^2+2x+5)^2} \)?  
Your answer is ______.

4. (1 pt) setAlgebra26PartialFraction/srw8_9_11.pg  
The partial fraction decomposition of \( \frac{20}{(x-1)(x+1)} \) can be written in the form of \( \frac{f(x)}{x-1} + \frac{g(x)}{x+1} \), where  
\( f(x) = \) _____  
\( g(x) = \) _____

5. (1 pt) setAlgebra26PartialFraction/srw8_9_17.pg  
The partial fraction decomposition of \( \frac{54}{x^2-4} \) can be written in the form of \( \frac{f(x)}{x-2} + \frac{g(x)}{x+2} \), where  
\( f(x) = \) _____  
\( g(x) = \) _____

6. (1 pt) setAlgebra26PartialFraction/srw8_9_21.pg  
The partial fraction decomposition of \( \frac{38x}{8x^2-10x+3} \) can be written in the form of \( \frac{f(x)}{2x} + \frac{g(x)}{4x-3} \), where  
\( f(x) = \) _____  
\( g(x) = \) _____

7. (1 pt) setAlgebra26PartialFraction/srw8_9_25.pg  
The partial fraction decomposition of \( \frac{x^2+11}{x(x+1)^2} \) can be written in the form of \( \frac{f(x)}{x} + \frac{g(x)}{x+1} + \frac{h(x)}{x^2+1} \), where  
\( f(x) = \) _____  
\( g(x) = \) _____  
\( h(x) = \) _____
1. (1 pt) setAlgebra27Conics/hyperinfo.pg
A hyperbola has a vertical transverse axis of length 10 and asymptotes of $y = \frac{7}{2}x + 6$ and $y = -\frac{7}{2}x + 4$. Find the center of the hyperbola, its focal length, and its eccentricity.
The center of the hyperbola is (______, ______).
The focal length is ______.
The eccentricity is ______.

2. (1 pt) setAlgebra27Conics/matching.pg

Match each graph to its equation.
(For all graphs on this page, if you are having a hard time seeing the picture clearly, click on it. It will expand to a larger picture on its own page so that you can inspect it more closely.)
Match each graph to its equation.

(For all graphs on this page, if you are having a hard time seeing the picture clearly, click on it. It will expand to a larger picture on its own page so that you can inspect it more closely.)

A. \((y - 1)^2 = 2(x + 1)\)
B. \(x^2 + \frac{(y-1)^2}{4} = 1\)
C. \(x^2 = -2y\)
D. \(\frac{(x-1)^2}{16} + y^2 = 1\)
E. \(y^2 - 4x^2 = 1\)
F. \(4x^2 - y^2 = 1\)
G. \(y^2 = -2x\)
H. \(x^2 - 4y^2 = 1\)
I. \(\frac{(x+1)^2}{4} + (y - 1)^2 = 1\)
J. \(4y^2 - x^2 = 1\)
Find an equation of the parabola that has a focus at \((7, 9)\) and a vertex at \((7, 6)\):

A. \((y - 1)^2 = 2(x + 1)\)
B. \((x - 1)^2 = 2(y + 1)\)
C. \(x^2 = 2y\)
D. \(y^2 = -2x\)
E. \(y^2 = 2x\)
F. \((y - 1)^2 = -2(x - 1)\)
Find an equation of its directrix:
\[ y = \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \]

5. (1 pt) setAlgebra27Conics/ur_geo_3_3.pg

Find the vertex, focus, and directrix for the following functions.
(a) \((y - 5)^2 = 12(x - 7)\)
   vertex: (__, __)
   focus: (__, __)
   directrix \(x = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)
(b) \(y^2 - 14y = 12x - 7^2\)
   vertex: (__, __)
   focus: (__, __)
   directrix \(x = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)
(c) \((x - 7)^2 = 12(y - 3)\)
   vertex: (__, __)
   focus: (__, __)
   directrix \(y = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)
(d) \(x^2 + 24x = 4y - 20\)
   vertex: (__, __)
   focus: (__, __)
   directrix \(y = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)

6. (1 pt) setAlgebra27Conics/ur_geo_3_4.pg

Write equations for each parabola (If you have a hard time seeing the picture clearly, click on the picture so that you can inspect it more closely.)

(a) \((y - K)^2 = A(x - H)\)
   where \(K = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)
   where \(H = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)
   where \(A = \_ \_ \_ \_ \_ \_ \_ \_ \_ \)

(b) \((y - K)^2 = A(x - H)\)

7. (1 pt) setAlgebra27Conics/ur_geo_3_5.pg

Match each graph to its equation.
(For all graphs on this page, if you are having a hard time seeing the picture clearly, click on it. It will expand to a larger picture on its own page so that you can inspect it more closely.)
Find the center, vertices, and foci of each ellipse.

(a) \( \frac{x^2}{36} + \frac{y^2}{25} = 1 \)
- Center: (____,____)
- Right vertex: (____,____)
- Left vertex: (____,____)
- Top vertex: (____,____)
- Bottom vertex: (____,____)
- Right focus: (____,____)
- Left focus: (____,____)

(b) \( \frac{(x+10)^2}{16} + \frac{(y-6)^2}{49} = 1 \)
- Center: (____,____)
- Right vertex: (____,____)
The equation of the ellipse that has a center at \((3, 1)\), a focus at \((6, 1)\), and a vertex at \((8, 1)\), is

\[
\frac{(x - C)^2}{A^2} + \frac{(y - D)^2}{B^2} = 1
\]

where

A = ____,
B = ____,
C = ____,
D = _____

Write equations for each ellipse (If you have a hard time seeing the picture clearly, click on the picture so that you can inspect it more closely.)

\[
\frac{(y - A)^2}{B^2} + \frac{(x - C)^2}{D^2} = 1
\]

where A = ____,
where B = ____,
where C = ____,
where D = _____

Match each graph to its equation. (For all graphs on this page, if you are having a hard time seeing the picture clearly, click on it. It will expand to a larger picture on its own page so that you can inspect it more closely.)

where A = ____,
where B = ____,
where C = ____,
where D = _____
The equation of the hyperbola that has a center at \((8, 5)\), a focus at \((13, 5)\), and a vertex at \((4, 5)\), is
\[
\frac{(x - C)^2}{A^2} - \frac{(y - D)^2}{B^2} = 1
\]
where
\[
A = \_, \quad B = \_, \quad C = \_, \quad D = 
\]

Write equations for each hyperbola (If you have a hard time seeing the picture clearly, click on the picture so that you can inspect it more closely.)
(a)
where \( A = \) \\
where \( B = \) \\
where \( C = \) \\
where \( D = \) 

(b) 

Solve the system by graphing each equation and finding the point of intersection.

\[
\begin{align*}
y &= \frac{-12}{x+2} + 6 \\
y - 14 &= (x + 1)^2
\end{align*}
\]

15. (1 pt) setAlgebra27Conics/ur_geo_3_13.png

The parabola given by the equation \( x = y^2 + 14y + 27 \) has its vertex at \((h, k)\) for:

\( h = \) \\
and \( k = \) 

16. (1 pt) setAlgebra27Conics/ur_geo_3_14.png

The parabola given by the equation \( y = -x^2 + 8x - 5 \) has its vertex at \((h, k)\) for:

\( h = \) \\
and \( k = \) 

17. (1 pt) setAlgebra27Conics/ur_geo_3_15.png

The parabola given by the equation \( x = -2y^2 + 24y - 72 \) has its vertex at \((h, k)\) for:

\( h = \) \\
and \( k = \) 

18. (1 pt) setAlgebra27Conics/ur_geo_3_16.png

The parabola given by the equation \( y = 3x^2 - 30x + 13 \) has its vertex at \((h, k)\) for:

\( h = \) \\
and \( k = \) 

19. (1 pt) setAlgebra27Conics/ur_geo_3_17.png

The parabola given by the equation \( 2y - 9 = x^2 + 2x \) has its vertex at \((h, k)\) for:

\( h = \) \\
and \( k = \) 

20. (1 pt) setAlgebra27Conics/ur_geo_3_18.png

The parabola given by the equation \( 5x - 2y = y^2 + 16 \) has its vertex at \((h, k)\) for:

\( h = \) \\
and \( k = \) 

21. (1 pt) setAlgebra27Conics/ur_geo_3_19.png

Match each equation for a parabola with the direction that the parabola opens.

IMPORTANT!! You only have 4 attempts to get this problem right!

1. \( y = -9(x + 3)^2 - 10 \)  
A. up  
B. left  
C. right  
D. down  

2. \( y = 9(x + 3)^2 - 10 \)  
3. \( x = -9(y + 3)^2 - 10 \)  
4. \( x = 9(y + 3)^2 - 10 \)
Match each equation for a parabola with the direction that the parabola opens.

**IMPORTANT!!** You only have 4 attempts to get this problem right!

1. \( x = \frac{1}{7}(y - 5)^2 + 9 \)  
   - A. up  
   - B. down  
   - C. left  
   - D. right

2. \( y = \frac{1}{7}(x - 5)^2 + 9 \)  
3. \( y = -\frac{1}{7}(x - 5)^2 + 9 \)  
4. \( x = \frac{1}{7}(y - 5)^2 + 9 \)

Match each equation for a parabola with the direction that the parabola opens.

**IMPORTANT!!** You only have 4 attempts to get this problem right!

1. \( y = -9(x - 8)^2 - 9 \)  
   - A. left  
   - B. right  
   - C. down  
   - D. up

2. \( x = 9(y - 8)^2 - 9 \)

3. \( x = -9(y - 8)^2 - 9 \)

4. \( y = 9(x - 8)^2 - 9 \)

Match each equation for a parabola with the direction that the parabola opens.

**IMPORTANT!!** You only have 4 attempts to get this problem right!

1. \( x = \frac{1}{4}(y + 10)^2 + 10 \)  
2. \( y = -\frac{1}{4}(x + 10)^2 + 10 \)  
3. \( x = -\frac{1}{4}(y + 10)^2 + 10 \)  
4. \( y = \frac{1}{4}(x + 10)^2 + 10 \)

\[ x^2 + y^2 + 10x - 14y + 25 = 0 \] is the equation of a circle with center \((h, k)\) and radius \(r\) for:

\[ h = \quad \text{and} \quad k = \quad \text{and} \quad r = \]

\[ x^2 + y^2 - 14x + 12y + 60 = 0 \] is the equation of a circle with center \((h, k)\) and radius \(r\) for:

\[ h = \quad \text{and} \quad k = \quad \text{and} \quad r = \]

\[ 3x^2 + 3y^2 - 12x - 6y + 12 = 0 \] is the equation of a circle with center \((h, k)\) and radius \(r\) for:

\[ h = \quad \text{and} \quad k = \quad \text{and} \quad r = \]

\[ 2x^2 + 2y^2 + 8x + 4y + 2 = 0 \] is the equation of a circle with center \((h, k)\) and radius \(r\) for:

\[ h = \quad \text{and} \quad k = \quad \text{and} \quad r = \]
1. (1 pt) setAlgebra28ExpFunctions/sw6_1_3.pg
For the function \( f(x) = 5^x \), calculate the following function values:
\[
\begin{align*}
f(-3) &= \quad \\
f(-1) &= \quad \\
f(0) &= \quad \\
f(1) &= \quad \\
f(3) &= \quad 
\end{align*}
\]

2. (1 pt) setAlgebra28ExpFunctions/sw6_1_5.pg
For the function \( f(x) = \left( \frac{1}{2} \right)^x \), calculate the following function values:
\[
\begin{align*}
f(-3) &= \quad \\
f(-1) &= \quad \\
f(0) &= \quad \\
f(1) &= \quad \\
f(3) &= \quad 
\end{align*}
\]

3. (1 pt) setAlgebra28ExpFunctions/c6s1p15_20/c6s1p15_20.png
Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function. (Click on image for a larger view)
- 1. \( f(x) = 5^{x-3} \)
- 2. \( f(x) = 5^{x+1} - 4 \)
- 3. \( f(x) = 5^x \)
- 4. \( f(x) = -5^x \)
- 5. \( f(x) = 5^x + 3 \)

4. (1 pt) setAlgebra28ExpFunctions/c4s1p13_18/c4s1p13_18.png
Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function. (Click on image for a larger view)
- 1. \( f(x) = 5^x + 3 \)
- 2. \( f(x) = 5^{x-3} \)
- 3. \( f(x) = -5^x \)
- 4. \( f(x) = 5^{-x} \)
- 5. \( f(x) = 5^{x+1} - 4 \)

5. (1 pt) setAlgebra28ExpFunctions/swr4_1_1.png
Starting with the graph of \( f(x) = 2^x \), write the equation of the graph that results from
- (a) shifting \( f(x) \) 8 units downward. \( y = \quad \)
- (b) shifting \( f(x) \) 6 units to the left. \( y = \quad \)
- (c) reflecting \( f(x) \) about the x-axis and the y-axis. \( y = \quad \)
- (d) reflecting \( f(x) \) about the line \( y = 2 \). \( y = \quad \)

6. (1 pt) setAlgebra28ExpFunctions/swr4_1_3.png
For the function \( f(x) = 5^x \), calculate the following function values:
\[
\begin{align*}
f(-3) &= \quad \\
f(-1) &= \quad \\
f(0) &= \quad \\
f(1) &= \quad \\
f(3) &= \quad 
\end{align*}
\]

7. (1 pt) setAlgebra28ExpFunctions/swr4_1_5.png
For the function \( f(x) = \left( \frac{1}{2} \right)^x \), calculate the following function values:
\[
\begin{align*}
f(-3) &= \quad \\
f(-1) &= \quad \\
f(0) &= \quad \\
f(1) &= \quad \\
f(3) &= \quad 
\end{align*}
\]

8. (1 pt) setAlgebra28ExpFunctions/swr4_1_3.png
For the function \( f(x) = 2e^x \), calculate the following function values:
\[
\begin{align*}
f(-3) &= \quad \\
f(-1) &= \quad \\
f(0) &= \quad \\
f(1) &= \quad \\
f(3) &= \quad 
\end{align*}
\]

9. (1 pt) setAlgebra28ExpFunctions/swr4_1_9.png
Find the exponential function \( f(x) = a^x \) whose graph goes through the point \((3, 125)\).
\( a = \quad \)

10. (1 pt) setAlgebra28ExpFunctions/swr4_1_11.png
Find the exponential function \( f(x) = a^x \) whose graph goes through the point \(2, 1/25\).
\( a = \quad \)

11. (1 pt) setAlgebra28ExpFunctions/swr4_1_21.png
The graph of the function \( f(x) = 7^x - 9 \) can be obtained from the graph of \( g(x) = 7^x \) by one of the following actions:
- (a) shifting the graph of \( g(x) \) to the right 9 units;
- (b) shifting the graph of \( g(x) \) to the left 9 units;
- (c) shifting the graph of \( g(x) \) upward 9 units;
- (d) shifting the graph of \( g(x) \) downward 9 units;
- (e) reflecting the graph of \( g(x) \) in the x-axis;
- (f) reflecting the graph of \( g(x) \) in the y-axis;
Your answer is (input a, b, c, d, e, or f) ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No) ________
The range of the function \( f(x) \) is \((A, \infty)\), the value of \( A \) is ________

12. (1 pt) setAlgebra28ExpFunctions/sw6_1.29.png
The graph of the function \( f(x) = e^{-x^2} - 3 \) can be obtained from the graph of \( g(x) = e^x \) by one of the following actions:
(a) reflecting the graph of \( g(x) \) in the \( x \)-axis;
(b) reflecting the graph of \( g(x) \) in the \( y \)-axis;
your answer is (input a or b) ________
then, by one of the following actions:
(a) shifting the resulting graph to the right 3 units;
(b) shifting the resulting graph to the left 3 units;
(c) shifting the resulting graph upward 3 units;
(d) shifting the resulting graph downward 3 units;
Your answer is (input a, b, c, or d) ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No) ________
The range of the function \( f(x) \) is \((A, \infty)\), the value of \( A \) is ________

13. (1 pt) setAlgebra28ExpFunctions/sw6_1.31.png
The graph of the function \( f(x) = 3^{x-9} \) can be obtained from the graph of \( g(x) = 3^x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 9 units;
(b) shifting the graph of \( g(x) \) to the left 9 units;
(c) shifting the graph of \( g(x) \) upward 9 units;
(d) shifting the graph of \( g(x) \) downward 9 units;
(e) reflecting the graph of \( g(x) \) in the \( x \)-axis;
(f) reflecting the graph of \( g(x) \) in the \( y \)-axis;
your answer is (input a, b, c, d, e, or f) ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No) ________
The range of the function \( f(x) \) is \((A, \infty)\), the value of \( A \) is ________

14. (1 pt) setAlgebra28ExpFunctions/sw6_1.33.png
Find the exponential function \( f(x) = Ca^x \) whose graph goes through the points \((0, 2)\) and \((2, 8)\).
\( a = \) ________
\( C = \) ________

15. (1 pt) setAlgebra28ExpFunctions/sw6_1.21.png
The graph of the function \( f(x) = -6^x \) can be obtained from the graph of \( g(x) = 6^x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 6 units;
(b) shifting the graph of \( g(x) \) to the left 6 units;
(c) shifting the graph of \( g(x) \) upward 6 units;
(d) shifting the graph of \( g(x) \) downward 6 units;
(e) reflecting the graph of \( g(x) \) in the \( x \)-axis;
(f) reflecting the graph of \( g(x) \) in the \( y \)-axis;
your answer is (input a, b, c, d, e, or f) ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No) ________
The range of the function \( f(x) \) is \((-\infty, A)\), the value of \( A \) is ________

16. (1 pt) setAlgebra28ExpFunctions/sw6_1.23.png
The graph of the function \( f(x) = 4^x - 4 \) can be obtained from the graph of \( g(x) = 4^x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 4 units;
(b) shifting the graph of \( g(x) \) to the left 4 units;
(c) shifting the graph of \( g(x) \) upward 4 units;
(d) shifting the graph of \( g(x) \) downward 4 units;
(e) reflecting the graph of \( g(x) \) in the \( x \)-axis;
(f) reflecting the graph of \( g(x) \) in the \( y \)-axis;
your answer is (input a, b, c, d, e, or f) ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No) ________
The range of the function \( f(x) \) is \((A, \infty)\), the value of \( A \) is ________

17. (1 pt) setAlgebra28ExpFunctions/sw6_1.24.png
The graph of the function \( f(x) = 8^{x-6} \) can be obtained from the graph of \( g(x) = 8^x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 6 units;
(b) shifting the graph of \( g(x) \) to the left 6 units;
(c) shifting the graph of \( g(x) \) upward 6 units;
(d) shifting the graph of \( g(x) \) downward 6 units;
(e) reflecting the graph of \( g(x) \) in the \( x \)-axis;
(f) reflecting the graph of \( g(x) \) in the \( y \)-axis;
your answer is (input a, b, c, d, e, or f) ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No) ________
The range of the function \( f(x) \) is \((A, \infty)\), the value of \( A \) is ________

18. (1 pt) setAlgebra28ExpFunctions/sw6_2.3.png
The graph of the function \( f(x) = -e^x \) can be obtained from the graph of \( g(x) = e^x \) by one of the following actions:
(a) reflecting the graph of \( g(x) \) in the \( y \)-axis;
(b) reflecting the graph of \( g(x) \) in the \( x \)-axis;
your answer is (input a or b) ________
The range of the function \( f(x) \) is \( f(x) < A \), find \( A \).
The value of \( A \) is ________
Is the domain of the function \( f(x) \) still \((-\infty, \infty)\)?
your answer is (input Yes or No) ________

19. (1 pt) setAlgebra28ExpFunctions/sw6_2.5.png
The graph of the function \( f(x) = e^{-x^2} - 3 \) can be obtained from the graph of \( g(x) = e^x \) by two of the following actions:
(a) reflecting the graph of \( g(x) \) in the \( y \)-axis;
(b) reflecting the graph of \( g(x) \) in the \( x \)-axis;
(c) shifting the graph of \( g(x) \) to the right 3 units;
(d) shifting the graph of \( g(x) \) to the left 3 units;
(e) shifting the graph of \( g(x) \) upward 3 units;
(f) shifting the graph of \( g(x) \) downward 3 units;
your answer: Apply the action ________ (input a, b, c, d, e, or f) then apply the action ________ (please give your answer in the order the changes are applied, e.g. a first, then b second)
The range of the function \( f(x) \) is \( f(x) > A \), find \( A \).
The value of $A$ is _______.
Is the domain of the function $f(x)$ still $(-\infty, \infty)$?
Your answer is (input Yes or No) ________

20. (1 pt) setAlgebra28ExpFunctions/sw6_2_9.pg
If 10000 dollars is invested at an interest rate of 8 percent per year, compounded semiannually, find the value of the investment after the given number of years.
(a) 5 years:
Your answer is ________
(b) 10 years:
Your answer is ________
(c) 15 years:
Your answer is ________

21. (1 pt) setAlgebra28ExpFunctions/sw6_2_11.pg
If 35000 dollars is invested at an interest rate of 9 percent per year, find the value of the investment at the end of 5 years for the following compounding methods.
(a) Annual:
Your answer is ________
(b) Semiannual:
Your answer is ________
(c) Monthly:
Your answer is ________
(d) Daily:
Your answer is ________
(e) Continuously:
Your answer is ________

22. (1 pt) setAlgebra28ExpFunctions/sw6_2_13.pg
Which of the given interest rates and compounding periods would provide the best investment?
(a) 8 1/2 percent per year, compounded semiannually;
Your answer is (input a, b, or c) ________
(b) 8 1/4 percent per year, compounded quarterly;
Your answer is (input a, b, or c) ________
(c) 8 percent per year, compounded continuously.
Your answer is (input a, b, or c) ________

You have only one chance to input your answer!!!

