28 Graphs of Rational Functions.

Due: 12/14/2015 at 06:00am EST.

Students will be able to:

• Identify domain of rational functions
• Identify vertical asymptotes of rational functions
• Identify horizontal or slanted asymptotes of rational functions
• Identify any holes a graph of rational function might have
• Find the x-intercepts and y-intercepts of a graph of rational function
• Identify the graph of rational function
• Produce a possible formula for a rational function based on the given graph

Functions and symbols that WeBWorK understands.

Links to some useful WeBWorK pages for students

1. (1 pt) This is a warmup for the next problem.

For the following functions, use "x" to indicate that the x-axis is an asymptote, "h" to indicate a horizontal asymptote other than the x axis, "v" to indicate a vertical asymptote, "s" to indicate a slanted asymptote, and "n" the lack of an asymptote. If the graph of a function has several types of asymptotes indicate them all in alphabetical order.

For example, the function

\[ f(x) = \frac{x^3}{x^2 - 1} \]

has a slanted asymptote since the degree of the numerator is one more than the degree of the denominator, and it also has two vertical asymptotes (at \( x = \pm 1 \)). So you would enter "sv" (without the double quotation marks). The graph of

\[ f(x) = \frac{1}{x} \]

has vertical asymptote (the y-axis) and the x-axis as an asymptote, so you would enter "vx". On the other hand, the graph of

\[ f(x) = \frac{x^3}{x^2 + 1} \]

has only a slanted asymptote, so you would enter just "s".

\[ f(x) = \frac{1}{x^3} \]
\[ f(x) = \frac{x^3}{x^2 - 1} \]

It may not be clear from the picture that the green graph (of \( f(x) = \frac{x^3}{x^2 + 1} \)) has a slanted asymptote, to make this clearer the Figure also contains the (red) graph of its asymptote defined by the equation \( y = x \). The yellow graph is the graph of \( f(x) = \frac{1}{x^3} \), the blue graph is the graph of \( f(x) = \frac{1}{x^2} \).

2. (1 pt)

Match the graphs shown above with the functions listed below. Enter "r" for red, "g" for green, "p" for purple, "b" for blue, and "y" for yellow.

\[ f(x) = \frac{1}{x} \]
\[ f(x) = \frac{1}{x^{3/2} + 1} \]

\[ f(x) = \frac{1}{x^{3/2} - 1} \]

\[ f(x) = \frac{1}{x^3 + 1} \]
\[ f(x) = \frac{1}{x^2 + 1} \]
3. (1 pt) For the following functions, use "x" to indicate that the x-axis is an asymptote, "h" to indicate a horizontal asymptote other than the x axis, "v" to indicate a vertical asymptote, "s" to indicate a slanted asymptote, and "n" the lack of an asymptote. If the graph of a function has several types of asymptotes indicate them all in alphabetical order.

\[
\begin{align*}
f(x) &= \frac{x}{x^2 - 1}, \\
f(x) &= \frac{x^2}{x^2 + 1}, \\
f(x) &= \frac{x^2 - 1}{x^3}, \\
f(x) &= \frac{x^3}{x^2 + 1}.
\end{align*}
\]

**Hint:** Again, look for various kinds of asymptotes.

4. (1 pt) Let \( t \) be the time in weeks. At time \( t = 0 \), organic waste is dumped into a pond. The oxygen level in the pond at time \( t \) is given by

\[ f(t) = \frac{t^2 - t + 1}{t^2 + 1}. \]

Assume \( f(0) = 1 \) is the normal level of oxygen.

(a) On a separate piece of paper, graph this function.

(b) What will happen to the oxygen level in the lake as time goes on?

(c) Approximately how many weeks must pass before the oxygen level returns to 75% of its normal level?

5. (1 pt) Find a possible formula for the function graphed below. Assume the function has only one x-intercept at the origin, and the point marked on the graph below is located at \((4, -\frac{1}{2})\). The asymptotes are \( x = -4 \) and \( x = 3 \). Give your formula as a reduced rational function.

\[ f(x) = \frac{2}{x^2 - 1}. \]

(Click on graph to enlarge)

6. (1 pt) More Graphing. Know how to graph rational functions and to compute asymptotes and intercepts. For example the graph of

\[ f(x) = \frac{2x - 1}{x - 1} \]

has a horizontal asymptote \( y = ____ \) and a vertical asymptote \( x = ____ \).

Its y intercept is \( y = ____ \) and its x intercept is \( x = ____ \). You should also draw the graph.

7. (1 pt) Perhaps the most central concept in all of mathematics is that of a **function**. You need to understand the concepts of **rule**, **domain**, and **range**, and what it means to **evaluate** a function at a number or an algebraic expression that may itself be defined by a function.

For this and the next two problems let

\[ f(x) = \frac{x + 1}{x^2 - 5x + 6}. \]

Two numbers **not** in the domain of \( f \) are ___ and ___. (Enter the numbers in increasing size.)

8. (1 pt) Find the horizontal asymptote, if it exists, of the rational function below. If the function does not have a horizontal asymptote, enter **NONE**.

\[ g(x) = \frac{(-9 - x)(6 + 8x)}{8x^2 + 1} \]

The horizontal asymptote has equation ________

9. (1 pt) Let \( r(x) = \frac{p(x)}{q(x)} \), where \( p \) and \( q \) are polynomials of degrees \( m \) and \( n \) respectively.

(a) If \( r(x) \to 0 \) as \( x \to \infty \), then

- A. \( m = n \)
- B. \( m < n \)
- C. \( m > n \)
- D. None of the above

(b) If \( r(x) \to k \) as \( x \to \infty \), with \( k \neq 0 \), then

- A. \( m > n \)
- B. \( m = n \)
- C. \( m < n \)
- D. None of the above