28 Graphs of Rational Functions.

Due:
12/14/2015 at 06:00am EST.

Students will be able to:

- Identify domain of rational functions
- Identify vertical asymptotes of rational functions
- Identify horizontal or slanted asymptotes of rational functions
- Identify any holes a graph of rational function might have
- Find the x-intercepts and y-intercepts of a graph of rational function
- Identify the graph of rational function
- Produce a possible formula for a rational function based on the given graph

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Functions and symbols that WeBWorK understands.

Links to some useful WeBWorK pages for students

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1. (1 pt) This is a warmup for the next problem.

For the following functions, use "x" to indicate that the x-axis is an asymptote, "h" to indicate a horizontal asymptote other than the x axis, "v" to indicate a vertical asymptote, "s" to indicate a slanted asymptote, and "n" the lack of an asymptote. If the graph of a function has several types of asymptotes indicate them all in alphabetical order.

For example, the function

$$f(x) = \frac{x^3}{x^2-1}$$

has a slanted asymptote since the degree of the numerator is one more than the degree of the denominator, and it also has two vertical asymptotes (at $x = \pm 1$). So you would enter "sv" (without the double quotation marks. The graph of

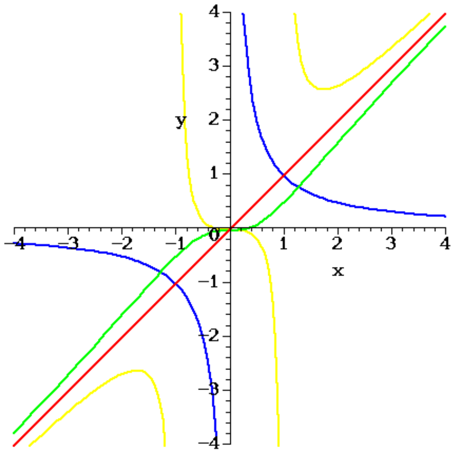
$$f(x) = \frac{1}{x}$$

has vertical asymptote (the y-axis) and the x-axis as an asymptote, so you would enter "vx". On the other hand, the graph of

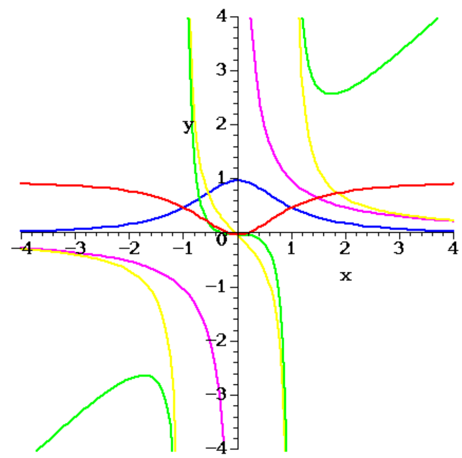
$$f(x) = \frac{x^3}{x^2+1}$$

has only a slanted asymptote, so you would enter just "s".

___ $f(x) = \frac{1}{x}$.
___ $f(x) = \frac{x^3}{x^2-1}$.
___ $f(x) = \frac{x^3}{x^2+1}$.
To make this clear the following picture shows the graphs involved:



It may not be clear from the picture that the green graph (of $f(x) = \frac{x^3}{x^2+1}$ has a slanted asymptote, to make this clearer the Figure also contains the (red) graph of its asymptote defined by the equation $y = x$. The yellow graph is the graph of $f(x) = \frac{x^3}{x^2-1}$, the blue graph is the graph of $f(x) = \frac{1}{x}$.



2. (1 pt)

Match the graphs shown above with the functions listed below. Enter "r" for red, "g" for green, "p" for purple, "b" for blue, and "y" for yellow.

___: $f(x) = \frac{1}{x}$.
___: $f(x) = \frac{1}{x^2+1}$.

___: $f(x) = \frac{x}{x^2-1}$.
___: $f(x) = \frac{x^2}{x^2+1}$.
___: $f(x) = \frac{x^3}{x^2-1}$.

Hint: Again, look for various kinds of asymptotes.