23. (1 pt) setAlgebra28ExpFunctions/sw6_2_17.pg
The number of bacteria in a culture is given by the function $n(t) = 1000e^{0.35t}$ where $t$ is measured in hours.
(a) What is the relative rate of growth of this bacterium population?
Your answer is ________ percent
(b) What is the initial population of the culture (at $t=0$)?
Your answer is ________
(c) How many bacteria will the culture contain at time $t=5$?
Your answer is ________

24. (1 pt) setAlgebra28ExpFunctions/sw6_2_23.pg
The population of the world in 1987 was 5 billion and the relative growth rate was estimated at 2 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 1995.
Your answer is ________ billion

25. (1 pt) setAlgebra28ExpFunctions/sw6_2_27.pg
Certain radioactive material decays in such a way that the mass remaining after $t$ years is given by the function $m(t) = 185e^{-0.045t}$ where $m(t)$ is measured in grams.
(a) Find the mass at time $t = 0$.
Your answer is ________
(b) How much of the mass remains after 30 years?
Your answer is ________

26. (1 pt) setAlgebra28ExpFunctions/beth1.pg
Complete the table below giving the amount $P$ that must be invested at interest rate 10.5% compounded weekly to obtain a balance of $A = $80000 in $t$ years.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

27. (1 pt) setAlgebra28ExpFunctions/beth2.pg
Complete the table below giving the amount $P$ that must be invested at interest rate 8% compounded continuously to obtain a balance of $A = $80000 in $t$ years.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

28. (1 pt) setAlgebra28ExpFunctions/ur_le_2_13.pg
Starting with the graph of $f(x) = 8^x$, write the equation of the graph that results from
(a) shifting $f(x)$ 9 units downward, $y =$ ________
(b) shifting $f(x)$ 5 units to the right. $y =$ ________
(c) reflecting $f(x)$ about the x-axis. $y =$ ________

29. (1 pt) setAlgebra28ExpFunctions/pexp.pg
Starting with the graph of $f(x) = 3^x$, write the equation of the graph that results from
(a) shifting $f(x)$ 8 units downward. $y =$ ________
(b) shifting $f(x)$ 8 units to the right. $y =$ ________
(c) reflecting $f(x)$ about the x-axis and the y-axis. $y =$ ________

30. (1 pt) setAlgebra28ExpFunctions/Test1_20.pg
You invest $6000 in Acme Inc. on January 1, 2000. Your investment returns 6.5% compounded monthly. How much money will you have on June 30, 2006?
You will have $______
After what month and year will you have at least $15,000?
You will have at least $15,000 after ____ (month) ____ (year). Please capitalize the month and do not use any abbreviation.

31. (1 pt) setAlgebra28ExpFunctions/simplifying.pg
Simplify the following expressions. Give exact answers with the fewest number of $e$’s possible. Then give a decimal approximation.

a.) $e^{-5} e^{0} e^{4} =$

_____ , which is approximately _____

b.) $\frac{e^{10} - e^{-10}}{e^{5} - e^{-5}} =$

_____ , which is approximately _____

c.) $(e^{8} - 1) (e^{7} + 9) =$

_____ , which is approximately _____

32. (1 pt) setAlgebra28ExpFunctions/ur_le_1_5.pg
Find the exponential function $f(x) = a \cdot 2^{bx}$ whose graph is shown below.
1. (1 pt) setAlgebra29LogFunctions/sw6_3_1.pg
Express the equation in exponential form
(a) \( \log_2 4 = 2 \).
That is, write your answer in the form \( 2^A = B \). Then
\[ A = \] and \( B = \).
(b) \( \log_5 3125 = 5 \).
That is, write your answer in the form \( 5^C = D \). Then
\[ C = \] and \( D = \).

2. (1 pt) setAlgebra29LogFunctions/sw6_3_3.pg
Express the equation in exponential form
(a) \( \log_2 4 = \frac{1}{2} \).
That is, write your answer in the form \( 8^A = B \). Then
\[ A = \] and \( B = \).
(b) \( \log_2 \frac{1}{32} = -5 \).
That is, write your answer in the form \( 2^C = D \). Then
\[ C = \] and \( D = \).

3. (1 pt) setAlgebra29LogFunctions/sw6_3_7.pg
Express the equation in logarithmic form
(a) \( 2^5 = 32 \).
That is, write your answer in the form \( \log_2 A = B \). Then
\[ A = \] and \( B = \).
(b) \( 10^{-4} = 0.000100 \).
That is, write your answer in the form \( \log C = D \). Then
\[ C = \] and \( D = \).

4. (1 pt) setAlgebra29LogFunctions/sw6_3_9.pg
Express the equation in logarithmic form
(a) \( 4^2 = 1024 \).
That is, write your answer in the form \( \log_4 A = B \). Then
\[ A = \] and \( B = \).
(b) \( 10^{-2} = 0.01 \).
That is, write your answer in the form \( \log_{10} C = D \). Then
\[ C = \] and \( D = \).

5. (1 pt) setAlgebra29LogFunctions/sw6_3_13.pg
Evaluate the expression
(a) \( \log_9 9^5 \)
Your answer is \( \) \( ) \)
(b) \( \log_2 8 \)
Your answer is \( \) \( ) \)
(c) \( \log_3 2 \)
Your answer is \( \) \( ) \)

6. (1 pt) setAlgebra29LogFunctions/sw6_3_15.pg
Evaluate the expression, reduce to simplest form.
(a) \( \log_4 4^2 \)
Your answer is \( \) \( ) \)
(b) \( \log_2 256 \)
Your answer is \( \) \( ) \)
(c) \( \log_4 4 \)
Your answer is \( \) \( ) \)

7. (1 pt) setAlgebra29LogFunctions/sw6_3_17.pg
Evaluate the expression, reduce to simplest form.
(a) \( \log_3 (\frac{1}{3}) \)
Your answer is \( \) \( ) \)
(b) \( \log \sqrt[10]{10} \)
Your answer is \( \) \( ) \)
(c) \( \log 0.001 \)
Your answer is \( \) \( ) \)

8. (1 pt) setAlgebra29LogFunctions/sw6_3_19.pg
Evaluate the expression, reduce to simplest form.
(a) \( 2^{\log_2 4} \)
Your answer is \( \) \( ) \)
(b) \( 10^{\log_6 6} \)
Your answer is \( \) \( ) \)
(c) \( e^{\ln 6} \)
Your answer is \( \) \( ) \)

9. (1 pt) setAlgebra29LogFunctions/sw6_3_45.pg
The graph of the function \( f(x) = \log_4 (x - 9) \) can be obtained from the graph of \( g(x) = \log_4 x \) by one of the following actions:
- (a) shifting the graph of \( g(x) \) to the right 9 units;
- (b) shifting the graph of \( g(x) \) to the left 9 units;
- (c) shifting the graph of \( g(x) \) upward 9 units;
- (d) shifting the graph of \( g(x) \) downward 9 units;
Your answer is (input a, b, c, or d) \( ) \)
The domain of the function \( f(x) = x > A \), find \( A \) The value of \( A \) is \( ) \)
Is the range of the function \( f(x) \) still \( (-\infty, \infty) \)?
Your answer is (input Yes or No) \( ) \)

10. (1 pt) setAlgebra29LogFunctions/sw6_3_49.pg
The graph of the function \( f(x) = 4 + \log_3 x \) can be obtained from the graph of \( g(x) = \log_3 x \) by one of the following actions:
- (a) shifting the graph of \( g(x) \) to the right 4 units;
- (b) shifting the graph of \( g(x) \) to the left 4 units;
- (c) shifting the graph of \( g(x) \) upward 4 units;

(d) shifting the graph of \( g(x) \) downward 4 units;
Your answer is (input a, b, c, or d) ________
The domain of the function \( f(x) = x > A \), find \( A \)
The value of \( A \) is ________
Is the range of the function \( f(x) \) still \( (-\infty, \infty) \)?
Your answer is (input Yes or No) ________

11. (1 pt) setAlgebra29LogFunctions/sw6.4.1.pg
Use the Laws of logarithms to rewrite the expression
\[
\log_2(3x(x-6))
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we will have:
\[
\log_2(3x(x-6)) = \log_2 A + \log_2 x + \log_2 f(x)
\]
with the constant
\[ A = \]
and the function
\[ f(x) = \]

12. (1 pt) setAlgebra29LogFunctions/sw6.4.3.pg
Use the Laws of logarithms to rewrite the expression
\[
\log 9^{18}
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have:
\[
\log 9^{18} = A \log 9
\]
with the constant
\[ A = \]\n
13. (1 pt) setAlgebra29LogFunctions/sw6.4.5.pg
Use the Laws of logarithms to rewrite the expression
\[
\log_2(x^6 y^{13})
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[
\log_2(x^6 y^{13}) = A \log_2 x + B \log_2 y
\]
with the constant
\[ A = \]
and the constant
\[ B = \]

14. (1 pt) setAlgebra29LogFunctions/sw6.4.7.pg
Use the Laws of logarithms to rewrite the expression
\[
\log_3(x^{8/3} y^{15})
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[
\log_3(x^{8/3} y^{15}) = A \log_3 x + B \log_3 y
\]
with the constant
\[ A = \]
and the constant
\[ B = \]

15. (1 pt) setAlgebra29LogFunctions/sw6.4.9.pg
Use the Laws of logarithms to rewrite the expression
\[
\log_5 \sqrt[4]{x^2 + 11}
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[
\log_5 \sqrt[4]{x^2 + 11} = A \log_5 f(x)
\]
with the constant
\[ A = \]
and the function
\[ f(x) = \]

16. (1 pt) setAlgebra29LogFunctions/sw6.4.11.pg
Use the Laws of logarithms to rewrite the expression
\[
\ln \sqrt[3]{xy}
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[
\ln \sqrt[3]{xy} = A \log x + B \log y
\]
with the constant
\[ A = \]
and the constant
\[ B = \]

17. (1 pt) setAlgebra29LogFunctions/sw6.4.13.pg
Use the Laws of logarithms to rewrite the expression
\[
\log(\sqrt[6]{x^{12} y^{16} / z^6})
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[
\log(\sqrt[6]{x^{12} y^{16} / z^6}) = A \log x + B \log y + C \log z
\]
with the constant
\[ A = \]
the constant
\[ B = \]
and the constant
\[ C = 2 \]

18. (1 pt) setAlgebra29LogFunctions/sw6.4.17.pg
Use the Laws of logarithms to rewrite the expression
\[
\ln(x^5 \sqrt[16]{y^{10} / z})
\]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[
\ln(x^5 \sqrt[16]{y^{10} / z}) = A \ln x + B \ln y + C \ln z
\]
with the constant
\[ A = \]
the constant
\[ B = \]
and the constant
\[ C = \]
19. Evaluate the expression, reducing to simplest form.
\( \log_5 \sqrt{125} = \) 

20. Evaluate the expression, reduce to simplest terms.
\( \log 2^5 + \log 5^3 = \) 

21. Rewrite the expression
\[ \log_2 x + 3 \log_2 y - 3 \log_2 z \]
as a single logarithm \( \log_2 A \). Then the function
\( A = \) 

22. Rewrite the expression
\[ 3 \log x - 3 \log (x^2 + 1) + 5 \log (x - 1) \]
as a single logarithm \( \log A \). Then the function
\( A = \) 

23. Rewrite the expression
\[ \ln (a + b) + 3 \ln (a - b) - 5 \ln c \]
as a single logarithm \( \ln A \). Then the function
\( A = \) 

24. Rewrite the expression
\[ \ln 3 + 3 \ln x + 6 \ln (x^2 + 6) \]
as a single logarithm \( \ln A \). Then the function
\( A = \) 

25. Express the equation in exponential form
(a) \( \ln 3 = x \) is equivalent to \( e^x = B \).
\( A = \) and 
\( B = \) 
(b) \( \ln x = 3 \) is equivalent to \( e^C = D \).
\( C = \) and 
\( D = \) 

26. Express the equation in logarithmic form:
(a) \( 4^3 = 64 \) is equivalent to \( \log_4 A = B \).
\( A = \) and 
\( B = \) 
(b) \( 10^{-3} = 0.001 \) is equivalent to \( \log_{10} C = D \).
\( C = \) and 
\( D = \) 

27. Express the equation in logarithmic form:
(a) \( e^3 = 4 \) is equivalent to \( \ln A = B \). Then
\( A = \) 

28. Evaluate the expression
(a) \( \log_3 8^3 \)
Your answer is 
(b) \( \log_2 243 \)
Your answer is 
(c) \( \log_3 3 \)
Your answer is 

29. Evaluate the expression, reduce to simplest form.
(a) \( \log_3 \left( \frac{4}{5} \right) \)
Your answer is 
(b) \( \log \sqrt[3]{10} \)
Your answer is 
(c) \( \log 0.01 \)
Your answer is 

30. Evaluate the expression, reduce to simplest form.
(a) \( \ln e^{-1} \)
Your answer is 
(b) \( \ln e^3 \)
Your answer is 
(c) \( \ln (1/e) \)
Your answer is 

31. The graphs of the functions \( y = a^x \) and \( y = \log_a x \) are symmetric with respect to the line \( y = \) 

32. The graph of the function \( f(x) = \log_2 (x - 9) \) can be obtained from the graph of \( g(x) = \log_2 x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 9 units; 
(b) shifting the graph of \( g(x) \) to the left 9 units; 
(c) shifting the graph of \( g(x) \) upward 9 units; 
(d) shifting the graph of \( g(x) \) downward 9 units; 
Your answer is (input a, b, c, or d) 
The domain of the function \( f(x) \) is \( x > A \), find \( A \) 
The value of \( A \) is 
Is the range of the function \( f(x) \) still \( (-\infty, \infty) \)? 
Your answer is (input Yes or No) 

33. The graph of the function \( f(x) = 3 + \log_3 x \) can be obtained from the graph of \( g(x) = \log_3 x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 3 units; 
(b) shifting the graph of \( g(x) \) to the left 3 units; 
(c) shifting the graph of \( g(x) \) upward 3 units; 
(d) shifting the graph of \( g(x) \) downward 3 units; 

Your answer is (input a, b, c, or d)
The domain of the function \( f(x) \) is \( x > A \), find A
The value of A is
Is the range of the function \( f(x) \) still \((-\infty, \infty)\)?
Your answer is (input Yes or No)

34. (1 pt) setAlgebra29LogFunctions/srw4_2_59.pg
The domain of the function \( g(x) = \log_a(x^2 - 25) \) is
\((-\infty, A) \) and \((A, \infty)\).

35. (1 pt) setAlgebra29LogFunctions/srw4_3_3.pg
Use the Laws of logarithms to rewrite the expression
\( \log_2(13x(x - 19)) \)
in a form with no logarithm of a product, quotient or power.
After rewriting we will have:
\[ \log_2(13x(x - 19)) = \log_2 A + \log_2 x + \log_2 f(x) \]
with the constant
A = 
and the function
\( f(x) = \)

36. (1 pt) setAlgebra29LogFunctions/srw4_3_5.pg
Use the Laws of logarithms to rewrite the expression
\( \log 16^{19} \)
in a form with no logarithm of a product, quotient or power.
After rewriting we have:
\[ \log 16^{19} = A \log 16 \]
with the constant
A = 

37. (1 pt) setAlgebra29LogFunctions/srw4_3_7.pg
Use the Laws of logarithms to rewrite the expression
\( \log_2(x^{13}y^{14}) \)
in a form with no logarithm of a product, quotient or power.
After rewriting we have:
\[ \log_2(x^{13}y^{14}) = A \log_2 x + B \log_2 y \]
with the constant
A = 
and the constant
B = 

38. (1 pt) setAlgebra29LogFunctions/srw4_3_9.pg
Use the Laws of logarithms to rewrite the expression
\( \log_3(x^{20}\sqrt[3]{y^{18}}) \)
in a form with no logarithm of a product, quotient or power.
After rewriting we have:
\[ \log_3(x^{20}\sqrt[3]{y^{18}}) = A \log_3 x + B \log_3 y \]
with the constant
A = 
and the constant
B = 

39. (1 pt) setAlgebra29LogFunctions/srw4_3_13.pg
Use the Laws of logarithms to rewrite the expression
\[ \ln \sqrt[\sqrt{xy}] \]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[ \ln \sqrt[\sqrt{xy}] = A \log x + B \log y \]
with the constant
A = 
and the constant
B = 

40. (1 pt) setAlgebra29LogFunctions/srw4_3_15.pg
Use the Laws of logarithms to rewrite the expression
\[ \log \left( \frac{x^{17}}{z^{11}} \right) \]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[ \log \left( \frac{x^{17}}{z^{11}} \right) = A \log x + B \log y + C \log(z) \]
with
A = 
B = 
C = 

41. (1 pt) setAlgebra29LogFunctions/srw4_3_17-20.pg
Evaluate the following expressions.
(a) \( \log_7 7^{12} = \)
(b) \( \log_3 243 = \)
(c) \( \log_4 64 = \)
(d) \( \log_2 2^8 = \)

42. (1 pt) setAlgebra29LogFunctions/srw4_3_19.pg
Use the Laws of logarithms to rewrite the expression
\[ \ln \left( \frac{x^{17} \sqrt[15]{y^{11}}}{z^{11}} \right) \]
in a form with no logarithm of a product, quotient or power.
After rewriting we have
\[ \ln \left( \frac{x^{17} \sqrt[15]{y^{11}}}{z^{11}} \right) = A \ln x + B \ln y + C \ln(z) \]
with the constant
A = 
the constant
B = 
and the constant
C = 

43. Use the Laws of logarithms to rewrite the expression
\[ \log \sqrt[9]{(x^2 + 13)(x^3 - 2)^9} \]
in a form with no logarithm of a product, quotient or power. After rewriting we have
\[ \log \sqrt[9]{(x^2 + 13)(x^3 - 2)^9} = A \log (x^2 + 13) + B \log (x^2 + 7) + C \log (x^3 - 2) \]
with the constant \( A = \) 
the constant \( B = \) 
and the constant \( C = \)

44. Evaluate the following expressions.
(a) \( \log_2 \left( \frac{1}{2} \right) = \)
(b) \( \log_6 1 = \)
(c) \( \log_5 \sqrt[3]{625} = \)
(d) \( 9^{\log_9 14} = \)

45. Use the Laws of logarithms to rewrite the expression
\[ \ln \left( \frac{13 \sqrt{x - 1}}{3x - 16} \right) \]
in a form with no logarithm of a product, quotient or power. After rewriting we have
\[ \ln \left( \frac{13 \sqrt{x - 1}}{3x - 16} \right) = A \ln x + B \ln (x - 1) + C \ln (3x - 16) \]
with the constant \( A = \) 
the constant \( B = \) 
and the constant \( C = \)

46. Evaluate the expression, reducing to simplest form
\( \log_4 \sqrt{256} = \)

47. Evaluate the following expressions.
\( \log_4 0.00390625 = \)
\( \log_{10} 0.001 = \)
\( \log_{3125} \sqrt[5]{5} = \)
\( \log_4 8 = \)

48. Evaluate the expression, reduce to simplest terms
\( \log_2 6 + \log_5 6 = \)

49. Evaluate the following expressions.
(a) \( \ln e^{-9} = \)
(b) \( e^{\ln 2} = \)
(c) \( e^{\ln \sqrt[4]{2}} = \)
(d) \( \ln \left( \frac{1}{e^3} \right) = \)

50. Evaluate the expression, reducing to simplest form
\( \log(\log 10000^{10000}) = \) + \( \log \) 

Your answers must be integers.

