Problem 4 of the Assignment for web-conference II:

Find all critical points of $g(x) = x + 1/x$.

This compilation of 16 problems from the NPL shows several approaches to stating (and programming) an algorithmic adaptation of that textbook problem. (The 17th problem is a generalization.)

Note: some of these examples have near-duplicates in the NPL — I have indicated that with a bracketed tally after the problem number. (The set definition file Rational.def has commented-out links to them.)

1) solve an equation
2) [2] evaluate derivative, write equation for tangent
3) endpoints of important intervals
4) [4] endpoints, identify incr/decr
5,6,7) intervals, identify local extremes
8) identify concavity for each half-line
9) [4] endpoints, incr/decr, concavity
10) [5] locate and evaluate local extremes
11) deprecated tactic to handle list of values
12) [2] constrained optimization
13) [2] minimize perimeter for fixed area (*includes a solution*)
14-15) [3] poster with margins
16) model for fuel consumption
17) model: $a\,v^2 + b\,v^{-2}$ (*includes a solution*)

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1. (1 pt) Solve the equation $x + \dfrac{16}{x} = 8$. If there is more than one correct answer, enter your answers as a comma separated list. If there are no solutions, enter *NONE.*

$x =$ __________

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2. (1 pt) If $f(x) = 2x + \frac{2}{x}$,
$f'(5) =$ __________
Use this to find the equation of the tangent line to the curve $y = 2x + \frac{2}{x}$ at the point $(5, 10.4)$. Write your answer in the form: $y = mx + b$, where m is the slope and b is the y-intercept.
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3. (1 pt) Consider the function $f(x) = 5x + 7x^{-1}$. For this function there are four important intervals: $(-\infty, A]$, $[A, B)$,(B, C), and $[C, \infty)$ where A, B and C are either critical numbers or points at which $f(x)$ is undefined.
Find A _____
and B _____
and C _____

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4. (1 pt) Consider the function $f(x) = 8x + 8x^{-1}$. For this function there are four important intervals: $(-\infty, A]$, $[A, B)$,(B, C), and $[C, \infty)$ where A, and C are the critical numbers and the function is not defined at B.
Find A _____
and B _____
and C _____
For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).
$(-\infty, A]$: _____
$[A, B)$: _____
$(B, C]$: _____
$[C, \infty)$: _____

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5. (1 pt) Consider the function $f(x) = 4x + 8x^{-1}$. For this function there are three numbers $A < B < C$ which are either critical or not in the domain of the function:
$A =$ _____
$B =$ _____
and $C =$ _____
For each of the following intervals, tell whether $f'(x)$ is positive (type in $+$) or negative (type in $-$).
$(-\infty, A)$: _____
(A, B): _____
(B, C): _____
(C, ∞): _____
For each of the numbers A, B, C state whether it is a local maximum (type MAX), minimum (type MIN) or not in the domain of $f(x)$ (type DNE):
A: __________
B: __________
C: __________

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6. (1 pt) Let

$$f(x) = 2x + \frac{6}{x}.$$

(A) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.
Increasing: ________________________
(B) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: ________________________
(C) List the x values of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maximums = ______________
(D) List the x values of all local minima of f. If there are no local minima, enter 'NONE'.
x values of local minimums = ______________

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7. (1 pt) Let $f(x) = 4x + \dfrac{4}{x}$. Find the open intervals on which f is increasing (decreasing). Then determine the x-coordinates of all relative maxima (minima).

1. f is increasing on the intervals ____
2. f is decreasing on the intervals ____
3. The relative maxima of f occur at $x =$ ____
4. The relative minima of f occur at $x =$ ____

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".

In the last two, your answer should be a comma separated list of x values or the word "none".

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8. (1 pt) Consider the function $f(x) = 7x + 5x^{-1}$.
Note that this function has no inflection points, but $f''(x)$ is undefined at $x = B$ where
$B =$ ____
For each of the following intervals, tell whether $f(x)$ is concave up (type in CU) or concave down (type in CD).
$(-\infty, B)$: ____
(B, ∞): ____

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9. (1 pt) Consider the function $f(x) = 5x + 7x^{-1}$. For this function there are four important intervals: $(-\infty, A]$, $[A, B)$ $(B, C]$, and $[C, \infty)$ where A, and C are the critical numbers and the function is not defined at B.

