30 Polynomials with Real Zeros

Due: 12/14/2015 at 06:00am EST.

Students will be able to:
- Use Rational Zero Theorem to find zeros of polynomials
- Determine the maximum possible number of positive real zeros and negative real zeros of a polynomial
- Factor a polynomial using Factor Theorem
- Use Synthetic Division to divide polynomials

Functions and symbols that WeBWorK understands.

Links to some useful WeBWorK pages for students

1. (1 pt) Find all rational zeros of the polynomial

\[ P(x) = 4x^4 - 12x^3 - 12x^2 - 12x - 16 \]

Its rational zeros are \( x_1 = \), \( x_2 = \), \( x_3 = \) and \( x_4 = \) with \( x_1 \leq x_2 \leq x_3 \leq x_4 \)

If the polynomial has only three rational zeros, input them at \( x_1 \), \( x_2 \) and \( x_3 \); If the polynomial has only two rational zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one rational zero, input it at \( x_1 \).

2. (1 pt) Find all real zeros of the polynomial

\[ P(x) = x^3 - 4x^2 - 13x + 6 \]

Its real zeros are \( x_1 = \), \( x_2 = \) and \( x_3 = \) with \( x_1 \leq x_2 \leq x_3 \)

If the polynomial has only two real zeros, input them at \( x_1 \) and \( x_2 \); if the polynomial has only one real zero, input it at \( x_1 \).

3. (1 pt) For the function \( y = x^5 + 5x^3 - 24x \), find all real zeros.

Note: If there is more than one real zero, separate the answers by commas. Also, if you want to enter the square root of a number, like two, enter \( \text{sqrt}(2) \).

The real zeros are \( x = \)

4. (1 pt) List all possible rational roots for the function

\[ f(x) = 5x^3 - 4x^2 - 5x^2 + 9x + 55 \]

Give your list in increasing order. Beside each possible rational root, type "yes" if it is a root and "no" if it is not a root. Leave any unnecessary answer blanks empty.

Possible rational root: Is it a root? Yes.
Possible rational root: Is it a root? Yes.
Possible rational root: Is it a root? Yes.

5. (1 pt) Give all of the zeros of the polynomial

\[ P(x) = x^3 - 3x^2 - 7x - 15 \]

as a comma separated list.

6. (1 pt) \( f(x) = x^6 + 11x^5 + 20x^4 - 566x^3 - 5104x^2 - 13200x^3 + 9216x^2 + 61152x + 43264 \)

What is the maximum number of positive real roots for \( f(x) \)?

7. (1 pt) Find all rational zeros of the polynomial

\[ P(x) = 4x^4 + 9x^3 - 5x^2 + 9x - 9 \]

Give a comma separated list of the rational zeros. If there are no rational zeros, enter the word none.

8. (1 pt) Factor \( P(x) = x^3 + 5x^2 + 8x + 16 \) into linear and irreducible quadratic factors with real coefficients.

Let \( P(x) = (x + a)(x^2 + bx + c) \). Then

\[ a = \]
\[ b = \]
\[ c = \]

9. (1 pt) Use the Factor Theorem to show that \( x - 1/2 \) is a factor of

\[ P(x) = 2x^3 - 9x^2 + 8x - 2 \]

The function value \( P(1/2) = \)

Thus, \( x - 1/2 \) is a factor of \( P(x) \).
10. (1 pt) Use synthetic division and the Remainder Theorem to evaluate $P(c)$, where

$$P(x) = x^4 + 7x^3 + 3x^2 + 25x + 35, \quad c = -7.$$ 

The quotient is _________
The remainder is _________

$$P(c) = _________$$

11. (1 pt) Use the Factor Theorem to show that $x - 1$ is a factor of

$$P(x) = x^3 - 7x^2 + 14x - 8.$$ 

The function value $P(1) = _____$
Thus, $x - 1$ is a _________ of $P(x)$.

12. (1 pt) Use synthetic division and the Remainder Theorem to evaluate $P(c)$, where

$$P(x) = x^2 + 2x + 2, \quad c = -1.$$ 

The quotient is _________
The remainder is _________

$$P(c) = _________$$

13. (1 pt) Use synthetic division and the Remainder Theorem to evaluate $P(c)$, where

$$P(x) = x^3 - 6x^2 + 9x - 5, \quad c = 2.$$ 

The quotient is _________
The remainder is _________

$$P(c) = _________$$