51. Rewrite the expression
\( \log_2 x + 4 \log_2 y - 5 \log_2 z \)
as a single logarithm \( \log_2 A \). Then the function \( A = \)

52. Rewrite the expression
\( 2 \log x - 2 \log (x^2 + 1) + 3 \log (x - 1) \)
as a single logarithm \( \log A \). Then the function \( A = \)

53. Rewrite the expression
\( \ln (a + b) + 4 \ln (a - b) - 3 \ln c \)
as a single logarithm \( \ln A \). Then the function \( A = \)

54. Rewrite the expression
\( \ln 2 + 2 \ln x + 3 \ln(x^2 + 8) \)
as a single logarithm \( \ln A \). Then the function \( A = \)

55. Evaluate the expression, correct to six decimal places, by the Change of Base Formula and a calculator.
\( \log_2 9 = \)

56. Evaluate the expression, correct to six decimal places, by the Change of Base Formula and a calculator.
\( \log_4 89 = \)

57. Match the statements defined below with the letters labeling their equivalent expressions.
1. \( \ln(y^x) \)
2. \( \ln(xy) \)
3. \( \ln \frac{y}{x} \)
4. \( \ln(x^y) \)
A. \( x \ln y \)
B. \( y \ln x \)
C. \( \ln x + \ln y \)
D. \( \ln x - \ln y \)

58. Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.
You must get all of the answers correct to receive credit.
1. \( \ln a^b = b \ln a \)
2. \( \ln(x - y) = \ln x - \ln y \)
3. \( (\ln a)^b = b \ln a \)
4. \( \log_b b = \log_b a \)

59. (1 pt) setAlgebra29LogFunctions/srw4_4_78.pg
Rewrite the expression in terms of \( \ln \)

\[
\log_4 16 = \_
\]

60. (1 pt) setAlgebra29LogFunctions/srw4_4_80.pg
Rewrite the expression in terms of \( \ln \)

\[
\log_{16} 6 = \_
\]

61. (1 pt) setAlgebra29LogFunctions/c4s2p39_44/c4s2p39_44.png
Match the functions with their graphs. Enter the letter of the graph below which corresponds to the function. (Click on image for a larger view)

1. \( f(x) = \ln(-x) \)
2. \( f(x) = -\ln(-x) \)
3. \( f(x) = 2 + \ln x \)
4. \( f(x) = \ln(x - 2) \)
5. \( f(x) = -\ln x \)

A  B  C  D

62. (1 pt) setAlgebra29LogFunctions/beth1logfun.png
The graph of the function \( f(x) = \log_4(x - 1) \) can be obtained from the graph of \( g(x) = \log_4 x \) by one of the following actions:
(a) shifting the graph of \( g(x) \) to the right 1 units;
(b) shifting the graph of \( g(x) \) to the left 1 units;
(c) shifting the graph of \( g(x) \) upward 1 units;
(d) shifting the graph of \( g(x) \) downward 1 units;
Your answer is (input a, b, c, or d)

The domain of the function \( f(x) \) is

Note: Enter your answer using interval notation.

The range of the function \( f(x) \) is

Note: Enter your answer using interval notation.

The \( x \)-intercept of the function \( f(x) \) is

The vertical asymptote of the function \( f(x) \) has equation:

63. (1 pt) setAlgebra29LogFunctions/evaluating_expressions.pg
If \( \ln a = 2 \), \( \ln b = 3 \), and \( \ln c = 5 \), evaluate the following:

(a) \( \ln(\frac{a^3}{b^2}) = \_
\)

(b) \( \ln \sqrt{b^{-2}c^3} = \_
\)

(c) \( \frac{\ln(a^2b^{-1})}{\ln(80)} = \_
\)

(d) \( \ln c^{-1} \) \( \left( \ln \frac{a}{b^2} \right)^{-4} = \_
\)

64. (1 pt) setAlgebra29LogFunctions/problem1.pg
Evaluate the following expressions. Your answers must be exact and in simplest form.

(a) \( \log_7 7^4 = \_
\)

65. (1 pt) setAlgebra29LogFunctions/problem2.pg
Evaluate the following expressions. Your answers must be exact and in simplest form.

(a) \( \log_5 \left( \frac{1}{125} \right) = \_
\)

(b) \( \log_{13} 1 = \_
\)

(c) \( \log_8 \sqrt{8125} = \_
\)

(d) \( 2^{\log_2 9} = \_
\)

66. (1 pt) setAlgebra29LogFunctions/problem3.pg
Evaluate the following expressions.

\( \log_2 0.125 = \_
\)

\( \log_{10} 0.001 = \_
\)

\( \log_{625} \sqrt{5} = \_
\)

\( \log_4 8 = \_
\)

67. (1 pt) setAlgebra29LogFunctions/problem4.pg
Evaluate the following expressions. Your answers must be exact and in simplest form.

(a) \( \ln e^{-9} = \_
\)

(b) \( e^{\ln 3} = \_
\)

(c) \( e^{\ln 2} = \_
\)

(d) \( \ln(1/e^2) = \_
\)

68. (1 pt) setAlgebra29LogFunctions/problem10.pg
If \( \log_b 2 = x \) and \( \log_b 3 = y \), evaluate the following in terms of \( x \) and \( y \):

(a) \( \log_b 324 = \_
\)

(b) \( \log_b 162 = \_
\)

(c) \( \log_b \pi^7 = \_
\)

(d) \( \log_b 81/\log_b 16 = \_
\)

69. (1 pt) setAlgebra29LogFunctions/simplifying_expressions.pg
Simplify the following expressions. Your answers must be exact and in simplest form.

(a) \( \log_2 2^{1x+2} = \_
\)

(b) \( 9^{\log_9 x - 2 + 9y} = \_
\)

(c) \( \log_2 0.000244140625^k = \_
\)

(d) \( x^{3\log_5 10 - 10\log_5 3} = \_
\)

70. (1 pt) setAlgebra29LogFunctions/ur_le_1_4.pg
Simplify:

\( 16^{\log_2 8} = \_
\)

\( 3^{\log_3 6} = \_
\)

71. (1 pt) setAlgebra29LogFunctions/srw4_4_9.pg
\[
\ln(r^2s^2\sqrt{r^2s^2})
\]

is equal to

\( A \ln r + B \ln s \)

where \( A = \_
\)

and \( B = \_
\)
1. Find $x$.
   (a) $\log_6 x = 4$
   Your answer is __________
   (b) $\log_2 8 = x$
   Your answer is __________

2. Find $x$.
   (a) $\log x = 2$
   $x = __________$
   (b) $\log_5 27 = 3$
   $x = __________$

3. Find the solution of the exponential equation $3^x = 16$ in terms of logarithms, or correct to at least four decimal places. $x = __________$

4. Find the solution of the exponential equation $13^{1-x} = 12$ in terms of logarithms, or correct to four decimal places. $x = __________$

5. Find the solution of the exponential equation $3e^x = 17$ in terms of logarithms, or correct to four decimal places. $x = __________$

6. Find the solution of the exponential equation $e^{1-4x} = 11$ in terms of logarithms, or correct to four decimal places. $x = __________$

7. Find the solution of the exponential equation $2 + 7^x = 26$ in terms of logarithms, or correct to four decimal places. $x = __________$

8. Find the solution of the exponential equation $3^x + 4 - 2x - 4^x = 0$ has two roots. The smaller root is ____ and the bigger root is ________

9. Find the solution of the exponential equation $20^{-x/7} = 3$ in terms of logarithms, or correct to four decimal places. $x = __________$

10. Find the solution of the exponential equation $e^{2x+1} = 11$ in terms of logarithms, or correct to four decimal places. $x = __________$

11. Find the solution of the logarithmic equation $\ln x = 9$ in terms of logarithms, or correct to four decimal places. Your answer is $x = __________$

12. Find the solution of the logarithmic equation $\log (3x - 3) = 2$ in terms of logarithms, or correct to four decimal places. Your answer is $x = __________$

13. Find the solution of the logarithmic equation $\log (x^2 - 1x - 16) = 2$ in terms of logarithms, or correct to decimal places. Your answers are $x_1 = ________$ and $x_2 = ________$ with $x_1 \leq x_2$

14. Find the solution of the logarithmic equation $\log x + \log(x - 5) = \log(17x)$ in terms of logarithms, or correct to four decimal places. Your answer is $x = __________$

15. Find the solution of the logarithmic equation $\ln(x + 4) + \ln(x - 4) = 0$ in terms of logarithms, or correct to four decimal places. Your answer is $x = __________$

16. The equation $x^2 + 4^x - 2x + 4^x = 0$ has two roots. The smaller root is ____ and the bigger root is ________

17. The equation $3x^2e^{-3x} - 8x^3e^{-8x} = 0$ has two roots.
The smaller root is _____
and the bigger root is _____

18. (1 pt) setAlgebra30LogExpEqns/srw4_2_3.pg
Express the equation in exponential form
(a) \( \log_3 2 = \frac{1}{3} \).
That is, write your answer in the form \( 8^A = B \). Then
\[ A= \]
and
\[ B= \]
(b) \( \log_2 \frac{1}{4} = -3 \).
That is, write your answer in the form \( 2^C = D \). Then
\[ C= \]
and
\[ D= \]

19. (1 pt) setAlgebra30LogExpEqns/srw4_2_23.pg
Find \( x \).
(a) \( \log_8 x = 4 \)
Your answer is __________
(b) \( \log_3 16 = x \)
Your answer is __________

20. (1 pt) setAlgebra30LogExpEqns/srw4_2_27.pg
Find \( x \).
(a) \( \log x = 2 \)
\( x= \)
(b) \( \log_3 x = 4 \)
\( x= \)

21. (1 pt) setAlgebra30LogExpEqns/srw4_2_30.pg
Find \( x \).
(a) \( \log_6 64 = 3 \)
\( x= \)
(b) \( \log_5 25 = 2 \)
\( x= \)

22. (1 pt) setAlgebra30LogExpEqns/srw4_2_35.pg
The graph of the function \( y = \log_a x \) goes through \((46, 1)\).
Then \( a = \) _____

23. (1 pt) setAlgebra30LogExpEqns/srw4_2_37.pg
The graph of the function \( y = \log_a x \) goes through \((8, -1)\).
Then \( a = \) __________

24. (1 pt) setAlgebra30LogExpEqns/srw4_3_37-38.pg
(a) If \( \log_3 x = 3 \), then \( x = \) ______
(b) If \( \log_4 x = 2 \), then \( x = \) ______

25. (1 pt) setAlgebra30LogExpEqns/srw4_3_41-42.pg
(a) If \( \log_4 4 = 2 \), then \( x = \) ______
(b) If \( \log_6 64 = 3 \), then \( x = \) ______

26. (1 pt) setAlgebra30LogExpEqns/srw4_3_43-44.pg
(a) If \( 3^x = 39 \), then \( x = \) ______
(b) If \( 19^{-x} = 2 \), then \( x = \) ______

27. (1 pt) setAlgebra30LogExpEqns/srw4_3_46.pg
Solve the given equation for \( x \).
\( 3^{x-2} = 41 \)
\( x = \) ______

28. (1 pt) setAlgebra30LogExpEqns/srw4_3_48.pg
If \( \ln(6x + 3) = 4 \), then \( x = \) ______

29. (1 pt) setAlgebra30LogExpEqns/srw4_3_51.pg
If \( e^{3x} = 23 \), then \( x = \) ______

30. (1 pt) setAlgebra30LogExpEqns/srw4_4_1.pg
Find the solution of the exponential equation
\( 2^x = 14 \)
in terms of logarithms, or correct to at least four decimal places.
\( x = \) ______

31. (1 pt) setAlgebra30LogExpEqns/srw4_4_5.pg
Find the solution of the exponential equation
\( 11^{1-x} = 13 \)
in terms of logarithms, or correct to four decimal places.
\( x = \) ______

32. (1 pt) setAlgebra30LogExpEqns/srw4_4_7.pg
Find the solution of the exponential equation
\( 9e^x = 16 \)
in terms of logarithms, or correct to four decimal places.
\( x = \) ______

33. (1 pt) setAlgebra30LogExpEqns/srw4_4_9.pg
Find the solution of the exponential equation
\( e^{1-4x} = 13 \)
in terms of logarithms, or correct to four decimal places.
\( x = \) ______

34. (1 pt) setAlgebra30LogExpEqns/srw4_4_11.pg
Find the solution of the exponential equation
\( -5 + 7^{5x} = 21 \)
correct to at least four decimal places.
\( x = \) ______

35. (1 pt) setAlgebra30LogExpEqns/srw4_4_15.pg
Find the solution of the exponential equation
\( 7^{-x/18} = 12 \)
in terms of logarithms, or correct to four decimal places.
\( x = \) ______

36. (1 pt) setAlgebra30LogExpEqns/srw4_4_17.pg
Find the solution of the exponential equation
\( e^{2x+1} = 30 \)
in terms of logarithms, or correct to four decimal places.
\( x = \) ______

37. (1 pt) setAlgebra30LogExpEqns/srw4_4_21.pg
Find the solution of the exponential equation
\( 2^{2x+14} = 3^{x-37} \)
in terms of logarithms, or correct to four decimal places.

\[ x = \]  

38. (1 pt) setAlgebra30LogExpEqns/srw4_4_25.pg
Find the solution of the exponential equation

\[ 1000(1.04)^2 = 5000 \]
in terms of logarithms, or correct to four decimal places.

\[ x = \]  

39. (1 pt) setAlgebra30LogExpEqns/srw4_4_27.pg
Find the solutions of the exponential equation

\[ x^2 2^x - 2^x = 0 \]

\[ x_1 = \text{ and } x_2 = \text{ with } x_1 < x_2. \]

40. (1 pt) setAlgebra30LogExpEqns/srw4_4_31.pg
Find the solutions of the exponential equation

\[ e^{2x} - 5e^x + 6 = 0. \]
Enter your answer as a comma-separated list, and enter none if there are no solutions.

41. (1 pt) setAlgebra30LogExpEqns/srw4_4_32-34.pg
Evaluate the following expressions.

(a) \[ e^{\ln 3} = \]

(b) \[ 10^{\log_{10} 5} = \]

(c) \[ \log_3 27^4 = \]

[NOTE: Your answers cannot be algebraic expressions.]

42. (1 pt) setAlgebra30LogExpEqns/srw4_4_33.pg
Find the solutions of the exponential equation

\[ e^{2x} + 3e^x - 4 = 0. \]

\[ x_1 = \text{ and } x_2 = \text{ with } x_1 < x_2. \]

If there is only one solution, input it at \( x_1 \).

43. (1 pt) setAlgebra30LogExpEqns/srw4_4_35.pg
Find the solution of the logarithmic equation

\[ \ln x = 9 \]
in terms of logarithms, or correct to four decimal places.
Your answer is

\[ x = \]

44. (1 pt) setAlgebra30LogExpEqns/srw4_4_39.pg
Find the solution of the logarithmic equation

\[ \log(3x + 5) = 3 \]
in terms of logarithms, or correct to four decimal places.
Your answer is

\[ x = \]

45. (1 pt) setAlgebra30LogExpEqns/srw4_4_41.pg
Find the solution of the logarithmic equation

\[ 13 - \ln(4 - x) = 0 \]
correct to four decimal places.
Your answer is

\[ x = \]

46. (1 pt) setAlgebra30LogExpEqns/srw4_4_42.pg
Find the solution(s) of the logarithmic equation

\[ \log_2(x^2 + 5x - 46) = 2 \]
correct to four decimal places.
If there is more than one solution write them separated by commas.

\[ x = \]

47. (1 pt) setAlgebra30LogExpEqns/srw4_4_45.pg
Find the solution(s) of the logarithmic equation

\[ \log x + \log(x - 15) = \log(20x) \]
correct to four decimal places.
If there is more than one solution write them separated by commas.

\[ x = \]

48. (1 pt) setAlgebra30LogExpEqns/srw4_4_50.pg
Find the solution(s) of the logarithmic equation

\[ \ln(x + 8) + \ln(x - 8) = 0 \]
correct to four decimal places.
If there is more than one solution write them separated by commas.

\[ x = \]

49. (1 pt) setAlgebra30LogExpEqns/srw4_4_51.pg
For what value of \( x \) is the following true?

\[ \log(x + 6) = \log x + \log 6. \]
Your answer is

\[ x = \]

50. (1 pt) setAlgebra30LogExpEqns/srw4_4_58.pg
If \( \ln x + \ln(x - 4) = \ln 5x \), then \( x = \)

51. (1 pt) setAlgebra30LogExpEqns/srw4_4_62.pg
Solve the given equation for \( x \).

\[ \log_{10} x + \log_{10}(x + 15) = 2 \]

\[ x = \]

52. (1 pt) setAlgebra30LogExpEqns/srw4_4_65.pg
Solve the given equation for \( x \).

\[ 3^{x-4} = 3 \]

\[ x = \]

53. (1 pt) setAlgebra30LogExpEqns/srw4_4_68.pg
Solve the given equation for \( x \).

\[ 4^{x/5} = 5 \]

\[ x = \]

54. (1 pt) setAlgebra30LogExpEqns/srw4_4_72.pg
Solve the given equation for \( x \).

\[ \left( \frac{2}{5} \right)^x = 2 \]

\[ x = \]

55. (1 pt) setAlgebra30LogExpEqns/mec1.png
The equation \( e^{2x} - 7e^x + 12 = 0 \) has two solutions.

The smaller one is: ______
and the larger one is: ______
56. (1 pt) setAlgebra30LogExpEqns/mec2.pg
If \( e^{2x} - 2e^x = +3 \), then \( x = \) ________

57. (1 pt) setAlgebra30LogExpEqns/ur_log_1.pg
For each of the following, find the base \( b \) if the graph of \( y = b^x \) contains the given point.

\[
(0.5, 1) \ b = \\
(1, 0.5) \ b = \\
(3, 125) \ b = \\
(1, 3) \ b = \\
(-1, 2) \ b = \\
(-4, 16) \ b = \\
(3, 27) \ b = \\
(4, 81) \ b = \\
(4, 0.0625) \ b = 
\]

58. (1 pt) setAlgebra30LogExpEqns/ur_log_1.pg
Determine the smallest integer \( x \) that satisfies the given inequality.

\[
8 \sqrt[3]{x} > 26
\]

59. (1 pt) setAlgebra30LogExpEqns/rbth2logfun.pg
Find the solution of the exponential equation

\[
17e^x - 6 = 7
\]
in terms of logarithms, or correct to four decimal places.