Find A ____

and B ____
and C ____

For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).
$(-\infty, A]$: ____
$[A, B)$: ____
$(B, C]$: ____
$[C, \infty)$ ____

Note that this function has no inflection points, but we can still consider its concavity. For each of the following intervals, tell whether $f(x)$ is concave up (type in CU) or concave down (type in CD).
$(-\infty, B)$: ____
(B, ∞): ____

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10. (1 pt) The function $f(x) = 7x + 7x^{-1}$ has one local minimum and one local maximum.
This function has a local maximum at $x =$ ____ with value ____
and a local minimum at $x =$ ____ with value ____

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11. (1 pt) Let

$$f(x) = 6x + \frac{4}{x}.$$

Use either the first derivative test or the second derivative test to find the following:

(A) The average of the x values of all local maxima of f.
Note: If there are no local maxima, enter -1000.
Average of x values = ____

(B) The average of the x values of all local minima of f.
Note: If there are no local minima, enter -1000.
Average of x values = ____

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12. (1 pt) Find the x-coordinate of the absolute maximum and absolute minimum for the function

$$f(x) = 9x + \frac{4}{x}, \qquad x > 0.$$

Enter *None* for any absolute extrema that does not exist.

x-coordinate of absolute maximum = ____

x-coordinate of absolute minimum = ____

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13. (1 pt) A rancher wants to fence in an area of 2000000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use? ____

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14. (1 pt) A printed poster is to have a total area of 542 square inches with top and bottom margins of 3 inches and side margins of 2 inches. What should be the dimensions of the poster so that the printed area be as large as possible?

To solve this problem let x denote the width of the poster and let y denote the length. We need to maximize the following function of x and y:

____

We can reexpress this as the following function of x alone:
$f(x) =$ ____
We find that $f(x)$ has a critical number at $x =$ ____
To verify that $f(x)$ has a maximum at this critical number we compute the second derivative $f''(x)$ and find that its value at the critical number is ____, a negative number.
Thus the optimal dimensions of the poster are ____ inches in width and ____ inches in height giving us a maximumal printed area of ____ square inches.

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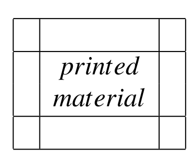
15. (1 pt) The top and bottom margins of a poster are 4 cm and the side margins are each 5 cm. If the area of printed material on the poster is fixed at 386 square centimeters, find the dimensions of the poster with the smallest area.



Width = ____ (include **units**)
Height = ____ (include **units**)

16. (1 pt) A truck has a minimum speed of 10 mph in high gear. When traveling x mph, the truck burns diesel fuel at the rate of

$$0.0046966\left(\frac{961}{x}+x\right)\frac{\text{gal}}{\text{mile}}$$

Assuming that the truck can not be driven over 63 mph and that diesel fuel costs \$1.96 a gallon, find the following.

(a) The steady speed that will minimize the cost of the fuel for a 620 mile trip.

$x =$ ______

(b) The steady speed that will minimize the total cost of the trip if the driver is paid \$19 an hour.

$x =$ ______

(c) The steady speed that will minimize the total cost of a 510 mile trip if the driver is paid \$32 an hour.

$x =$ ______

17. (1 pt) It takes a certain power P to keep a plane moving along at a speed v. The power needs to overcome air drag which increases as the speed increases, and it needs to keep the plane in the air which gets harder as the speed decreases. So assume the power required is given by

$$P = cv^2 + \frac{d}{v^2}$$

where c and d are positive constants. (They depend on your plane, your altitude, and the weather, among other things.) Enter here ______ the choice of v that will minimize the power required to keep moving at speed v.

Suppose you have a certain amount of fuel and the fuel flow required to deliver a certain power is proportional to to that power. What is the speed v that will maximize your range (i.e., the distance you can travel at that speed before your fuel runs out)? Enter your speed here ______

Finally, enter here ______ the ratio of the speed that maximizes the distance and the speed that minimizes the required power.