

## hw-04a-review-of-complex-numbers

Due: 12/12/2015 at 06:00am EST.

Students will be able to:

- Add and Subtract Complex Numbers
- Multiply and Divide Complex Numbers

**Functions and symbols that WeBWorK understands.**

**Links to some useful WeBWorK pages for students**

1. (1 pt) Evaluate the expression  $(9 + 4i) + (7 + 6i)$  and write the result in the form  $a + bi$ .

The sum is \_\_\_\_\_.

2. (1 pt) Evaluate the expression  $(-5 + 6i) + (-4 + 6i)$  and write the result in the form  $a + bi$ .

The real number  $a$  equals \_\_\_\_\_

The real number  $b$  equals \_\_\_\_\_

3. (1 pt) Evaluate the expression  $(-6 - 4i) - (-8 - 4i)$  and write the result in the form  $a + bi$ .

The real number  $a$  equals \_\_\_\_\_

The real number  $b$  equals \_\_\_\_\_

4. (1 pt) Evaluate the expression  $(-6 + 2i) - (6 + 7i)$  and write the result in the form  $a + bi$ .

The difference is \_\_\_\_\_.

5. (1 pt) Evaluate the expression  $(9 - 7i)(-8 - 5i)$  and write the result in the form  $a + bi$ .

The product is \_\_\_\_\_.

6. (1 pt) Write the following numbers in  $a + bi$  form:

(a)  $(3 + i)(5 + 5i)(-3 + 3i) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$ ,

(b)  $((-4 - 4i)^2 - 3)i = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$ .

7. (1 pt) Complete the following equations. Your answers will be algebraic expressions.

$$(a + bi)(a - bi) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$$

$$(a + bi)^2 - (a - bi)^2 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$$

Hint: You can use some of the results in the preceding problem and the discussion of **complex numbers**.

8. (1 pt) Complete the following equation. Your answers will be algebraic expressions.

$$(a + bi)^3 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$$

Hint: Think of  $i$  as an ordinary variable and then replace  $i^2$  with  $-1$ .

9. (1 pt) For some practice working with complex numbers:

Calculate

$$(4 - 6i) + (2 - 5i) = \underline{\hspace{1cm}},$$

$$(4 - 6i) - (2 - 5i) = \underline{\hspace{1cm}},$$

$$(4 - 6i)(2 - 5i) = \underline{\hspace{1cm}}.$$

The complex conjugate of  $(1 + i)$  is  $(1 - i)$ . In general to obtain the complex conjugate reverse the sign of the imaginary part. (Geometrically this corresponds to finding the "mirror image" point in the complex plane by reflecting through the  $x$ -axis. The complex conjugate of a complex number  $z$  is written with a bar over it:  $\bar{z}$  and read as "z bar".

Notice that if  $z = a + ib$ , then

$$(z)(\bar{z}) = |z|^2 = a^2 + b^2$$

which is also the square of the distance of the point  $z$  from the origin. (Plot  $z$  as a point in the "complex" plane in order to see this.)

If  $z = 4 - 6i$  then  $\bar{z} = \underline{\hspace{1cm}}$  and  $|z| = \underline{\hspace{1cm}}$ .

You can use this to simplify complex fractions. Multiply the numerator and denominator by the complex conjugate of the denominator to make the denominator real.

$$\frac{4 - 6i}{2 - 5i} = \underline{\hspace{1cm}} + i \underline{\hspace{1cm}}.$$

Two convenient functions to know about pick out the real and imaginary parts of a complex number.

$Re(a + ib) = a$  (the real part (coordinate) of the complex number), and

$Im(a + ib) = b$  (the imaginary part (coordinate) of the complex number.  $Re$  and  $Im$  are linear functions – now that you know about linear behavior you may start noticing it often.

10. (1 pt) Evaluate the expression

$$\frac{8 - 7i}{-8 - 6i}$$

and write the result in the form  $a + bi$ .

The quotient is \_\_\_\_\_.

11. (1 pt) Write the following numbers in  $a + bi$  form:

(a)  $\left(\frac{2 + i}{5i - (-5 - 2i)}\right)^2 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$ ,

(b)  $(i)^2(-5 + i)^2 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} i$ .

12. (1 pt) Calculate the following:

(a)  $i^3 = \underline{\hspace{1cm}}$ ,

(b)  $i^4 = \underline{\hspace{1cm}}$ ,

(c)  $i^5 = \underline{\hspace{1cm}}$ ,

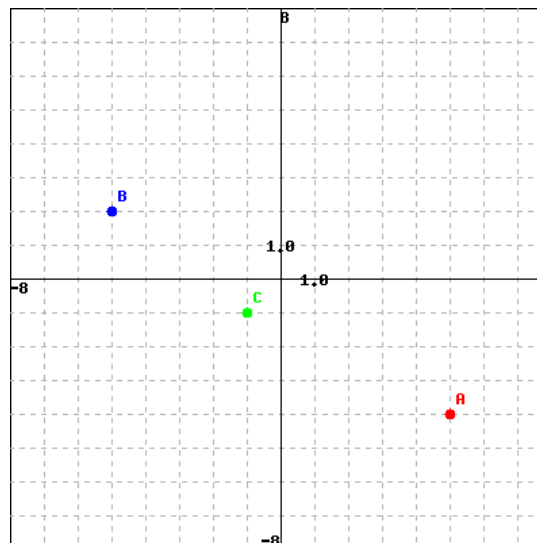
(d)  $i^6 = \underline{\hspace{1cm}}$ ,

(e)  $i^{59} = \underline{\hspace{1cm}}$ ,

- (f)  $i^0 = \underline{\hspace{1cm}}$ ,
- (g)  $i^{-1} = \underline{\hspace{1cm}}$ ,
- (h)  $i^{-2} = \underline{\hspace{1cm}}$ ,
- (i)  $i^{-3} = \underline{\hspace{1cm}}$ ,
- (j)  $i^{-89} = \underline{\hspace{1cm}}$ .

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**13.** (1 pt) Enter the complex coordinates of the following points:



- A:       ,
- B:       ,
- C:       .