60. (1 pt) setAlgebra30LogExpEqns/rbth2logfun.pg
Express the equation in exponential form

(a) \( \log_2 2 = \frac{1}{2} \).

That is, write your answer in the form \( A^B = C \). Then

\[
A = , \ B = , \ C = 
\]

(b) \( \log_2 \frac{1}{32} = -5 \).

That is, write your answer in the form \( D^E = F \). Then

\[
D = , \ E = , \ F = 
\]

61. (1 pt) setAlgebra30LogExpEqns/rbth2logfun.pg
Express the equation in logarithmic form

(a) \( 2^x = 16 \).

That is, write your answer in the form \( \log_A B = C \). Then

\[
A = , \ B = , \ C = 
\]

(b) \( 10^{-2} = 0.010000 \).

That is, write your answer in the form \( \log D = E \). Then

\[
D = \ \text{and} \ E = 
\]

62. (1 pt) setAlgebra30LogExpEqns/problem1.pg
Solve for \( x \):

\[
125^{9x-5} = \left( \frac{1}{625} \right)^{2x-4}
\]

63. (1 pt) setAlgebra30LogExpEqns/problem2.pg
Solve for \( x \):

\[
x = 8^{\log_2 x + \log_5 x}
\]

64. (1 pt) setAlgebra30LogExpEqns/problem3.pg
Solve for \( x \):

\[
\log_4 \frac{1}{512} = x
\]

Your answer must be exact and in simplest form.

65. (1 pt) setAlgebra30LogExpEqns/problem4.pg
Solve for \( x \):

\[
\frac{9}{8} \log_b x = 8
\]

66. (1 pt) setAlgebra30LogExpEqns/problem5.pg
Solve for \( x \):

\[
(\log_4 (\log_4 x)) = -3
\]

67. (1 pt) setAlgebra30LogExpEqns/problem6.pg
(a) If \( \log_9 x = 7 \), then \( x = \) ________

(b) If \( \log_3 x = 9 \), then \( x = \) ________

68. (1 pt) setAlgebra30LogExpEqns/problem7.pg
(a) If \( \log_5 512 = 3 \), then \( x = \) ________

(b) If \( \log_4 16 = 2 \), then \( x = \) ________

69. (1 pt) setAlgebra30LogExpEqns/problem6a.pg
Solve for \( x \):

\[
\log_3 x^3 = -8
\]

70. (1 pt) setAlgebra30LogExpEqns/problem7a.pg
Solve for \( x \):

\[
\log_6 x + \log_6 (x + 4) = \log_6 1
\]

71. (1 pt) setAlgebra30LogExpEqns/problem8.pg
Solve for \( x \):

\[
\log x + \log (x + 5) = 4
\]

72. (1 pt) setAlgebra30LogExpEqns/problem9.pg
Solve for \( x \):

\[
4^x = 38
\]
Solve for $x$:

$x = \log_6 793$

$x = \underline{\hphantom{3456}}$

74. (1 pt) setAlgebra30LogExpEqns/problem11.pg

Solve for $x$:

$7 \cdot 6^{x-5} = 65$

$x = \underline{\hphantom{3456}}$

75. (1 pt) setAlgebra30LogExpEqns/problem12.pg

Solve for $x$:

$\log (x^3) = (\log x)^2$

Note, there are 2 solutions, $A$ and $B$, where $A < B$.

$A = \underline{\hphantom{3456}}$

$B = \underline{\hphantom{3456}}$

76. (1 pt) setAlgebra30LogExpEqns/problem13a.pg

Solve for $x$ in terms of $k$.

$\log_4 x^6 - \log_4 x^8 = k$.

$x = \underline{\hphantom{3456}}$

Find $x$ if $k = 7$. \underline{\hphantom{3456}}

77. (1 pt) setAlgebra30LogExpEqns/problem13.pg

Solve for $x$ in terms of $k$.

$\log_7 x + \log_7 (x + 4) = k$.

$x = \underline{\hphantom{3456}}$

Find $x$ if $k = 6$. \underline{\hphantom{3456}}

78. (1 pt) setAlgebra30LogExpEqns/problem14.pg

Solve for $x$ in terms of $k$.

$\log_3 x - \log_3 (x + 7) = \log_3 k$.

$x = \underline{\hphantom{3456}}$

Find $x$ if $k = 1/7$. \underline{\hphantom{3456}}

79. (1 pt) setAlgebra30LogExpEqns/problem15.pg

Solve for $x$ in terms of $a$ and $b$.

$\log x = -9(\log a + \log b) + 4\log b^7 + 2(\log b - \log a)$

$x = \underline{\hphantom{3456}}$

80. (1 pt) setAlgebra30LogExpEqns/problem16.pg

Solve for $x$ in each of the following.

(a) If $\log_4 8 = \frac{4}{3}$, then $x = \underline{\hphantom{3456}}$

(b) If $5^{8x+4} = 7$, then $x = \underline{\hphantom{3456}}$

81. (1 pt) setAlgebra30LogExpEqns/solve_easy_eqn.pg

Find the largest value of $x$ that satisfies:

$\log_4 (x^2) - \log_4 (x + 2) = 6$

$x = \underline{\hphantom{3456}}$

82. (1 pt) setAlgebra30LogExpEqns/solve_difference_of_logs.pg

The equation $4x^3e^{-3x} - 3x^4e^{-3x} = 0$ has two roots.

The smaller root is \underline{\hphantom{3456}}

and the bigger root is \underline{\hphantom{3456}}

83. (1 pt) setAlgebra30LogExpEqns/solve_nested_logs.pg

Solve for $x$:

$4^{2x-9} = 8^{7x-7}$

$x = \underline{\hphantom{3456}}$

84. (1 pt) setAlgebra30LogExpEqns/solve_in_exponent.pg

Solve for $x$:

$(\log_4 (\log_4 x)) = 2$

$x = \underline{\hphantom{3456}}$

85. (1 pt) setAlgebra30LogExpEqns/solve_nested_logs.pg

The equation $e^{2x} - 10x^2 + 24 = 0$ has two solutions.

The smaller one is: \underline{\hphantom{3456}} and the larger one is: \underline{\hphantom{3456}}

86. (1 pt) setAlgebra30LogExpEqns/solve_in_exponent.pg

Solve for $x$:

$\log_3 x + \log_3 (x - 3) = 7$

There are two potential roots, $A$ and $B$, where $A \leq B$.

$A = \underline{\hphantom{3456}}$

$B = \underline{\hphantom{3456}}$

Is $A$ actually a root? (yes or no) \underline{\hphantom{3456}}

Is $B$ actually a root? (yes or no) \underline{\hphantom{3456}}

87. (1 pt) setAlgebra30LogExpEqns/solve_sum_of_logs.pg

Solve for $x$:

$x = \frac{7}{2} (\log_6 \sqrt{108} - 3 \log_6 (\frac{2}{\sqrt{3}}) + 2 \log_6 9 + \log_6 512)$

Note: Your answer must be a decimal or fraction.

$x = \underline{\hphantom{3456}}$

88. (1 pt) setAlgebra30LogExpEqns/simplify_rules.pg

Solve for $x$:

$e^{x+6} = e^x + 2$

$x = \underline{\hphantom{3456}}$

89. (1 pt) setAlgebra30LogExpEqns/solve_nested_logs.pg

Solve the equation $e^{x+6} = e^x + 2$

$x = \underline{\hphantom{3456}}$

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, © UR
1. (1 pt) setAlgebra31LogExpApplications/srw4_2_1.pg

A bacteria culture initially contains 2000 bacteria and doubles every half hour.

Find the size of the bacterial population after 40 minutes.

Find the size of the bacterial population after 10 hours.

2. (1 pt) setAlgebra31LogExpApplications/srw4_2_2.pg

The doubling period of a bacterial population is 15 minutes. At time \( t = 120 \) minutes, the bacterial population was 80000. What was the initial population at time \( t = 0 \)?

Find the size of the bacterial population after 4 hours.

3. (1 pt) setAlgebra31LogExpApplications/srw4_2_7.pg

The half-life of Radium-226 is 1590 years. If a sample contains 100 mg, how many mg will remain after 1000 years?

4. (1 pt) setAlgebra31LogExpApplications/srw4_2_9.pg

The half-life of Palladium-100 is 4 days. After 16 days a sample of Palladium-100 has been reduced to a mass of 2 mg. What was the initial mass (in mg) of the sample?________

What is the mass 5 weeks after the start?________

5. (1 pt) setAlgebra31LogExpApplications/srw4_2_13.pg

If 6000 dollars is invested in a bank account at an interest rate of 5 percent per year, find the amount in the bank after 13 years if interest is compounded annually________

Find the amount in the bank after 13 years if interest is compounded quarterly________

Find the amount in the bank after 13 years if interest is compounded monthly________

Finally, find the amount in the bank after 13 years if interest is compounded continuously________

6. (1 pt) setAlgebra31LogExpApplications/srw4_4_57.pg

Find the time required for an investment of 5000 dollars to grow to 8400 dollars at an interest rate of 7.5 percent per year, compounded quarterly.

Your answer is \( t = \) ________ years.

7. (1 pt) setAlgebra31LogExpApplications/srw4_5_2.pg

The pH scale for acidity is defined by \( \text{pH} = -\log_{10}[H^+] \) where \([H^+]\) is the concentration of hydrogen ions measured in moles per liter (M). A substance has a hydrogen ion concentration of \([H^+] = 3.6 \times 10^{-5} \text{M} \). Calculate the pH of the substance._____

8. (1 pt) setAlgebra31LogExpApplications/srw4_5_3.pg

The fox population in a certain region has a relative growth rate of 7 percent per year. It is estimated that the population in the year 2000 was 9800.

(a) Find a function that models the population \( t \) years after 2000 \((t = 0 \text{ for } 2000)\).

Your answer is \( P(t) = \) ________

(b) Use the function from part (a) to estimate the fox population in the year 2008.

Your answer is (the answer must be an integer) __________

9. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_5.pg

The population of a certain city was 161000 in 1998, and the observed relative growth rate is 2 percent per year.

(a) Find a function that models the population after \( t \) years.

Your answer is ________

(b) Find the projected population in the year 2004.

Your answer is ________

(c) In what year will the population reach 231916? Your answer is ________

10. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_8.pg

If a bacteria culture starts with 9000 bacteria and doubles every 30 minutes, how many minutes will it take the population to reach 36000?_____

11. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_9.pg

A culture starts with 20000 bacteria. After one hour the count is 11100.

(a) Find the relative growth rate of the bacteria. Give your answer to at least 4 decimal places.

Your answer is ________ per hour.

(b) Find the number of bacteria after 2 hours.

Your answer is (your answer must be an integer) ________

(c) After how many hours will the number of bacteria double? Your answer is ________ hours.

12. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_10.pg

The count in a bacteria culture was 500 after 20 minutes and 1500 after 35 minutes. What was the initial size of the culture? ________

Find the doubling period. ________ Find the population after 70 minutes. ________ When will the population reach 11000. ________

13. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_13.pg

An infectious strain of bacteria increases in number at a relative growth rate of 230 percent per hour. When a certain critical number of bacteria are present in the bloodstream, a person becomes ill. If a single bacterium infects a person, the critical level to be reached if the same person is infected with 10 bacteria?

Your answer is ________ hours.

14. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_17.pg

The half-life of strontium-90 is 28 years. How long will it take for the critical level to be reached if the same person is infected with 10 bacteria?

Your answer is ________ hours.

15. (1 pt) setAlgebra31LogExpApplications/srw4_4_5_21.pg

A wooden artifact from an ancient tomb contains 35 percent of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years.)

Your answer is ________ years.
The rat population in a major metropolitan city is given by the formula \( n(t) = 35e^{0.04t} \) where \( t \) is measured in years since 1993 and \( n(t) \) is measured in millions.

What was the rat population in 1993? _________

What is the rat population going to be in the year 2010? _________

A certain bacteria population is known to triples every 90 minutes. Suppose that there are initially 90 bacteria.

What is the size of the population after \( t \) hours? _________

Students in a fifth-grade class were given an exam. During the next 2 years, the same students were retested several times. The average score was given by the model

\[ f(t) = 82 - 19 \log_{10}(t + 1), \quad 0 \leq t \leq 24 \]

where \( t \) is the time in months.

(a) What is the average score on the original exam?

(b) What was the average score after 6 months?

(c) What was the average score after 18 months?

At the beginning of an experiment, a scientist has 212 grams of radioactive goo. After 225 minutes, her sample has decayed to 3.3125 grams.

What is the half-life of the goo in minutes? _________

Find a formula for \( G(t) \), the amount of goo remaining at time \( t \).

\[ G(t) = \text{__________} \]

How many grams of goo will remain after 2 minutes? _________

The half-life of Palladium-100 is 4 days. After 12 days a sample of Palladium-100 has been reduced to a mass of 4 mg. What was the initial mass (in mg) of the sample? _________

What is the mass 8 weeks after the start? _________

The doubling period of a bacterial population is 15 minutes. At time \( t = 110 \) minutes, the bacterial population was 90000. For some constant \( A \), the formula for the population is \( p(t) = Ae^{kt} \) where \( k = \frac{\ln 2}{15} \). What was the initial population at time \( t = 0? \)

Find the size of the bacterial population after 4 hours. _________

Find the size of the bacterial population after 8 hours. _________

The 1906 San Francisco earthquake had a magnitude of 8.3 on the Richter scale. At the same time in South America there was an earthquake with magnitude 5 that caused only minor damage. How many times more intense was the San Francisco earthquake than the South American one? _________

The 1985 Mexico City earthquake had a magnitude of 8.1 on the Richter scale. The 1976 earthquake in Tangshan, China, was 1.26 times as intense. What was the magnitude of the Tangshan earthquake? _________

An earthquake with magnitude 5 that caused only minor damage. How many times more intense was the South American earthquake than the Tangshan earthquake? _________

If one earthquake is 20 times as intense as another, how much larger is its magnitude on the Richter scale? _________

Your answer is some constant \( A \), the formula for the population is \( \text{__________} \)

\[ \text{__________} \]

The pH reading of a sample of each substances is given. Calculate the hydrogen ion concentration of the substance.

(a) Vinegar: pH = 3.0.

Your answer is _________.

(b) Milk: pH = 6.5.

Your answer is _________.

A roasted turkey is taken from an oven when its temperature has reached 185 Fahrenheit and is placed on a table in a room where the temperature is 75 Fahrenheit.

(a) If the temperature of the turkey is 145 Fahrenheit after half an hour, what is its temperature after 45 minutes?

Your answer is _________ Fahrenheit.

(b) When will the trukey cool to 100 Fahrenheit?

Your answer is _________. hours.

The pH of a sample of each substance is given. Calculate the hydrogen ion concentration of the substance.

(a) Vinegar: pH = 3.0.

(b) Milk: pH = 6.5.

A bacteria culture initially contains 2500 bacteria and doubles every half hour. The formula for the population is \( p(t) = 2500e^{kt} \) for some constant \( k \). (You will need to find \( k \) to answer the following.)

Find the size of the baterial population after 60 minutes. _________

Find the size of the baterial population after 8 hours. _________

The rat population in a major metropolitan city is given by the formula \( n(t) = 35e^{0.04t} \) where \( t \) is measured in years since 1993 and \( n(t) \) is measured in millions.

What was the rat population in 1993? _________

What is the rat population going to be in the year 2010? _________

A certain bacteria population is known to triples every 90 minutes. Suppose that there are initially 90 bacteria.

What is the size of the population after \( t \) hours? _________

Students in a fifth-grade class were given an exam. During the next 2 years, the same students were retested several times. The average score was given by the model

\[ f(t) = 82 - 19 \log_{10}(t + 1), \quad 0 \leq t \leq 24 \]

where \( t \) is the time in months.

(a) What is the average score on the original exam?

(b) What was the average score after 6 months?

(c) What was the average score after 18 months?

At the beginning of an experiment, a scientist has 212 grams of radioactive goo. After 225 minutes, her sample has decayed to 3.3125 grams.

What is the half-life of the goo in minutes? _________

Find a formula for \( G(t) \), the amount of goo remaining at time \( t \).

\[ G(t) = \text{__________} \]

How many grams of goo will remain after 2 minutes? _________

The half-life of Palladium-100 is 4 days. After 12 days a sample of Palladium-100 has been reduced to a mass of 4 mg. What was the initial mass (in mg) of the sample? _________

What is the mass 8 weeks after the start? _________

The rule of 72 states that if an investment earns \( \text{__________} \) interest annually.

According to the rule of 72, what is the doubling time, in years, for this investment _________

Use the doubling time to find a formula for \( V(t) \), the value of your investment at time \( t \).

\[ V(t) = \text{__________} \]

According to the doubling time, how much will your investment be worth after 50 years? _________

Use the compound interest formula to find how much the investment will be worth after 50 years. _________

You may notice that your two values for the investment’s worth after 50 years are different. That is because the doubling time
you found with the rule of 72 is only an approximation. If the approximation were better, the two values would be the same.

29. (1 pt) setAlgebra31LogExpApplications/growth2.png
The doubling period of a bacterial population is 10 minutes. At time \( t = 100 \) minutes, the bacterial population was 70000. What was the initial population at time \( t = 0? \)
Find the size of the bacterial population after 3 hours.

30. (1 pt) setAlgebra31LogExpApplications/infection1.png
The town of Sickville, with a population of 21500 is exposed to the Blue Moon Virus, against which there is no immunity. The number of people infected when the virus is detected is 85. Suppose the number of infections grows logistically, with \( k = 0.72. \)
Find \( A. \)
Find the formula for the number of people infected after \( t \) days.
\[ N(t) = \]
Find the number of people infected after 19 days.

31. (1 pt) setAlgebra31LogExpApplications/interest.png
If 4000 dollars is invested in a bank account at an interest rate of 6 percent per year, find the amount in the bank after 10 years if interest is compounded annually
Find the amount in the bank after 10 years if interest is compounded quarterly.
Find the amount in the bank after 10 years if interest is compounded monthly.
Finally, find the amount in the bank after 10 years if interest is compounded continuously.

32. (1 pt) setAlgebra31LogExpApplications/terminalvelocity.png
Let \( P(t) = 40(1 - e^{-kt}) + 59 \) represent the expected score for a student who studies \( t \) hours for a test. Suppose \( k = 0.3 \) and test scores must be integers.
What is the highest score the student can expect?
If the student does not study, what score can he expect?

33. (1 pt) setAlgebra31LogExpApplications/cooling.png
You are taking a road trip in a car without A/C. The temperature in the car is 94 degrees F. You buy a cold pop at a gas station. Its initial temperature is 45 degrees F. The pop’s temperature reaches 60 degrees F after 44 minutes.
Given that
\[ \frac{T - A}{T_0 - A} = e^{-kt} \]
where \( T = \) the temperature of the pop at time \( t. \)
\( T_0 = \) the initial temperature of the pop.
\( A = \) the temperature in the car.
\( k = \) a constant that corresponds to the warming rate.
and \( t = \) the length of time that the pop has been warming up.
How long will it take the pop to reach a temperature of 83.75 degrees F?
It will take ________ minutes.

34. (1 pt) setAlgebra31LogExpApplications/investing_equity.png
8000 dollars is invested in a bank account at an interest rate of 10 percent per year, compounded continuously. Meanwhile, 39000 dollars is invested in a bank account at an interest rate of 5 percent compounded annually. To the nearest year, When will the two accounts have the same balance?
The two accounts will have the same balance after ________ years.