3. (1 pt) For the following functions, use "x" to indicate that the x-axis is an asymptote, "h" to indicate a horizontal asymptote other than the x axis, "v" to indicate a vertical asymptote, "s" to indicate a slanted asymptote, and "n" the lack of an asymptote. If the graph of a function has several types of asymptotes indicate them all in alphabetical order.

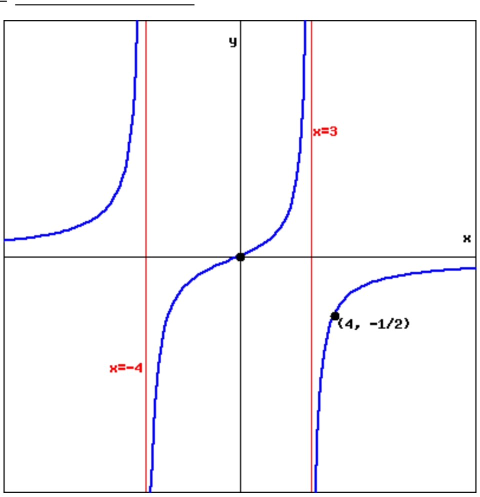
___ $f(x) = \frac{x}{x-1}$.
___ $f(x) = \frac{x}{x^2-1}$.
___ $f(x) = \frac{x-1}{x^2+1}$.
___ $f(x) = \frac{x^4}{x^2+1}$.

4. (1 pt) Let t be the time in weeks. At time $t = 0$, organic waste is dumped into a pond. The oxygen level in the pond at time t is given by

$$f(t) = \frac{t^2 - t + 1}{t^2 + 1}.$$

Assume $f(0) = 1$ is the normal level of oxygen.
(a) On a separate piece of paper, graph this function.
(b) What will happen to the oxygen level in the lake as time goes on?
?
(c) Approximately how many weeks must pass before the oxygen level returns to 75% of its normal level?
_______ weeks (Round to at least two decimal places.)

5. (1 pt) Find a possible formula for the function graphed below. Assume the function has only one x-intercept at the origin, and the point marked on the graph below is located at $\left(4, \frac{-1}{2}\right)$. The asymptotes are $x = -4$ and $x = 3$. Give your formula as a reduced rational function.
$f(x) =$ _______________



(Click on graph to enlarge)

6. (1 pt) **More Graphing.** Know how to graph rational functions and to compute asymptotes and intercepts. For example the graph of

$$f(x) = \frac{2x-1}{x-1}$$

has a horizontal asymptote $y =$ _____ and a vertical asymptote $x =$ _____.
Its y intercept is $y =$ _____ and its x intercept is $x =$ _____. You should also draw the graph.

7. (1 pt) Perhaps the most central concept in all of mathematics is that of a **function**. You need to understand the concepts of **rule**, **domain**, and **range**, and what it means to **evaluate** a function at a number or an **algebraic expression** that may itself be defined by a function.

For this and the next two problems let

$$f(x) = \frac{x+1}{x^2 - 5x + 6}.$$

Two numbers **not** in the domain of f are ___ and ___. (Enter the numbers in increasing size.)

8. (1 pt) Find the horizontal asymptote, if it exists, of the rational function below. If the function does not have a horizontal asymptote, enter *NONE.*

$g(x) = \dfrac{(-9-x)(6+8x)}{8x^2+1}$
The horizontal asymptote has equation __________

9. (1 pt) Let $r(x) = \dfrac{p(x)}{q(x)}$, where p and q are polynomials of degrees m and n respectively.
(a) If $r(x) \to 0$ as $x \to \infty$, then

- A. $m = n$
- B. $m < n$
- C. $m > n$
- D. None of the above

(b) If $r(x) \to k$ as $x \to \infty$, with $k \neq 0$, then

- A. $m > n$
- B. $m = n$
- C. $m < n$
- D. None of the above