35. (1 pt) setAlgebra31LogExpApplications/problem7.png
The pH scale for acidity is defined by \( \text{pH} = -\log_{10}[H^+] \) where \([H^+]\) is the concentration of hydrogen ions measured in moles per liter (M). A substance has a hydrogen ion concentration of \([H^+] = 8.2 \times 10^{-7}M. \) Calculate the pH of the substance.
The pH is ________

36. (1 pt) setAlgebra31LogExpApplications/problem8.png
If 5000 dollars is invested in a bank account at an interest rate of 8 percent per year, find the amount in the bank after 5 years if interest is compounded annually.
Find the amount in the bank after 5 years if interest is compounded quarterly.
Find the amount in the bank after 5 years if interest is compounded monthly.
Finally, find the amount in the bank after 5 years if interest is compounded continuously.

37. (1 pt) setAlgebra31LogExpApplications/problem9.png
If \( p = x \) and \( q = y \), evaluate the following in terms of \( x \) and \( y \):
(a) \( \log(p^{-9}q^{-6}) = \)
(b) \( \log \sqrt{p^4q^3} = \)
(c) \( \log \frac{p^2}{q} = \)
(d) \( \log p^{-2} = \)
(e) \( \log (p^{-2})^2 = \)

38. (1 pt) setAlgebra31LogExpApplications/problem11.png
The pH scale for acidity is defined by \( \text{pH} = -\log_{10}[H^+] \) where \([H^+]\) is the concentration of hydrogen ions measured in moles per liter (M).
A solution has a pH of 3.95.
Calculate the concentration of hydrogen ions in moles per liter (M).
The concentration of hydrogen ions is ________ moles per liter.

39. (1 pt) setAlgebra31LogExpApplications/problem12.png
If 5000 dollars is invested in a bank account at an interest rate of 6 percent per year, compounded continuously. How many years will it take for your balance to reach 10000 dollars?
NOTE: Give your answer to the nearest tenth of a year.

40. (1 pt) setAlgebra31LogExpApplications/radioactive_dye.png
You go to the doctor and he gives you 13 milligrams of radioactive dye. After 12 minutes, 7.25 milligrams of dye remain in your system. To leave the doctor’s office, you must pass through a radiation detector without sounding the alarm. If the detector will sound the alarm if more than 2 milligrams of the dye are in
your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived? Give your answer to the nearest minute.

You will spend ________ minutes at the doctor’s office.
1. Find all solutions of the system
   \[ y + x^2 = 2x, \]
   \[ y + 2x = 4. \]
   The solution of the system is: (1,2), (3,4).
   If there is more than one point, type the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

2. Use the method of elimination to solve the system
   \[ x - 3y = 9, \]
   \[ -3x - y = 3. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, type the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

3. Use a calculator solve the system
   \[ x^2 + y^2 = 7, \]
   \[ x + y = 1. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

4. Solve the system
   \[ 2x - 6y = 3, \]
   \[ -3x + 9y = -6. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, type the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

5. Use the method of substitution to solve the system
   \[ x^2 + y^2 = 15, \]
   \[ x + y = 1. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
   If there is no solution, type none in the answer blank.

6. Use the substitution method to solve the system
   \[ -x + y = 3, \]
   \[ 4x - 3y = -12. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

7. Use the substitution method to solve the system
   \[ y = x^2, \]
   \[ y = 5x - 4. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, type the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

8. Use the substitution method to solve the system
   \[ x + y^2 = 0, \]
   \[ 2x + 5y^2 = 27. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, type the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

9. Use the elimination method to find all solutions of the system
   \[ 5x + 2y = 8, \]
   \[ 7x + 3y = 11. \]
   Your answer is (1,2), (3,4).
   If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
   If the system has no solutions, type none in the answer blank.

10. Use the elimination method to find all solutions of the system
    \[ x^2 - 2y = 5, \]
    \[ x^2 + 5y = 19. \]
    The two solutions of the system are: the one with \( x < 0 \) is (1,2), (3,4).
    The one with \( x > 0 \) is (1,2), (3,4).
    Your answer is (1,2), (3,4).
    If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
    If the system has no solutions, type none in the answer blank.

11. Use the elimination method to find all solutions of the system
    \[ 3x^2 - y^2 = 11, \]
    \[ x^2 + 4y^2 = 8. \]
    The four solutions of the system are: \((-a, -b), (-a, b), (a, -b), (a, b))\) with positive numbers \(a = \) and \(b = \).
    Your answer is (1,2), (3,4).
    If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
    If the system has no solutions, type none in the answer blank.

12. Use the elimination method to find all solutions of the system
    \[ x^2 - y^2 + 3 = 0, \]
    \[ 2x^2 + y^2 - 4 = 0. \]
    Your answer is (1,2), (3,4).
    If there is more than one point, write the points separated by a comma (i.e.: (1,2),(3,4)).
    If the system has no solutions, type none in the answer blank.
The four solutions of the system are: \((-a, -b), (-a, b), (a, -b),\) and \((a, b)\) with positive numbers 
\[ a = \quad \text{and} \quad b = \quad . \]

13. (1 pt) setAlgebra32EqnSystems/srw8_1_17.pg

Use the elimination method to find all solutions of the system
\[ y + x^2 = 4x, \]
\[ y + 4x = 16. \]

The solution of the system is:
\[ x = \quad \]
\[ y = \quad \]

14. (1 pt) setAlgebra32EqnSystems/srw8_1_20.pg

Find all solutions of the system
\[ y = 49 - x^2, \]
\[ y = x^2 - 49. \]

The two solutions of the system are:
the one with \(x < 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x > 0\) is 
\[ x = \quad \]
\[ y = \quad \]

15. (1 pt) setAlgebra32EqnSystems/srw8_1_21.pg

Use the substitution method to find all solutions of the system
\[ y = x - 2, \]
\[ xy = 3. \]

The solutions of the system are:
\[ x_1 = \quad y_1 = \quad \text{and} \quad x_2 = \quad y_2 = \quad \text{with} \quad x_1 < x_2. \]

16. (1 pt) setAlgebra32EqnSystems/srw8_1_25.pg

Use the elimination method to find all solutions of the system
\[ x^2 + y^2 = 7, \]
\[ x^2 - y^2 = 2. \]

The four solutions of the system are:
the one with \(x < 0, y < 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x < 0, y > 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x > 0, y < 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x > 0, y > 0\) is 
\[ x = \quad \]
\[ y = \quad \]

17. (1 pt) setAlgebra32EqnSystems/srw8_1_27.pg

Use the elimination method to find all solutions of the system
\[ x^2 + y^2 = 6, \]
\[ x^2 - y^2 = 3. \]

The four solutions of the system are:
the one with \(x < 0, y < 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x < 0, y > 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x > 0, y < 0\) is 
\[ x = \quad \]
\[ y = \quad \]
the one with \(x > 0, y > 0\) is 
\[ x = \quad \]
\[ y = \quad \]
23. Use the elimination method to find all solutions of the system
\[5x + 2y = 1, \quad 7x + 3y = 2.\]
Your answer is
\[x = \quad \]
\[y = \quad \]

24. Use the elimination method to find all solutions of the system
\[x^2 - 2y = 17, \quad x^2 + 5y = -11.\]
The two solutions of the system are:
the one with \(x < 0\) is
\[x = \quad \]
\[y = \quad \]
the one with \(x > 0\) is
\[x = \quad \]
\[y = \quad \]

25. Use the elimination method to find all solutions of the system
\[y + x^2 = 4x,\]
\[y + 4x = 16.\]
The solution of the system is:
\[x = \quad \]
\[y = \quad \]

26. Find all solutions of the system
\[y = 81 - x^2,\]
\[y = x^2 - 81.\]
The two solutions of the system are:
the one with \(x < 0\) is
\[x = \quad \]
\[y = \quad \]
the one with \(x > 0\) is
\[x = \quad \]
\[y = \quad \]

27. Use the elimination method to find all solutions of the system
\[x^2 + y^2 = 5,\]
\[x^2 - y^2 = 3.\]
The four solutions of the system are:
the one with \(x < 0, y < 0\) is
\[x = \quad \]
\[y = \quad \]
the one with \(x < 0, y > 0\) is
\[x = \quad \]
\[y = \quad \]
the one with \(x > 0, y < 0\) is
\[x = \quad \]
\[y = \quad \]
the one with \(x > 0, y > 0\) is

28. Solve the system
\[-x + y = -1,\]
\[4x - 3y = 6.\]
If the system has infinitely many solutions, express your answer in the form \(x = x\) and \(y = y\) as a function of \(x\)
Your answer is
\[x = \quad \]
\[y = \quad \]

29. Solve the system
\[x + 2y = 5,\]
\[5x - y = 3.\]
If the system has infinitely many solutions, express your answer in the form \(x = x\) and \(y = y\) as a function of \(x\)
Your answer is
\[x = \quad \]
\[y = \quad \]

30. Solve the system
\[3x + 2y = 15,\]
\[x - 2y = -3.\]
If the system has infinitely many solutions, express your answer in the form \(x = x\) and \(y = y\) as a function of \(x\)
Your answer is
\[x = \quad \]
\[y = \quad \]

31. Solve the system
\[x + 4y = 6,\]
\[3x + 12y = 18.\]
If the system has infinitely many solutions, express your answer in the form \(x = x\) and \(y = y\) as a function of \(x\)
Your answer is
\[x = \quad \]
\[y = \quad \]

32. Solve the system
\[2x - 6y = -16,\]
\[-3x + 9y = 24.\]
If the system has infinitely many solutions, express your answer in the form \(x = x\) and \(y = y\) as a function of \(x\)
Your answer is
\[x = \quad \]
\[y = \quad \]
33. Solve the system
\[6x + 4y = 2, \quad 9x + 6y = 3.\]
If the system has infinitely many solutions, express your answer in the form \(x = \ldots\) and \(y = \ldots\) as a function of \(x\).

Your answer is
\[x = \ldots, \quad y = \ldots\]

34. Solve the following system of equations. If there are no solutions, type "No Solution" for both \(x\) and \(y\). If there are infinitely many solutions, type "\(x\)" for \(x\), and an expression in terms of \(x\) for \(y\).

\[-1x + 2y = 11 \quad 2x + 2y = 2\]

\(x = \ldots, \quad y = \ldots\)

35. Solve the following system of equations. If there are no solutions, type "No Solution" for both \(x\) and \(y\). If there are infinitely many solutions, type "\(x\)" for \(x\), and an expression in terms of \(x\) for \(y\).

\[1x - 1y = -1 \quad 3x - 3y = -3\]

\(x = \ldots, \quad y = \ldots\)

36. Solve the following system of equations.
If there are no solutions, type "No Solution" for all \(x\) and \(y\) values.
If there are solutions, enter them in increasing order of the \(x\) values. Type "\(x\)" for any entry that you do not need.

\[x^2 + y^2 = 16 \quad y = 1.2x^2 - 5\]

\(x_1 = \ldots, \quad y_1 = \ldots\)

\(x_2 = \ldots, \quad y_2 = \ldots\)

\(x_3 = \ldots, \quad y_3 = \ldots\)

\(x_4 = \ldots, \quad y_4 = \ldots\)

37. Solve the following system of equations.
If there are no solutions, type "No Solution" for all \(x\) and \(y\) values.
If there is only one solution, use \(x_1\) and \(y_1\) for your answers. Type "No Solution" for the other \(x\) and \(y\) values.
If there are two solutions, use \(x_1\) and \(y_1\) for the solution with the smallest \(x\) value.

\[-7x - 5y = 8 \quad y = -8x^2 + 2x + 5\]

\(x_1 = \ldots, \quad y_1 = \ldots\)

\(x_2 = \ldots, \quad y_2 = \ldots\)

38. Charlie is trying to allocate his study time this weekend. He can spend time working with either his math tutor or his chemistry tutor to prepare for next week’s tests. His math tutor charges $30 per hour. His chemistry tutor charges $50 per hour. He has $250 to spend on tutoring, but each hour with the math tutor requires 4 aspirin and 1 hour of sleep to recover. Each hour with the chemistry tutor requires 2 aspirin and 4 hours of sleep to recover. Charlie has only 30 aspirin left, and can only afford to sleep for 16 hours this weekend. If each hour of math tutoring will improve his grade by 4 points and each hour of chemistry tutoring will improve his grade by 1 point, how many hours should he spend with each tutor in order to improve his grades the most?

Charlie should spend _____ hours with his math tutor and _____ hours with his chemistry tutor to improve his grades by a total of _____ points.

39. A company that makes thing-a-ma-bobs has a start up cost of $36191. It costs the company $2.19 to make each thing-a-ma-bob. Let \(x\) denote the number of thing-a-ma-bobs produced. Write the cost function for this company.

\[C(x) = \ldots\]

Write the revenue function for this company.

\[R(x) = \ldots\]

What is the minimum number of thing-a-ma-bobs that the company must produce and sell to make a profit?

40. Find the point of equilibrium for the following supply and demand equations where \(x\) is number of units and \(p\) is the price per unit.

Demand: \(p = 250 - 0.000010x\)

Supply: \(p = 215 + 0.000340x\)

Number of units for equilibrium = \ldots

Price per unit at equilibrium = \ldots

41. A rectangle has an area of 228 cm² and a perimeter of 62 cm. What are its dimensions?

Its length is __________

Its width is __________

42. You are offered two different sales jobs. The first company offers a straight commission of 7% of the sales. The second company offers a salary of $270 per week plus 2% of the sales. How much would you have to sell in a week in order for the straight commission offer to be at least as good?

43. A rectangle has an area of 144 cm² and a perimeter of 50 cm. What are its dimensions?

Its length is __________
Its width is ____________

The perimeter of a rectangle is 70 and its diagonal is 25. Find its length and width.
Its length is ____________
Its width is ____________

A circular piece of sheet metal has a diameter of 20 in. The edges are to be cut off to form a rectangle of area 160 in² (see the figure below). What are the dimensions of the rectangle?
Its length is ____________
Its width is ____________

Click on the graph to view an enlarged graph

Find two numbers \( a \) and \( b \) whose sum \( a + b \) is 1 and whose difference \( a - b \) is -1.
Your answer is
\( a = \) ____________
\( b = \) ____________

A man has 24 coins in his pocket, all of which are dimes and quarters. If the total value of his change is 525 cents, how many dimes and how many quarters does he have?
Your answer is
number of dimes equals ____________
number of quarters equals ____________

The admission fee at an amusement park is 1.5 dollars for children and 4 dollars for adults. On a certain day, 274 people entered the park, and the admission fees collected totaled 836 dollars. How many children and how many adults were admitted?
Your answer is
number of children equals ____________
number of adults equals ____________

A circular piece of sheet metal has a diameter of 20 in. The edges are to be cut off to form a rectangle of area 160 in² (see the figure below). What are the dimensions of the rectangle?
Its length is ____________
Its width is ____________

Find two numbers \( a \) and \( b \) whose sum \( a + b \) is 1 and whose difference \( a - b \) is -1.
Your answer is
\( a = \) ____________
\( b = \) ____________

A man flies a small airplane from Fargo to Bismarck, North Dakota — a distance of 180 miles. Because he is flying into a head wind, the trip takes him 2 hours. On the way back, the wind is still blowing at the same speed, so the return trip takes only 1 hour 12 minutes. What is his speed in still air, and how fast is the wind blowing?
Your answer is
his speed equals ____________
the wind speed equals ____________

A man invests his savings in two accounts, one paying 6 percent and the other paying 10 percent simple interest per year. He puts twice as much in the lower-yielding account because it is less risky. His annual interest is 10230 dollars. How much did he invest at each rate?
Your answer is
total in the account paying 6 percent interest is ____________
total in the account paying 10 percent interest is ____________

Country Day’s scholarship fund receives a gift of $150000. The money is invested in stocks, bonds, and CDs. CDs pay 3 % interest, bonds pay 4.4 % interest, and stocks pay 8.4 % interest. Country Day invested $10000 more in bonds than in CDs. If the annual income from the investments is $9380, how much was invested in each vehicle?
Country Day invested $______ in stocks.
Country Day invested $______ in bonds.
Country Day invested $______ in CDs.

Given the table below, find a cubic equation in standard form for \( g(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>5</th>
<th>4</th>
<th>-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>-4</td>
<td>-660</td>
<td>-343</td>
<td>3596</td>
</tr>
</tbody>
</table>

\( g(x) = \) ____________
1. (1 pt) setAlgebra33SystemsIneq/feasible_region_1.pg

Given the system of inequalities below, determine the shape of the feasible region and find the vertices of the feasible region. Give the shape as "triangle", "quadrilateral", or "unbounded". Report your vertices starting with the one which has the smallest x-value. If more than one vertex has the same, smallest x-value, start with the one that has the smallest y-value. Proceed clockwise from the first vertex. Leave any unnecessary answer spaces blank.

\[
\begin{align*}
  x + y &\leq 8 \\
  4x + y &\leq 10 \\
  x &\geq 0 \\
  y &\geq 0 
\end{align*}
\]

The shape of the feasible region is (a) __________
The first vertex is (_________).
The second vertex is (_________).
The third vertex is (_________).
The fourth vertex is (_________).

2. (1 pt) setAlgebra33SystemsIneq/feasible_region_2.pg

Given the system of inequalities below, determine the shape of the feasible region and find the vertices of the feasible region. Give the shape as "triangle", "quadrilateral", or "unbounded". Report your vertices starting with the one which has the smallest x-value. If more than one vertex has the same, smallest x-value, start with the one that has the smallest y-value. Proceed clockwise from the first vertex. Leave any unnecessary answer spaces blank.

\[
\begin{align*}
  x + y &\leq 2 \\
  3x + y &\geq 3 \\
  x + 4y &\geq 3 \\
  x &\geq 0 \\
  y &\geq 0 
\end{align*}
\]

The shape of the feasible region is (a) __________
The first vertex is (_________).
The second vertex is (_________).
The third vertex is (_________).
The fourth vertex is (_________).

3. (1 pt) setAlgebra33SystemsIneq/feasible_region_3.pg

Given the system of inequalities below, determine the shape of the feasible region and find the vertices of the feasible region. Give the shape as "triangle", "quadrilateral", "pentagon", or "unbounded". Report your vertices starting with the one which has the smallest x-value. If more than one vertex has the same, smallest x-value, start with the one that has the smallest y-value. Proceed clockwise from the first vertex. Leave any unnecessary answer spaces blank. Also give the value of the objective function \( P = -4x + 8y \) for each vertex.

\[
\begin{align*}
  x + y &\geq 17 \\
  4y - x &\geq 43 \\
  4x - y &\leq 8 \\
  x &\geq 0 \\
  y &\geq 0 
\end{align*}
\]

The shape of the feasible region is (a) __________
The first vertex is (_________), \( P = \) __________
The second vertex is (_________), \( P = \) __________
The third vertex is (_________), \( P = \) __________
The fourth vertex is (_________), \( P = \) __________
The fifth vertex is (_________), \( P = \) __________

4. (1 pt) setAlgebra33SystemsIneq/linear_programming_1.pg

Given the system of inequalities below, determine the shape of the feasible region and find the vertices of the feasible region. Report your vertices starting with the one which has the smallest x-value. If more than one vertex has the same, smallest x-value, start with the one that has the smallest y-value. Proceed clockwise from the first vertex. Leave any unnecessary answer spaces blank.

\[
\begin{align*}
  x + y &\leq 8 \\
  5x + y &\geq 11 \\
  x &\geq 0 \\
  y &\geq 0 
\end{align*}
\]

The feasible region is (a) __________
The first vertex is (_________).
The second vertex is (_________).
The third vertex is (_________).
The fourth vertex is (_________).
1. (1 pt) setAlgebra34Matrices/id_entry.pg
If
\[ A = \begin{pmatrix} -1 & 8 & -3 & -4 \\ -2 & 7 & 1 & 1 \\ 1 & 7 & 4 & -3 \end{pmatrix} \]
then \( A_{12} \) is __________.

2. (1 pt) setAlgebra34Matrices/size.pg
If
\[ A = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \]
then the size of \( A \) is _____.

3. (1 pt) setAlgebra34Matrices/defined_ops.pg
If
\[ A = \begin{pmatrix} -2 & 9 & -1 & 7 \\ 0 & 7 & 4 & -1 \\ -7 & -1 & -3 & -3 \\ 3 & 5 & -2 & -6 \end{pmatrix}, \quad B = \begin{pmatrix} -6 \\ 7 \\ -3 \end{pmatrix}, \quad C = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \]
then decide if each of the following operations is defined (answer yes or no)
- \( A + B \) ______
- \( A + C \) ______
- \( B + C \) ______
- \( AB \) ______
- \( BA \) ______
- \( AC \) ______
- \( CA \) ______
- \( BC \) ______
- \( CB \) ______

4. (1 pt) setAlgebra34Matrices/scalarmult3.pg
If
\[ A = \begin{pmatrix} -3 & 0 & 3 \\ 1 & 1 & -3 \\ 2 & -2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} -4 & 0 & -2 \\ 1 & -1 & -3 \\ -1 & 0 & -4 \end{pmatrix} \]
Then
\[ 4A + B = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix}, \quad A^T = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix} \]

5. (1 pt) setAlgebra34Matrices/scalarmult3a.pg
If
\[ A = \begin{pmatrix} -3 & 0 & -1 \\ -2 & -3 & 3 \\ -2 & 2 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 3 \\ 3 & -4 & -2 \\ 2 & 0 & 1 \end{pmatrix} \]
Then
\[ 4A + B = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix} \]

6. (1 pt) setAlgebra34Matrices/matrixmult2.pg
If
\[ A = \begin{pmatrix} 4 + 2i & 2 - 4i \\ 1 & 3 - 4i \end{pmatrix}, \quad B = \begin{pmatrix} -3 + 2i & -4i \\ -3 + 3i & 3 + i \end{pmatrix} \]
Then
\[ AB = \begin{pmatrix} \text{_____} & \text{_____} \\ \text{_____} & \text{_____} \end{pmatrix}, \quad BA = \begin{pmatrix} \text{_____} & \text{_____} \\ \text{_____} & \text{_____} \end{pmatrix} \]

7. (1 pt) setAlgebra34Matrices/matrixmult3.pg
If
\[ A = \begin{pmatrix} -9 & -9 & -2 \\ 0 & 3 \\ -3 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & -1 & 1 \\ 1 & -1 & -3 \\ -1 & -4 & -3 \end{pmatrix} \]
Then
\[ AB = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix}, \quad BA = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix} \]

8. (1 pt) setAlgebra34Matrices/matrixmult3a.pg
If
\[ A = \begin{pmatrix} -3 & 3 & 1 \\ 2 & -4 & -2 \\ -1 & -3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -4 & 3 \\ 4 & 3 & 1 \\ -1 & -1 & 2 \end{pmatrix} \]
Then
\[ AB = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix}, \quad BA = \begin{pmatrix} \text{_____} & \text{_____} & \text{_____} \\ \text{_____} & \text{_____} & \text{_____} \end{pmatrix} \]

9. (1 pt) setAlgebra34Matrices/product_size.pg
If
\[ A = \begin{pmatrix} -7 & 7 & -4 & 3 & -1 \\ 5 & -2 & 4 & 1 & 4 \\ -5 & -6 & 9 & -9 & -5 \\ 6 & -8 & -4 & 8 & 5 \\ -8 & -9 & -4 & 5 & 9 \\ -7 & -4 & 5 & 8 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 9 & -7 & -2 \\ 2 & -4 & -9 \\ -6 & -6 & -1 \\ -3 & 0 & 7 \\ 5 & 1 & 8 \end{pmatrix} \]
then the size of \( AB \) is _____
and the size of \( BA \) is _____.
NOTE: If either of the products is not defined, type UNDEFINED for you answer.
Given the matrices
\[ B = \begin{bmatrix} 1 & -4 & -2 \\ 3 & 1 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 4 & 4 \\ 1 & -4 & 3 \end{bmatrix}, \]
find \( B + C \). Write \( B + C \) as
\[ \begin{bmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \]
input your answer below:
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{13} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]
\[ a_{23} = \ldots \]

Given the matrices
\[ B = \begin{bmatrix} 5 & 5 & 3 \\ 3 & 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 4 \\ 1 & 4 & 3 \end{bmatrix}; \]
find \( C - B \). Write \( C - B \) as
\[ \begin{bmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \]
input your answer below:
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{13} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]
\[ a_{23} = \ldots \]

Given the matrices
\[ B = \begin{bmatrix} -3 & 1 & 3 \\ 5 & 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -4 & -1 & -3 \\ 4 & 1 & -1 \end{bmatrix}; \]
find \( 3B + 2C \). Write \( 3B + 2C \) as
\[ \begin{bmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \]
input your answer below:
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{13} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]
\[ a_{23} = \ldots \]

Given the matrices
\[ B = \begin{bmatrix} -5 & 3 & 2 \\ 3 & -4 & -4 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 5 & -3 \\ -2 & 5 & -2 \end{bmatrix}; \]
can the operation \( BC \) be performed? Your answer is (input Yes or No)
(a) does the inverse of the matrix exist? Your answer is (input Yes or No)
(b) if your answer is yes, write it as
\[ \begin{bmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \]
find
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]

Given the matrices
\[ B = \begin{bmatrix} -3 & 3 & 2 \\ 0 & 1 & 2 \end{bmatrix}, \quad F = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \]
can the operation \( BF \) be performed? Your answer is (input Yes or No)
If your answer is Yes, calculate \( BF \). Write \( BF \) as
\[ \begin{bmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \]
input your answer below:
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{13} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]
\[ a_{23} = \ldots \]

Given the matrix
\[ A = \begin{bmatrix} -2 & -1 \\ 0 & -3 \end{bmatrix}, \]
find \( A^3 \). Write \( A^3 \) as
\[ \begin{bmatrix} \ldots & \ldots \\ \ldots & \ldots \end{bmatrix} \]
input your answer below:
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]

Given the matrix
\[ A = \begin{bmatrix} 3 & 3 \\ 0 & 4 \end{bmatrix} \]
find \( A^3 \).
\[ A^3 = \begin{bmatrix} \ldots & \ldots \\ \ldots & \ldots \end{bmatrix} \]

Given the matrix
\[ \begin{bmatrix} 2 & 7 \\ 3 & 11 \end{bmatrix}, \]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No)
(b) if your answer is yes, write it as
\[ \begin{bmatrix} \ldots & \ldots \\ \ldots & \ldots \end{bmatrix} \]
find
\[ a_{11} = \ldots \]
\[ a_{12} = \ldots \]
\[ a_{21} = \ldots \]
\[ a_{22} = \ldots \]
18. (1 pt) setAlgebra34Matrices/sw7_5_5.pg
Given the matrix
\[
\begin{bmatrix}
2 & 9 \\
3 & 13
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ____
(b) if your answer is yes, write it as
\[
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix},
\]

\begin{align*}
da_{11} &= \\
da_{12} &= \\
da_{21} &= \\
da_{22} &=
\end{align*}

19. (1 pt) setAlgebra34Matrices/inverse2x2.pg
Given the matrix
\[
\begin{bmatrix}
4 & 8 \\
7 & -3
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ____
(b) if your answer is yes, write the inverse here:
\[
\begin{bmatrix}
& \\
&
\end{bmatrix},
\]

20. (1 pt) setAlgebra34Matrices/inverse2x2a.pg
Given the matrix
\[
\begin{bmatrix}
2 & 1 \\
5 & 2
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ____
(b) if your answer is yes, write the inverse here:
\[
\begin{bmatrix}
& \\
&
\end{bmatrix},
\]

21. (1 pt) setAlgebra34Matrices/inverse3x3.pg
Given the matrix
\[
\begin{bmatrix}
3 & -4 & 1 \\
-1 & 1 & -1 \\
1 & -1 & 0
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ____

22. (1 pt) setAlgebra34Matrices/inverse3x3a.pg
Given the matrix
\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix},
\]
find
\begin{align*}
a_{11} &= \\
a_{12} &= \\
a_{13} &= \\
a_{21} &= \\
a_{22} &= \\
a_{23} &= \\
a_{31} &= \\
a_{32} &= \\
a_{33} &=
\end{align*}

23. (1 pt) setAlgebra34Matrices/inverse3x3b.pg
Given the matrix
\[
\begin{bmatrix}
1 & 2 & 3 \\
-7 & 4 & 1 \\
-65 & 32 & 3
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ____
(b) if your answer is Yes, write the inverse here:
\[
\begin{bmatrix}
& \\
&
\end{bmatrix},
\]

\begin{align*}
& \\
&
\end{align*}
24. (1 pt) Given the matrix
\[
\begin{bmatrix}
-2 & 7 & 1 \\
1 & -1 & -1 \\
1 & 1 & 0
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ______
(b) if your answer is Yes, write the inverse as
\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix},
\]

find
\[
a_{11} = \\
a_{12} = \\
a_{13} = \\
a_{21} = \\
a_{22} = \\
a_{23} = \\
a_{31} = \\
a_{32} = \\
a_{33} = 
\]

25. (1 pt) Given the matrix
\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & -1 \\
1 & -1 & -10
\end{bmatrix},
\]
(a) does the inverse of the matrix exist? Your answer is (input Yes or No): ______
(b) if your answer is Yes, write the inverse as
\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix},
\]

find
\[
a_{11} = \\
a_{12} = \\
a_{13} = \\
a_{21} = \\
a_{22} = \\
a_{23} = \\
a_{31} = \\
a_{32} = \\
a_{33} = 
\]

26. (1 pt) Given the matrix
\[
\begin{bmatrix}
-1 & -2 \\
1 & -2
\end{bmatrix}
\]
(a) find its determinant; Your answer is : ______
(b) does the matrix have an inverse? Your answer is (input Yes or No): ______

27. (1 pt) Given the matrix
\[
A = \begin{bmatrix}
5 & 2 \\
-1 & -1
\end{bmatrix}
\]
find its determinant; The determinant of A is ____________

28. (1 pt) Given the matrix
\[
A = \begin{bmatrix}
-3 + 3i & 3 - i \\
2i & 3 + i
\end{bmatrix}
\]
find |A| = ____________

29. (1 pt) Given the matrix
\[
\begin{bmatrix}
0 & -1 & 4 \\
3 & 0 & -2 \\
0 & 4 & -3
\end{bmatrix}
\]
(a) find its determinant; Your answer is : ______
(b) does the matrix have an inverse? Your answer is (input Yes or No): ______

30. (1 pt) Given the matrix
\[
\begin{bmatrix}
0 & 0 & 4 \\
0 & -4 & 0 \\
2 & 0 & -3
\end{bmatrix}
\]
(a) find its determinant; Your answer is : ______
(b) does the matrix have an inverse? Your answer is (input Yes or No): ______

31. (1 pt) Given the matrix
\[
\begin{bmatrix}
-2 & 2 & 2 \\
2 & 0 & 5 \\
0 & -1 & -1
\end{bmatrix}
\]
(a) find its determinant; Your answer is : ______
(b) does the matrix have an inverse? Your answer is (input Yes or No): ______

32. (1 pt) Given the matrix
\[
\begin{bmatrix}
-4 & 0 & 3 \\
0 & 0 & -5 \\
5 & 0 & -2
\end{bmatrix}
\]
(a) find its determinant; Your answer is : ______
(b) does the matrix have an inverse?  
Your answer is (input Yes or No): ____________

33. (1 pt) setAlgebra34Matrices/determinant_3x3.png
Given the matrix
\[
\begin{bmatrix}
4 & -2 & -3 \\
-3 & 0 & 2 \\
0 & 5 & 4
\end{bmatrix}
\]
find its determinant.
The determinant is: ____________

34. (1 pt) setAlgebra34Matrices/determinant_3x3a.png
Given the matrix
\[
A = \begin{bmatrix}
a & 2 & 4 \\
a & -3 & 7 \\
5 & 3 & a
\end{bmatrix}
\]
find all values of \(a\) that make the \(|A| = 0\).  
Give your answers in increasing order.  
\(a\) can be ____, ____, or ____.
Note: Leave any unneeded answer spaces blank.

35. (1 pt) setAlgebra34Matrices/determinant_3x3b.png
Given the matrix
\[
\begin{bmatrix}
5 & 3 & 1 \\
2 & -1 & 5 \\
3 & 1 & 3
\end{bmatrix}
\]
find its determinant. Do not use a calculator.
The determinant is: ____________
1. (1 pt) setAlgebra35SystemMatrices/sw7_3.7.pg
Given the matrix
\[ A = \begin{bmatrix} 1 & -2 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}, \]
(a) determine whether the matrix \( A \) is in echelon form; 
Your answer is (input Yes or No) __________
(b) determine whether the matrix \( A \) is in reduced echelon form; 
Your answer is (input Yes or No) __________
You have only one chance to input your answer

2. (1 pt) setAlgebra35SystemMatrices/sw7_3.9.pg
Given the matrix
\[ A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]
(a) determine whether the matrix \( A \) is in echelon form; 
Your answer is (input Yes or No) __________
(b) determine whether the matrix \( A \) is in reduced echelon form; 
Your answer is (input Yes or No) __________
You have only one chance to input your answer

3. (1 pt) setAlgebra35SystemMatrices/augmentedmatrix.pg
Given the matrix
\[ A = \begin{bmatrix} 1 & 0 & 4 & 0 & 0 \\ 0 & 1 & 5 & 0 & -3 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]
Is the matrix in echelon form? (input Yes or No) __________
Is the matrix in reduced echelon form? (input Yes or No) __________
If this matrix were the augmented matrix for a system of linear equations, would the system be inconsistent, dependent, or independent? __________
You have only one chance to input your answer

4. (1 pt) setAlgebra35SystemMatrices,ID_row_ops.pg
Identify the elementary row operation used below. Write your answer with one space between every character.
\[
\begin{pmatrix}
-5 & 3 & 2 & 1 & 7 \\
0 & 7 & -9 & 4 & -6 \\
-1 & -4 & -8 & 2 & 9 \\
5 & 3 & 8 & -3 & -6 \\
1 & 7 & -8 & -5 & -4
\end{pmatrix}
\rightarrow
\begin{pmatrix}
-5 & 3 & 2 & 1 & 7 \\
0 & 7 & -9 & 4 & -6 \\
-1 & -4 & -8 & 2 & 9 \\
5 & 3 & 8 & -3 & -6 \\
1 & 7 & -8 & -5 & -4
\end{pmatrix}
\]
The system is __________

5. (1 pt) setAlgebra35SystemMatrices/ID_row_ops_2.pg
Identify the elementary row operation used below. Write your answer with one space between every character.
\[
\begin{pmatrix}
-5 & -4 & 7 & 0 \\
6 & 9 & -4 & -3 \\
-6 & -1 & 9 & -6 \\
5 & 7 & 6 & -6
\end{pmatrix}
\rightarrow
\begin{pmatrix}
5 & 7 & 6 & -6 \\
6 & 9 & -4 & -3 \\
-6 & -1 & 9 & -6 \\
-5 & -4 & 7 & 0
\end{pmatrix}
\]
The row operation is __________

6. (1 pt) setAlgebra35SystemMatrices/matrix_form.pg
Write the system of equations
\[
\begin{align*}
2x - 2y - 3z &= -2 \\
1x - 4y - 5z &= 1 \\
-1x + 4y - 5z &= -5
\end{align*}
\]
as a matrix equation, that is, rewrite it in the form
\[
\begin{pmatrix}
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \end{pmatrix} = \begin{pmatrix}
\_ \\
\_ \\
\_ \\
\end{pmatrix}
\]

7. (1 pt) setAlgebra35SystemMatrices/3x3_test.pg
Given the augmented matrix below, determine if the associated system of equations is independent, dependent, or inconsistent.
\[
\begin{pmatrix}
-9 & 5 & 7 \\
27 & 16 & 53 \\
-8 & 1 & -2
\end{pmatrix}
\begin{pmatrix}
9 \\
13 \\
-8
\end{pmatrix}
\]
The system is __________

8. (1 pt) setAlgebra35SystemMatrices/classify_3x3.pg
Given the augmented matrix below, determine if the associated system of equations is independent, dependent, or inconsistent.
If the system is independent, give the solution. If the system is dependent, label each variable as "free" or "fixed". If the system is inconsistent, label each variable "No Solution".
\[
\begin{pmatrix}
0 & 14 & -6 \\
-1 & -12 & -4 \\
-11 & -14 & 9
\end{pmatrix}
\begin{pmatrix}
-4 \\
0 \\
-3
\end{pmatrix}
\]
The system is ____________
The solution to the system is: ____________

9. (1 pt) setAlgebra35SystemMatrices/classify_4x4.pg
Given the augmented matrix below, determine if the associated system of equations is independent, dependent, or inconsistent.
The system is 

**10.** Solve the system associated with the augmented matrix below. If the system is inconsistent, type "No Solution" in each blank. If the system is dependent, use the variable "t" as your free variable.

\[
\begin{pmatrix}
5 & 0 & -1 & -7 \\
-85 & 18 & -10 & -7 \\
-5 & 2 & -2 & -7 \\
-3 & 4 & 2 & 7
\end{pmatrix}
\begin{pmatrix}
2 \\
47 \\
-7 \\
0
\end{pmatrix}
\]

The solution is ( )

**11.** Given the matrix below, solve the associated system of equations. For your variables, use \(x_1, x_2, x_3, x_4, x_5, \) and \(x_6.\)

\[
\begin{pmatrix}
1 & -1 & -4 & 5 & -7 & 1 \\
0 & 0 & 0 & 1 & -6 & -9 \\
0 & 0 & 0 & 0 & 1 & -4
\end{pmatrix}
\begin{pmatrix}
8 \\
2 \\
6
\end{pmatrix}
\]

The solution is ( )

**12.** Given the matrix below, solve the associated system of equations. For your variables, use \(x_1, x_2, x_3, x_4, x_5, x_6, x_7, \) and \(x_8.\)

\[
\begin{pmatrix}
1 & -8 & -4 & 9 & -5 & 4 & 7 & -7 \\
0 & 0 & 0 & 0 & 1 & -3 & -3 & -3 \\
0 & 0 & 0 & 0 & 0 & 1 & -6 & -7
\end{pmatrix}
\begin{pmatrix}
1 \\
4 \\
-5
\end{pmatrix}
\]

The solution is ( )

**13.** Solve the system of equations

\[
\begin{align*}
5x + 3y &= -24 \\
3x + 2y &= -15
\end{align*}
\]

by converting to a matrix equation and using the inverse of the coefficient matrix, as in Example 4 of the text.

\(x =\) 
\(y =\)

**14.** Solve the system of equations

\[
\begin{align*}
2x - 4y + z &= 5 \\
-x + y - z &= -3 \\
x - 2y &= 1
\end{align*}
\]

by converting to a matrix equation and using the inverse of the coefficient matrix, as in Example 4 of the text.

\(x =\) 
\(y =\)

**15.** Solve the system of equations

\[
\begin{align*}
3x - 1y &= 23 \\
-4x + 5y &= 28
\end{align*}
\]

by converting to a matrix equation and using the inverse of the coefficient matrix.

\(x =\) 
\(y =\)

**16.** Solve the system of equations

\[
\begin{align*}
2x - 4y + z &= -6 \\
-x + y - z &= 1 \\
x - 2y &= -3
\end{align*}
\]

by converting to a matrix equation and using the inverse of the coefficient matrix, as in Example 4 of the text.

\(x =\) 
\(y =\) 
\(z =\)

**17.** Write the system of equations

\[
\begin{align*}
4x + 3y - 2z &= 3 \\
1x - 2y - 1z &= 1 \\
-3x - 3y + 3z &= 1
\end{align*}
\]

as a matrix equation, that is, rewrite it in the form

\[
\begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= 
\begin{pmatrix}
b_1 \\
b_2 \\
b_3
\end{pmatrix}
\]

Input your answer below:

\(a_{11} =\) 
\(a_{12} =\) 
\(a_{13} =\) 
\(a_{21} =\) 
\(a_{22} =\) 
\(a_{23} =\) 
\(a_{31} =\) 
\(a_{32} =\) 
\(a_{33} =\) 
\(b_1 =\) 
\(b_2 =\) 
\(b_3 =\)
18. (1 pt) setAlgebra35SystemMatrices/sw7_3_11.pg
The system of equations
\[
\begin{align*}
x - 2y + z &= 7, \\
y + 2z &= -3, \\
x + y + 3z &= -2 \\
\end{align*}
\]
has a unique solution. Find the solution using Gaussian elimination method or Gauss-Jordan elimination method.
\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

19. (1 pt) setAlgebra35SystemMatrices/sw7_3_15.pg
The system of equations
\[
\begin{align*}
x + 2y - z &= 6, \\
x + z &= 2, \\
2x - y - z &= -2. \\
\end{align*}
\]
has a unique solution. Find the solution using Gaussian elimination method or Gauss-Jordan elimination method.
\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

20. (1 pt) setAlgebra35SystemMatrices/sw7_3_17.pg
The system of equations
\[
\begin{align*}
x_1 + 2x_2 - x_3 &= -2, \\
2x_1 - x_3 &= 4, \\
x_1 + 5x_2 + 2x_3 &= -4 \\
\end{align*}
\]
has a unique solution. Find the solution using Gaussian elimination method or Gauss-Jordan elimination method.
\[
\begin{align*}
x_1 &= \\
x_2 &= \\
x_3 &= \\
\end{align*}
\]

21. (1 pt) setAlgebra35SystemMatrices/sw7_3_19.pg
The system of equations
\[
\begin{align*}
2x - 3y - z &= 4, \\
-x + 2y - 5z &= -13, \\
5x - y - z &= 13 \\
\end{align*}
\]
has a unique solution. Find the solution using Gaussian elimination method or Gauss-Jordan elimination method.
\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

22. (1 pt) setAlgebra35SystemMatrices/sw7_3_21.pg
Given the system of equations
\[
\begin{align*}
x + y + z &= -6, \\
y - 3z &= 7, \\
2x + y + 5z &= -18, \\
\end{align*}
\]
(a) determine whether the system is inconsistent or dependent; Your answer is (input inconsistent or dependent) __________
(b) if your answer is dependent, find the complete solution. Write \(x\) and \(y\) as functions of \(z\).
\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

23. (1 pt) setAlgebra35SystemMatrices/sw7_3_23.pg
Given the system of equations
\[
\begin{align*}
2x - 3y - 9z &= 21, \\
x + 3z &= -9, \\
-3x + y - 4z &= 14, \\
\end{align*}
\]
(a) determine whether the system is inconsistent or dependent; Your answer is (input inconsistent or dependent) __________
(b) if your answer is dependent in (a), find the complete solution. Write \(x\) and \(y\) as functions of \(z\).
\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

24. (1 pt) setAlgebra35SystemMatrices/sw7_6_27.pg
Use Cramer’s rule to solve the system
\[
\begin{align*}
2x - y &= 1 \\
x + 2y &= 3 \\
\end{align*}
\]
\[
\begin{align*}
x &= \\
y &= \\
\end{align*}
\]

25. (1 pt) setAlgebra35SystemMatrices/sw7_6_29.pg
Use Cramer’s rule to solve the system
\[
\begin{align*}
x - 6y &= -4 \\
3x + 2y &= 8 \\
\end{align*}
\]
\[
\begin{align*}
x &= \\
y &= \\
\end{align*}
\]

26. (1 pt) setAlgebra35SystemMatrices/sw7_6_33.pg
Use Cramer’s rule to solve the system
\[
\begin{align*}
x - y + 2z &= -4 \\
3x + z &= -5 \\
-x + 2y &= -1 \\
\end{align*}
\]
\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

27. (1 pt) setAlgebra35SystemMatrices/sw7_6_35.pg
Use Cramer’s rule to solve the system
\[
\begin{align*}
2x_1 + 3x_2 - 5x_3 &= 14 \\
x_1 + x_2 - x_3 &= 4 \\
2x_2 + x_3 &= -2 \\
\end{align*}
\]
\[
\begin{align*}
x_1 &= \\
x_2 &= \\
x_3 &= \\
\end{align*}
\]

28. (1 pt) setAlgebra35SystemMatrices/cramer.pg
Use Cramer’s rule to find the value of \(z\) in the solution of the following system:
\[
\begin{align*}
-2x - 2y - 2z &= -8, \\
1x + 2y - 4z &= 61, \\
4x + 4y - 1z &= 56, \\
\end{align*}
\]
\[
\begin{align*}
z &= \\
\end{align*}
\]
1. (1 pt) setAlgebra36SeqSeries/evalfact.pg
Calculate each of the following. Your answer must be a number. No arithmetic operations are allowed in your answer. Please give 7 places after your decimal point if you use scientific notation.

\[
\begin{align*}
910! &= 910 \\
10!900! &= 10! \\
350! &= 350! \\
320!20! &= 320! \\
581! &= 581! \\
577! &= 577! \\
576! &= 576!
\end{align*}
\]

2. (1 pt) setAlgebra36SeqSeries/factorial2.pg
Evaluate

\[
\frac{5!}{1!} = 5!
\]

3. (1 pt) setAlgebra36SeqSeries/factorial3.pg
Simplify the expression

\[
\frac{(2n+4)!}{(2n-2)!} = \frac{(2n+4)!}{(2n-2)!}
\]

4. (1 pt) setAlgebra36SeqSeries/factorial1.pg
Find the first five (5) terms of

\[a_n = \frac{3n!}{n+1}\]

starting with \(n = 1\). Write your answer as a comma separated (i.e.: 1,2).

5. (1 pt) setAlgebra36SeqSeries/srw10_1_1.pg
For the sequence \(a_n = n + 1\),
its first term is ___;
is its second term is ___;
is its third term is ___;
is its fourth term is ___;
is its 100th term is ___

6. (1 pt) setAlgebra36SeqSeries/srw10_1_3.pg
For the sequence \(a_n = \frac{15}{n+1}\),
its first term is ___;
is its second term is ___;
is its third term is ___;
is its fourth term is ___;
is its 100th term is ___

7. (1 pt) setAlgebra36SeqSeries/srw10_1_5.pg
For the sequence \(a_n = \frac{(-1)^n10}{n!}\),
is its first term is ___;
is its second term is ___;
its third term is ___;
is its fourth term is ___;
is its 100th term is ___

8. (1 pt) setAlgebra36SeqSeries/srw10_1_7.pg
For the sequence \(a_n = 3 + (-1)^n\),
is its first term is ___;
is its second term is ___;
is its third term is ___;
is its fourth term is ___;
is its 100th term is ___

9. (1 pt) setAlgebra36SeqSeries/srw10_1_11.pg
For the sequence \(a_n = 2(a_{n-1} - 2)\) and \(a_1 = 3\),
is its first term is ___;
is its second term is ___;
is its third term is ___;
is its fourth term is ___;
is its 100th term is ___

10. (1 pt) setAlgebra36SeqSeries/srw10_1_15.pg
For the sequence \(a_n = a_{n-1} + a_{n-2}\) and \(a_1 = 2, a_2 = 3\),
is its first term is ___;
is its second term is ___;
is its third term is ___;
is its fourth term is ___;
is its fifth term is ___

11. (1 pt) setAlgebra36SeqSeries/srw10_1_17.pg
Use a graphing calculator to find the first 10 terms of the sequence \(a_n = 5n + 4\),
is its 9th term is ___;
is its 10th term is ___

12. (1 pt) setAlgebra36SeqSeries/srw10_1_19.pg
Use a graphing calculator to find the first 10 terms of the sequence \(a_n = \frac{23}{n}\).
is its 9th term is ___;
is its 10th term is ___

13. (1 pt) setAlgebra36SeqSeries/srw10_1_21.pg
Use a graphing calculator to find the first 10 terms of the sequence \(a_n = \frac{1}{a_{n-1}}\) and \(a_1 = 1\).
is its 9th term is ___;
is its 10th term is ___

14. (1 pt) setAlgebra36SeqSeries/srw10_1_23.pg
Use a graphing calculator to find the first 10 terms of the sequence \(a_n = \frac{5n}{n+12}\)
is its 9th term is ___;
is its 10th term is ___

15. (1 pt) setAlgebra36SeqSeries/ns8_1_5.pg
For each sequence, find a formula for the general term, \(a_n\). For example, answer \(n^2\) if given the sequence:
\[
\{1, 4, 9, 16, 25, 36, \ldots\}
\]
\[
\{1^2, 2^2, 3^2, 4^2, \ldots\}
\]
\[
\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}
\]
For each sequence, find a formula for the general term, \(a_n\). For example, answer \(n^2\) if given the sequence:

\[
\{1, 4, 9, 16, 25, 36, \ldots\}
\]

1. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
2. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{16}, \ldots\)

For each sequence, find a formula for the general term, \(a_n\). For example, answer \(n^2\) if given the sequence:

\[
\{1, 4, 9, 16, 25, 36, \ldots\}
\]

1. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
2. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{16}, \ldots\)

For the sequence 1, 4, 7, 10, \ldots, its \(n\)th term is

For the sequence \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\), its fifth partial sum \(S_5 = \ldots\)

For each sequence, find a closed formula for the general term, \(a_n\),

1. \(-2, -8, -18, -32, -50, \ldots\), \(a_n = \ldots\)
2. 37, 370, 3700, 37000, 370000, \(a_n = \ldots\)
3. \(-1, 2, 7, 14, 23, \ldots\), \(a_n = \ldots\)

For each sequence, find a closed formula for the general term, \(a_n\),

1. 2, 4, 8, 16, 32, \ldots, \(a_n = \ldots\)
2. \(-21, -26, -31, -36, -41, \ldots\), \(a_n = \ldots\)
3. 1, 4, 9, 16, 25, \ldots, \(a_n = \ldots\)

List the first four terms of each sequence.

\(a_n = 3n + 5: \ldots\)
\(b_n = (3)^n: \ldots\)
\(c_n = 1, c_n = 1 + 2: \ldots\)

For the sequence \(a_n = 4 + 6 \times (n - 1),\)
its first term is \(\ldots\)
its second term is \(\ldots\)
its third term is \(\ldots\)
its fourth term is \(\ldots\)
its fifth term is \(\ldots\)
its common difference \(d = \ldots\)

For the sequence \(a_n = 4 - 5 \times (n - 1),\)
its first term is \(\ldots\)
its second term is \(\ldots\)
its third term is \(\ldots\)
its fourth term is \(\ldots\)
its fifth term is \(\ldots\)

Is the sequence 5, 8, 11, 14, \ldots, arithmetic?
Your answer is (input yes or no) \(\ldots\)
if your answer is yes, its first term is \(\ldots\)
its common difference is \(\ldots\)

Is the sequence 3, \(\frac{1}{2}, 0, -\frac{1}{2}, \ldots\), arithmetic?
Your answer is (input yes or no) \(\ldots\)
if your answer is yes, its first term is \(\ldots\)
its common difference is \(\ldots\)

Is the sequence \(a_n = 3 + 6n\) arithmetic?
Your answer is (input yes or no) \(\ldots\)
if your answer is yes, its first term is \(\ldots\)
its common difference is \(\ldots\)

Is the sequence \(a_n = 6n - 7\) arithmetic?
Your answer is (input yes or no) \(\ldots\)
if your answer is yes, its first term is \(\ldots\)
its common difference is \(\ldots\)

Write the arithmetic sequence 8, 16, 24, 32, \ldots in the standard form:

\(a_n = \ldots\)

Write the arithmetic sequence \(-18, -10, -2, 6, \ldots\) in the standard form:

\(a_n = \ldots\)

Write the arithmetic sequence \(-14, -21, -28, -35, \ldots\) in the standard form:

\(a_n = \ldots\)

Write the arithmetic sequence 22, 17, 12, 7, \ldots in the standard form:
36. (1 pt) setAlgebra36SeqSeries/sw10_2_35.pg
If the 100th term of an arithmetic sequence is 492, and its common difference is 6, find:

- its first term \(a_1 = \) _______
- its second term \(a_2 = \) _______
- its third term \(a_3 = \) _______

37. (1 pt) setAlgebra36SeqSeries/sw10_2_37.pg
Which term of the arithmetic sequence 1, 5, 9, 13, … is 125? It is the ______th term.

38. (1 pt) setAlgebra36SeqSeries/sw10_2_39.pg
Find the partial sum \(S_{10}\) for the arithmetic sequence with \(a = 7, d = 2\).

\[S_{10} = \] _______

39. (1 pt) setAlgebra36SeqSeries/sw10_2_45.pg
The partial sum \(1 + 5 + 9 + 13 + \cdots + 77\) equals _______

40. (1 pt) setAlgebra36SeqSeries/ur_seq_4_2.pg
Find the 11th term of the arithmetic sequence

\[-5, -10, -15, …\]

Answer: _______

41. (1 pt) setAlgebra36SeqSeries/ur_seq_4_3.pg
Find the sum

\[5 + 12 + 19 + \cdots + (-2 + 7n)\]

Answer: _______

42. (1 pt) setAlgebra36SeqSeries/ur_seq_4_4.pg
Find the sum

\[3 + 1 - 1 + \cdots - 3\]

Answer: _______

43. (1 pt) setAlgebra36SeqSeries/ur_seq_4_5.pg
Find the common difference and write out the first four terms of the arithmetic sequence \(\left\{ \frac{1}{6}n - \frac{4}{5} \right\}\)

Common difference is _______

\(a_1 = \) _______

\(a_2 = \) _______

\(a_3 = \) _______

\(a_4 = \) _______

44. (1 pt) setAlgebra36SeqSeries/ur_seq_4_6.pg
Find the nth term of the arithmetic sequence whose initial term is 1 and common difference is 6.

\(a_n = \) _______ (Your answer must be a function of \(n\)).

45. (1 pt) setAlgebra36SeqSeries/ur_seq_4_7.pg
Find the first term and the common difference of the arithmetic sequence whose 11th term is \(-58\) and 15th term is \(-74\).

First term is _______

Common difference is _______

46. (1 pt) setAlgebra36SeqSeries/ur_seq_4_8.pg
Find \(x\) such that \(-8x - 1, 4x - 4,\) and \(-6x + 81\) are consecutive terms of an arithmetic sequence.

\(x = \) _______

47. (1 pt) setAlgebra36SeqSeries/sequence1.pg
Find a closed formula for \(a_n\) if

\[\sum_{k=1}^{n} a_k = 4n^2 + 6n\]

\(a_n = \) _______

48. (1 pt) setAlgebra36SeqSeries/sequence5.pg
Find the sum of the first 53 terms = _______

49. (1 pt) setAlgebra36SeqSeries/type1.pg
All sequences for this problem are arithmetic. Give all answers to the nearest thousandth.

If \(a_1 = 43\) and \(d = -4\), then \(a_{11} = \) _______.

If \(b_{16} = -80\) and \(b_{25} = -75\), then \(b_1 = \) _______.

If \(c_{15} = 7\) and \(c_{34} = -47\), then \(S_{10} = \) _______.

50. (1 pt) setAlgebra36SeqSeries/p12.pg
Find the 18th term of the geometric sequence with \(a_5 = -64/81\) and \(a_8 = -512/2187\)

\(a_{18} = \) _______

51. (1 pt) setAlgebra36SeqSeries/ur_seq_5_1.pg
Find the common ratio and write out the first four terms of the geometric sequence \(\left\{ \frac{4^n - 2}{7} \right\}\)

Common ratio is _______

\(a_1 = \) _______

\(a_2 = \) _______

\(a_3 = \) _______

\(a_4 = \) _______

52. (1 pt) setAlgebra36SeqSeries/ur_seq_5_2.pg
Find the 5th term of the geometric sequence

\(-2, -5, -12.5, …\)

Answer: _______

53. (1 pt) setAlgebra36SeqSeries/ur_seq_5_3.pg
Find the nth term of the geometric sequence whose initial term is 7 and common ratio is 6.

\(\) _______ (Your answer must be a function of \(n\)).

54. (1 pt) setAlgebra36SeqSeries/geometric1.pg
Find all values of \(x\) such that \(x - 3, x + 3,\) and \(9x - 3\) form a geometric sequence. Give your answers in increasing order. \(x\) can equal _______ or _______.

55. (1 pt) setAlgebra36SeqSeries/sequence6.pg
Find an explicit formula for \(a_n\) when

\(a_n = \) _______

\(a_{13} = \) _______

Note: Your answer to part one should be a function in terms of \(n\). Your answer to part two should be a decimal, with at least 5 significant figures.
56. (1 pt) setAlgebra36SeqSeries/sequence6A.pg
Given the geometric sequence: 5, 15, 45, \ldots
Find an explicit formula for \(a_n\).
\[ a_n = \quad \text{Find } a_{13} = \quad \]

57. (1 pt) setAlgebra36SeqSeries/sequence7A.pg
Given the geometric sequence: 36, 12, 4, \ldots
Find an explicit formula for \(a_n\).
\[ a_n = \quad \text{Find } a_9 = \quad \text{Find } S_5 = \quad \text{Find } S = \quad \]
Note: Your answer to part one should be a function in terms of \(n\). Your other answers should be decimals, accurate to at least 5 places.

58. (1 pt) setAlgebra36SeqSeries/sequence7A.pg
Given the geometric sequence: 18, \(\frac{18}{2}\), \(\frac{18}{2^2}\), \ldots
Find an explicit formula for \(a_n\).
\[ a_n = \quad \text{Find } a_7 = \quad \text{Find } S_7 = \quad \text{Find } S = \quad \]

59. (1 pt) setAlgebra36SeqSeries/type2.pg
All sequences for this problem are geometric. Give all answers to the nearest thousandth.
If \(a_1 = -42\) and \(r = -10\), then \(a_{16} = \quad \).
If \(b_{12} = 36\) and \(b_{49} = 5\), then \(b_1 = \quad \).
If \(c_{20} = 84\) and \(c_{36} = 97\), then \(S_{12} = \quad \)

60. (1 pt) setAlgebra36SeqSeries/jj1.pg
Find the sum
\[ \sum_{n=10}^{130} 9n = \quad \]

61. (1 pt) setAlgebra36SeqSeries/jj3.pg
Find the indicated sum.
\[ \sum_{n=0}^{13} 4 \left( \frac{10}{3} \right)^n = \quad \]

62. (1 pt) setAlgebra36SeqSeries/p11.pg
Find the infinite sum (if it exists):
\[ \sum_{i=0}^{\infty} -1 \cdot \left( \frac{1}{2} \right)^i \]
If the sum does not exist, type DNE in the answer blank.
sum = \quad \]

63. (1 pt) setAlgebra36SeqSeries/srw10_1_35.pg
For the sequence \(a_n = \frac{6}{n}\),
its fifth partial sum \(S_5 = \quad \).
its \(n\)th partial sum \(S_n = \quad \)

64. (1 pt) setAlgebra36SeqSeries/srw10_1_41.pg
\[ \sum_{k=1}^{3} \frac{1}{k} = \quad \]

65. (1 pt) setAlgebra36SeqSeries/srw10_1_43.pg
\[ \sum_{i=1}^{12} [1 + (-1)^i] = \quad \]

66. (1 pt) setAlgebra36SeqSeries/srw10_1_47.pg
\[ \sum_{k=1}^{18} k^2 = \quad \]

67. (1 pt) setAlgebra36SeqSeries/srw10_1_51.pg
\[ \sum_{n=0}^{28} (-1)^n2n = \quad \]

68. (1 pt) setAlgebra36SeqSeries/srw10_1_59.pg
Wirte the sum using sigma notation:
\[ 1 + 2 + 3 + 4 + \cdots + 86 = \sum_{n=1}^{A} B, \quad \text{where} \quad A = \quad \]
\[ B = \quad \]

69. (1 pt) setAlgebra36SeqSeries/srw10_1_63.pg
Wirte the sum using sigma notation:
\[ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \sum_{n=1}^{A} B, \quad \text{where} \quad A = \quad \]
\[ B = \quad \]

70. (1 pt) setAlgebra36SeqSeries/infsum1.pg
Evaluate the following sum. If the sum is not finite, type DOES NOT EXIST as your answer.
\[ \sum_{n=2}^{\infty} 5 \left( \frac{2}{9} \right)^n \]
The sum is \quad \]

71. (1 pt) setAlgebra36SeqSeries/infsum2.pg
Evaluate the following sum. If the sum is not finite, type DOES NOT EXIST as your answer.
\[ \sum_{n=1}^{\infty} 1 - 7n \]
The sum is \quad \]

72. (1 pt) setAlgebra36SeqSeries/sequence2.pg
Evaluate
\[ \sum_{k=1}^{7} (6k - 9)(9k + 9) \]

73. (1 pt) setAlgebra36SeqSeries/sequence3.pg
Insert 5 arithmetic means between 38 and 68.
First mean = \quad \]
Second mean = \quad \]
Third mean = \quad \]
Fourth mean = \quad \]
Fifth mean = \quad \]
Note: Your answers must be in decimal form, given to at least 5 places.

74. (1 pt) setAlgebra36SeqSeries/sequence4.pg
Insert 5 geometric means between 41 and 67.
First mean = \quad \]
Second mean = \quad \]
Third mean = \quad \]
Fourth mean = __________
Fifth mean = __________
Note: Your answers must be in decimal form, given to at least 5 places.

75. (1 pt) setAlgebra36SeqSeries/sum1.pg
Evaluate the following sum:
\[ \sum_{n=5}^{9} (-1)^n (7n^2 + 6n - 1) \]
The sum is ________.

76. (1 pt) setAlgebra36SeqSeries/sum2.pg
Solve the following equation over the real numbers. If no solution exists, type NO SOLUTION as your answer. Give your answers in increasing order to the nearest thousandth.
\[ \sum_{n=3}^{7} (-1)^{n-1} (4nx^2 - 3nx - 9) = 0 \]
x = _______ or x = _______.

77. (1 pt) setAlgebra36SeqSeries/sum3.pg
Evaluate the following sum without writing out all the terms:
\[ \sum_{n=7}^{35} (-8n^2 + 1n - 7) \]
The sum is ________.

78. (1 pt) setAlgebra36SeqSeries/ur_seq_A_9.pg
Write down the first five terms of the following recursively defined sequence.
\[ a_1 = 5; \ a_{n+1} = -2a_n - 5 \]

79. (1 pt) setAlgebra36SeqSeries/recursive1.pg
Suppose \( a_n = 3a_{n-1} + 6a_{n-2} - 4a_{n-3} \) and \( a_4 = 10, a_5 = 74, \) and \( a_6 = 258 \). Find \( a_1, a_2, \) and \( a_3 \).
\[ a_1 = \]
\[ a_2 = \]
\[ a_3 = \]

80. (1 pt) setAlgebra36SeqSeries/faris3.pg
Suppose you go to a company that pays 0.03 for the first day, 0.06 for the second day, 0.12 for the third day and so on. If the daily wage keeps doubling, what will your total income be for working 29 days?

Total Income = ________________

81. (1 pt) setAlgebra36SeqSeries/jj2.pg
Determine the seating capacity of an auditorium with 15 rows of seats if there are 15 seats in the first row, 17 seats in the second row, 19 seats in the third row, 21 seats in the forth row, and so on.

Total number of seats = ________________

82. (1 pt) setAlgebra36SeqSeries/srw10_2_51.pg
The purchase value of an office computer is 12690 dollars. Its annual depreciation is 1915 dollars. The value of the computer after 5 years is ________ dollars.

83. (1 pt) setAlgebra36SeqSeries/srw10_2_53.pg
A man gets a job with a salary of 31300 dollars a year. He is promised a 2400 dollars raise each subsequent year. His total earning for a 6-year period is ________ dollars.

84. (1 pt) setAlgebra36SeqSeries/ur_seq_5_4.pg
Scott and Amanda want to purchase a house. Suppose they invest 600 dollars per month into a mutual fund. How much will they have for a downpayment after 5 years if the per annum rate of return of the mutual fund is assumed to be 9 percent compounded monthly?

85. (1 pt) setAlgebra36SeqSeries/auditorium.pg
An auditorium has 35 rows of seats. The first row contains 60 seats. As you move to the rear of the auditorium, each row has 5 more seats than the previous row.
How many seats are in the 13th row? _______
How many seats are in the auditorium? _______

86. (1 pt) setAlgebra36SeqSeries/auditorium2pg
In a certain auditorium, each row has 7 more seats than the row in front of it. The first 6 rows contain 345 seats. How many rows does the auditorium have if it holds 17160 seats?
The auditorium has ________ rows.

87. (1 pt) setAlgebra36SeqSeries/sequence8.pg
Solve for \( x \):
\[ \sum_{n=1}^{\infty} 4x^{n-1} = 10 \]
\( x = \) __________

88. (1 pt) setAlgebra36SeqSeries/sequence9.pg
The hypotenuse of an isosceles right triangle is 9 inches. The midpoints of its sides are connected to form an inscribed triangle, and this process is repeated. Find the sum of the areas of these triangles if this process is continued infinitely.

\( S = \) __________

89. (1 pt) setAlgebra36SeqSeries/sequence10.pg
Solve for \( x \):
\[ \sum_{n=1}^{\infty} 6x^{5n} = 10 \]
\( x = \) __________

90. (1 pt) setAlgebra36SeqSeries/sequence11.pg
Chucky takes his first step on January 1, 2000. Every day after that, he takes 18 more steps than the day before. Tommy takes his first steps on February 1, 2000. On that day, Tommy takes 11 steps. Every day after that, Tommy takes twice as many steps as the day before.

Who walks farther on Valentine’s Day? _______
Who walks farther on Groundhog Day? _______
What is the last day in February that Chucky walks farther than Tommy? __________

Note: Your answer to parts one and two should be names. Your answer to part three should be the last day in February that Chucky takes more steps than Tommy.

91. (1 pt) setAlgebra36SeqSeries/sequence12.pg
A Super Happy Fun Ball is dropped from a height of 12 feet and rebounds \(6/7\) of the distance from which it fell. How many times will it bounce before its rebound is less than 1 foot?

It will bounce __________ times before its rebound is less than 1 foot.

92. (1 pt) setAlgebra36SeqSeries/superball.pg
A superball that rebounds \(2/5\) of the height from which it fell on each bounce is dropped from 46 meters.

How high does it rebound, in meters, on the 7th bounce? _____
How far does it travel, in meters, before coming to rest? _____

How far will the ball travel before it comes to rest on the ground?

It will travel __________ feet before it comes to rest on the ground.
1. (1 pt) setAlgebra37Binomial/binomial.pg
   Expand the expression using the Binomial Theorem:
   \[(2x + 1)^5 = \ldots x^5 + \ldots x^4 + \ldots x^3 + \ldots x^2 + \ldots x + \ldots\]

2. (1 pt) setAlgebra37Binomial/findcoeff.pg
   Find the coefficient of \(x^{16}y^9\) in the expansion of \((-5x^4 + y^3)^7\).
   The coefficient of \(x^{16}y^9\) is \ldots

3. (1 pt) setAlgebra37Binomial/findcoeff1.pg
   Find the coefficient of \(x^5y^8\) in the expansion of \((-4x - 3y)^{13}\).
   The coefficient of \(x^5y^8\) is \ldots

4. (1 pt) setAlgebra37Binomial/findcnst.pg
   Find the constant term in the expansion of \((3x^3 + \frac{-4}{x})^20\).
   The constant term is \ldots

5. (1 pt) setAlgebra37Binomial/whatterm.pg
   Which term of the expansion of \((x^6 + 3)^{34}\) contains \(x^{42}\).
   Term number \ldots contains \(x^{42}\).

6. (1 pt) setAlgebra37Binomial/whatterm1.pg
   Which term of the expansion of \((x + 3)^6\) contains \(x^4\)
   Term number \ldots contains \(x^4\).
A vendor sells ice cream from a cart on the boardwalk. He offers vanilla, chocolate, strawberry, blueberry, and pistachio ice cream, served on either a waffle, sugar, or plain cone. How many different single-scoop ice-cream cones can you buy from this vendor?

Your answer is: 

How many three-letter “words” can be made from 6 letters “FGHIJK” if repetition of letters

(a) is allowed?
Your answer is: 

(b) is not allowed?
Your answer is: 

How many different ways can a race with 7 runners be completed? (Assume there is no tie.)

Your answer is: 

5 different color dice are rolled, and the numbers showing are recorded. How many different outcomes are possible?

Your answer is: 

A girl has 4 skirts, 6 blouses, and 10 pairs of shoes. How many different skirt-blouse-shoe outfits can she wear? (Assume that each item matches all the others, so she is willing to wear any combination.)

Your answer is: 

A company has 800 employees. Each employee is to be given an ID number that consists of one letter followed by two digits. Is it possible to give each employee a different ID number using this scheme?

Your answer is (input Yes or No): 

You have only one chance to input your answer.

Standard automobile license plates in a country display 2 numbers, followed by 3 letters, followed by 2 numbers. How many different standard plates are possible in this system? (Assume repetitions of letters and numbers are allowed.)

Your answer is: 

A true-false test contains 8 questions. In how many different ways can this test be completed. (Assume we don’t care about our scores.)

Your answer is: 

5-letter “words” are formed using the letters A, B, C, D, E, F, G. How many such words are possible for each of the following conditions?

(a) No condition is imposed.
Your answer is: 

(b) No letter can be repeated in a word.
Your answer is: 

(c) Each word must begin with the letter A.
Your answer is: 

(d) The letter C must be at the end.
Your answer is: 

(e) The second letter must be a vowel.
Your answer is: 

Evaluate the expression \( P(18, 3) \).
Your answer is: 

Evaluate the expression \( P(170, 1) \).
Your answer is: 

How many three-letter “words” can be made from 7 letters “FGHIJKL”? (Letters may not be repeated.)

Your answer is: 

A pianist plans to play 5 pieces at a recital. In how many ways can she arrange these pieces in the program?

Your answer is: 

In how many ways can first, second, and third prizes be awarded in a contest with 850 contestants?

Your answer is: 

In how many ways can 5 students be seated in a row of 5 chairs if Jack insists on sitting in the first chair?

Your answer is: 

Find the number of distinguishable permutations of the given letters “AABCD”.

Your answer is: 

Find the number of distinguishable permutations of the given letters “AAABBBC”.

Your answer is: 

Evaluate the expression \( C(16, 3) \).
Your answer is: 

Evaluate the expression \( C(130, 1) \).
Your answer is: 

20. (1 pt) setAlgebra38Counting/sw10_2_39.pg
In how many ways can 2 books be chosen from a group of nine?
Your answer is: __________

21. (1 pt) setAlgebra38Counting/sw10_2_40.pg
In how many ways can 2 pizza toppings be chosen from 12 available toppings?
Your answer is: __________

22. (1 pt) setAlgebra38Counting/sw10_2_43.pg
How many different 5 card hands can be dealt from a deck of 52 cards?
Your answer is: __________

23. (1 pt) setAlgebra38Counting/sw10_2_46.pg
A pizza parlor offers a choice of 16 different toppings. How many 4-topping pizzas are possible?
Your answer is: __________

24. (1 pt) setAlgebra38Counting/sw10_2_49.pg
In how many ways can 3 students from a class of 19 be chosen for a field trip?
Your answer is: __________

25. (1 pt) setAlgebra38Counting/sw10_2_52.pg
In the 6/52 lottery game, a player picks six numbers from 1 to 52. How many different choices does the player have?
Your answer is: __________

26. (1 pt) setAlgebra38Counting/sw10_2_60.pg
A school dance committee is to consist of 2 freshmen, 3 sophomores, 4 juniors, and 5 seniors. If 6 freshmen, 8 sophomores, 7 juniors, and 9 seniors are eligible to be on the committee, in how many ways can the committee be chosen?
Your answer is: __________

27. (1 pt) setAlgebra38Counting/baseball.pg
There are nine different positions on a baseball team. If a team has 17 players how many different line-ups can the team make?
The team can make _____ different line-ups.
Baseball games consist of nine innings. A team wants to change its line-up every inning. If no game goes to extra innings, and a season consists of 153 games, how many complete seasons can the team play without repeating a line-up?
The team can play _____ complete seasons without repeating a line-up.

28. (1 pt) setAlgebra38Counting/outfits.pg
A boy owns 8 pairs of pants, 5 shirts, 7 ties, and 4 jackets. How many different outfits can he wear to school if he must wear one of each item?
He can wear ________ different outfits.

29. (1 pt) setAlgebra38Counting/wonka.pg
Willie Wonka gives everyone who visits his factory 5 pieces of candy to take home. He never gives a person 2 or more pieces of the same type of candy. If Mr. Wonka has 26 different types of candy, in how many different ways could Mr. Wonka give a visitor his candy?
Mr. Wonka can distribute candy in ________ different ways.
If 155 people visit Mr. Wonka’s factory each day, how many days could Mr. Wonka go without giving two visitors the same selection of candy?
Mr. Wonka can go for ________ days without repeating candy selections.
1. A die is rolled. Find the probability of the given event.
   (a) The number showing is a 3; The probability is: 
   (b) The number showing is an even number; The probability is: 
   (c) The number showing is greater than 1; The probability is: 

2. A card is drawn randomly from a standard 52-card deck. Find the probability of the given event.
   (a) The card drawn is 4; The probability is: 
   (b) The card drawn is a face card; The probability is: 
   (c) The card drawn is not a face card. The probability is: 

3. A ball is drawn randomly from a jar that contains 7 red balls, 2 white balls, and 9 yellow ball. Find the probability of the given event.
   (a) A red ball is drawn; The probability is: 
   (b) A white ball is drawn; The probability is: 
   (c) A yellow ball is drawn; The probability is: 

4. A poker hand, consisting of 2 cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains 2 hearts. Your answer is: 

5. A poker hand, consisting of 8 cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains 8 cards of the same suit. Your answer is: 

6. A poker hand, consisting of 2 cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains 2 face cards. Your answer is: 

7. A poker hand, consisting of 5 cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains an ace, king, queen, jack, and 10 of the same suit (royal flush). Your answer is: 

8. An American roulette wheel has 38 slots: two slots are numbered 0 and 00, and the remaining slots are numbered from 1 to 36. Find the probability that the ball lands in an odd-numbered slot. Your answer is: 

9. In the 6/49 lottery game, a player selects 6 numbers from 1 to 49. What is the probability of picking the 6 winning numbers? Your answer is: 

10. A jar contains 6 red marbles numbered 1 to 6 and 8 blue marbles numbered 1 to 8. A marble is drawn at random from the jar. Find the probability of the given event.
   (a) The marble is red; Your answer is: 
   (b) The marble is odd-numbered; Your answer is: 
   (c) The marble is red or odd-numbered; Your answer is: 
   (d) The marble is blue or even-numbered; Your answer is: 

11. A coin is tossed twice. Let $E$ be the event “the first toss shows heads” and $F$ the event “the second toss shows heads”.
   (a) Are the events $E$ and $F$ independent? Input Yes or No here: 
   (b) Find the probability of showing heads on both toss. Input your answer here: 

12. A die is rolled twice. What is the probability of showing a 3 on both rolls? Your answer is: 

13. A die is rolled twice. What is the probability of showing a(n) 1 on the first roll and an even number on the second roll? Your answer is: 

14. A baseball player has a batting average of 0.16. What is the probability that he has exactly 1 hits in his next 7 at bats? The probability is: 

15. 3 cards are drawn at random from a standard deck. Find the probability that all the cards are hearts. Find the probability that all the cards are face cards. Note: Face cards are kings, queens, and jacks. Find the probability that all the cards are even. (Consider aces to be 1, jacks to be 11, queens to be 12, and kings to be 13)
16. A poker hand consisting of 8 cards is dealt from a standard deck of 52 cards. Find the probability that the hand contains exactly 7 face cards.

The probability is ________

17. An algebra class has 8 students and 8 desks. For the sake of variety, students change the seating arrangement each day. How many days must pass before the class must repeat a seating arrangement?

______ days must pass before a seating arrangement is repeated.

Suppose the desks are arranged in rows of 4. How many seating arrangements are there that put Larry, Moe, Curly, and Shemp in the front seats?

There are ______ seating arrangements that put them in the front seats.

What is the probability that Larry, Moe, Curly and Shemp are sitting in the front seats?

The probability is ________

18. You flip a fair coin 10 times. What is the probability that it lands on heads exactly 5 times?

The probability of exactly 5 heads is ________

What is the probability that it lands on heads at least 5 times?

The probability of at least 5 heads is ________.

19. An algebra class has 20 students and 20 desks. For the sake of variety, students change the seating arrangement each day. How many days must pass before the class must repeat a seating arrangement?

______ days must pass before a seating arrangement is repeated.

Suppose the desks are arranged in rows of 4. How many seating arrangements are there that put Larry, Moe, Curly, and Shemp in the front seats?

There are ______ seating arrangements that put them in the front seats.

What is the probability that Larry, Moe, Curly and Shemp are sitting in the front seats?

The probability is ________

20. The letters in the word MATHEMATICS are arranged randomly.

What is the probability that the first letter is E? ______

What is the probability that the first letter is M? ______

21. A bag contains 8 red marbles, 6 white marbles, and 6 blue marbles. You draw 5 marbles out at random, without replacement.

What is the probability that all the marbles are red?

The probability that all the marbles are red is ______

What is the probability that exactly two of the marbles are red?

The probability that exactly two of the marbles are red is ______

What is the probability that none of the marbles are red?

The probability of picking no red marbles is ______

22. You own 18 CDs. You want to randomly arrange 6 of them in a CD rack. What is the probability that the rack ends up in alphabetical order?

The probability that the CDs are in alphabetical order is ______

23. Find the number of distinguishable permutations of the given letters “AAABBCD”.

There are: ______ permutations.

If a permutation is chosen at random, what is the probability that it begins with at least 2 A's?

The probability is ______

24. Suppose a number is chosen at random from the set 0,1,2,3,...,195.

What is the probability that the number is a perfect cube?

The probability of choosing a perfect cube is ______

Note: Your answer must be a fraction or a decimal number.

25. What is the probability that if 3 letters are typed, no letters are repeated?

The probability that no letters are repeated is ______
1. (1 pt) setAlgebra40SolveForVariables/Fraction.pg
Solve for \( a \):
\[
\frac{a - 8b}{a + 8b} = 3k - 1
\]
\( a = \)\

2. (1 pt) setAlgebra40SolveForVariables/circle.pg
Solve for \( a \):
\[
(x + 8a)^2 + (y + 6b)^2 = 25
\]
There are two solutions, \( a_1 \) and \( a_2 \), where \( a_1 \leq a_2 \).
\( a_1 = \) \( a_2 = \)

3. (1 pt) setAlgebra40SolveForVariables/perfect_square.pg
Solve for \( a \):
\[
(3r^2 + 3a^2) = 68
\]
There are two solutions, \( a_1 \) and \( a_2 \), where \( a_1 \leq a_2 \).
\( a_1 = \) \( a_2 = \)

4. (1 pt) setAlgebra40SolveForVariables/surface_area.pg
Solve for \( a \):
\[
S = 5r^2 + 5r \sqrt{10r^2 + 4a^2}
\]
There are two solutions, \( a_1 \) and \( a_2 \), where \( a_1 \leq a_2 \).
\( a_1 = \) \( a_2 = \)